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Prediction of magic numbers of heavy and super heavy nuclei from the behavior of α-decay half-lives

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Abstract: *In the framework of the preformed-cluster model, a simple method for calculating α-decay half-lives ($T_{1/2}$) of even-even nuclei in the range $90 \le Z_p \le 122$ and $112 \le N_p \le 190$ is derived using the WKB approximation Then, the neutron number variation of $log(T_{1/2})$ is studied to explore nucleon magic numbers. As a result, the predicted neutron and proton magic numbers are N=126, 162, 178 and 184 and Z=108, 114, 118 and 120, respectively, which are found in consistent with those predicted in other studies.*

Keywords*:α-decay, Super heavy nuclei, Half-life*

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I. Introduction

Alpha(α) particle emission is one of the most important decay channels for unstable heavy and super heavy nuclei [1, 2].This phenomenonwas discovered by Rutherford [3, 4]in 1899.Actually, there are two natural forces governing the process of α-decay within the nucleus. The first one is the short-range, strong, attractive nuclear force which binds the nucleons together within the nucleus. The second one is thelong-range, repulsive Coulomb force between the protons in the alpha particle and those in the daughter nucleus. The balance between these two forces comprises a well-like Coulomb barrier. Forα particle to escape from the nucleus, it has to penetrate this potential barrier.In 1928, the Russian-American physicist Gamow[5], and then, independently, Gurney and Condon[6], proposed the mechanism of the α- decay using the idea of quantum tunneling in which the barrier penetrability was calculated using Wentzel-Kramers-Brillouin (WKB) approximation[7, 8].Later on, many theoretical models such as cluster-model[9-11],Coulomb and proximity potential model[12] and fission-like model[13, 14]were performed to describe the α-radioactivity, to extract a varity of detailed information about the nuclear structure[15-18]and to predict the absolute of α- decay width. In the unified fission model[13, 19], the α- decay width is simply the product of the assault frequency ν(the number of collisions of alpha particle per unit time with the barrier, calculated using the classical method[20]) and the barrier penetrability *P*calculated using the WKB approximation.

According to preformed cluster-model[9-11], α particle is assumed to be preformed in the parent nucleus before penetrating the barrier.As a result, a spectroscopic factor, called preformation factor, S_α , was introducedto describe the preformation probability to find an alpha particle inside the nucleus at the nuclear surface. It can be calculated by dividing the experimental α-decay width by the barrier penetrability. Besides,the decay constant could be defined as the product of the assault frequency, the barrier penetrability, and the α preformation factor. However, the preformed cluster- modelneeds heavy numerical calculations. So, a number of authors[21-30]replaced this model by deriving simple formulae for the α-decay half-lives ($T_{1/2}$). Subsequently, the present paper introducesa simple new model for calculating α-decay half-lives and testsits ability to produce the more complicated calculations based on the density dependent cluster model used frequently in α- decay calculations.

It is known that the possible existence and the location of the stability of heavy and super heavy nuclei play very important rule not only for checking the present theoretical models but also extending the nuclide chart in nuclear physics. In this contribution, the study of the behavior of the calculated $T_{1/2}$ for heavy and super heavy nuclei was used to get information about the stability of nuclei. In 1960_s a number of theoretical calculations[31-36]were performed and indicated to the existence of an island of long-lived super heavy elements at *Z*=126, *N*=184. Using axially deformed relativistic mean field calculations, Patra *et al*[37]predicted the existence of magic numbers at *Z*=120 and *N* =172 or 184. Recently, Ismail *et al*[16]predicted stabilities at *Z*=100, 104, and 108 and *N*=152, 162,178 and 184, owing to the stability of parent nuclei against alpha decayduring the calculations of α-decay half-lives of heavy and super heavy nuclei in the framework of the preformed alpha model. Presently, we will focus on exploring proton and neutron magic numbers for heavy and super heavy nuclei using oursimple model of calculation.

The paper is organized as follows. In Sect.2 we present the formulas and the parameters of the simple model for calculatingthe alpha half-lives and the summary and discussion are in Sect.3.Finally, in Sect.4 we give a brief conclusion.

II. Theoretical Framework

In the present work, a simple method for α-decay half-lives ($T_{1/2}$) of heavy and super heavy nuclei is derived using the WKB approximation for the penetration of the Coulomb barrier with the Woods-Saxon potential for the nuclear part and the Langer modified centrifugal potential for the centrifugal part.Furthermore, a set of parameters of the potentials were obtained by fitting the experimental data for several fusion reactions[38].According to preformed cluster-model[9-11], an alpha particle is assumed to preformed in the parent nucleus of a ground state which is assumed to be an alpha cluster orbiting the daughter nucleus. The α-daughter interaction*V(R)*[16, 39] can be written as;

$$V(R) = V_N(R) + V_C(R) + V_{Cf}(R) \qquad (1)$$

where R is the distance between the centers of alpha particle and the daughter nucleus and the potentials V_N, V_C and V_{Cf} are the nuclear, the Coulomb and the centrifugal potentials, respectively. For the nuclear potential we adopt the Woods-Saxon formwhich is characterized by a depth $V_0(A_d, Z_d, Q_\alpha)$ and diffuseness a_0, thus

$$V_N(R) = \frac{V_0(A_d, Z_d, Q_\alpha)}{1 + Exp(\frac{R - R_m}{a_0})}. \qquad (2)$$

Also the Coulomb potential is adopted to be given by[10]

$$V_C(R) = \begin{cases} \dfrac{2Z_d e^2}{R} & \to \text{ R>R}_m \\ \dfrac{Z_d e^2}{R_m}\left[3 - \dfrac{R^2}{R_m^2}\right] & \to \text{ R} \le \text{R}_m \end{cases} \qquad (3)$$

here the parameters of these potentials were obtained by fitting the experimental data for several fusion reactions[38].

$$R_m = 1.5268 + R_0,$$

$$R_0 = R_p\left(1 + \frac{3.0909}{R_p^2}\right) + 0.12430\left(\frac{A_d - 2Z_d}{A_d} - \frac{0.4A_d}{A_d + 200}\right), \qquad (4)$$

$$R_p = 1.2A_d^{1/3}\left(1 + \frac{1.646}{A_d} - \frac{0.191(A_d - 2Z_d)}{A_d}\right),$$

$$V_0(A_d, Z_d, Q_\alpha) = -\left[30.275 - 0.45838Z_d / A_d^{1/3} + 58.270\frac{(A_d - 2Z_d)}{A_d} - 0.24244Q_\alpha\right], \qquad (5)$$

$$a_0 = 0.49290 \qquad (6)$$

where A_d and Z_d are the mass number and atomic number for the daughter nucleus, respectively, and $Q_\alpha$is the alpha decay energy. For the centrifugal potential we adopt the Langer modified centrifugal potential[40],

$$V_{Cf}(R) = \frac{\hbar^2 (l + 1/2)^2}{2\mu R^2}, \qquad (7)$$

where l is the angular momentum which will be carried by an emitted alpha particle. The value of l is taken to be the minimal value of alpha particle during alpha transition and will be obtained using the following spin-parity selection rule,

$$|J_p - J_d| \le l \le J_p + J_d \quad \text{and} \quad \pi_p = (-1)^l \pi_d$$

where J_p, π_p and J_d, π_d are the spin and parity of the parent and daughter nuclei, respectively. When the decay energy, Q_α, of alpha particle is less than the Coulomb barrier, the penetrability, $P(Q_\alpha)$, can be calculated using the WKB approximation as;

$$P(Q_\alpha) = EXP\left[\frac{-2}{\hbar}\int_{Rin}^{Rout}\sqrt{2\mu(V(R)-Q_\alpha)}dR\right] \qquad (8)$$

where R_{in} and R_{out} are the inner and outer classical turning points of the barrier at which $V(R = R_{in}, R_{out}) = Q_\alpha$ and their values are calculated numerically, $\mu = M_{nuc}\frac{4A_d}{4 + A_d}$ is the reduced mass of α particle and the daughter nucleus. With A_d are the atomic mass number for the daughter nucleus, and M_{nuc} is the nuclear mass unit, $M_{nuc} = 931\ \mathrm{MeV/c^2}$ [41].Then the α-decay half-life ($T_{1/2}$) is expressed as [31, 38]

$$T_{1/2} = \frac{\hbar \ln 2}{\Gamma} \qquad (9)$$

here Γ is the decay width which is related to the penetrability of α-decay by:

$$\Gamma = \hbar \nu S_\alpha P(Q_\alpha) = \hbar \xi P(Q_\alpha) \qquad (10)$$

where ν is the assault frequency of alpha particle at the barrier, S_α is the preformation factorwhich was taken to equal to 1.0 for even-even heavy and super heavy nucleiand $P(Q_\alpha)$ is the penetrability of alpha particle. For spherical nuclei, the factor ξ is parametrized as[38],

$$\xi = \left(6.1814 + 0.2988 A_d^{-1/6}\right)\times 10^{19}. \qquad (11)$$

III. Results

The purpose of this investigation is to explore proton and neutron magic numbers in heavy and super heavy nuclei using the present simple model of calculations given in Sect.2. This is performed from studying the behavior of the decimal logarithm of half-lives $log(T_{1/2})$ for different isotopes of nuclei as a function of neutron number of the daughter nuclei. We consider the even (Z_p)-even (N_p) nuclei with atomic numbers within the range $90 \leq Z_p \leq 122$ and neutron numbers range $112 \leq N_p \leq 190$. The Q_α values of the alpha decay are taken from [42] and the Q_α values missed in [42] are taken from [41, 43]. It is assumed that the stability of the daughter (parent) nuclei is proportional to the depth (height) of the minimum (maximum) value in log $T_{1/2}$.Thus, we will focus on the clear minimum and maximum values of $\log T_{1/2}$ which occur at specific values of neutron number of the daughter nuclei. Indeed, for the given group of isotopes, $T_{1/2}$ reaches a minimum value for the parent isotope with the daughter nucleus having a larger stability or nucleon magic number. This results from the role of the shell effect in α-decay transition. As a consequence, the present study predicted neutron magic numbers N=126, 162, 178 and 184 and proton magic numbers Z=108, 114, 118 and 120.

IV. Discussion

Figure 1 shows the variation of decimal logarithm of α-decay half-lives ($\log T_{1/2}$) for even-even nuclei with $90 \leq Z_p \leq 99$ as a function of neutron number of the daughter nuclei. It can be seen from fig.1 that $\log T_{1/2}$ reaches a minimum value at N_d =126 (well-known magic number) for all Z_p-values corresponding to $N_p = N_d$ +2 (neutron number of the parent nucleus). To illustrate, when the nucleon number of the parent nucleus becomes more than nucleon magic number by more than two nucleons, the parent nucleus tends to be more stable by emitting α-particle. Subsequently, $\log T_{1/2}$ reaches a minimum value.Thus, the closer the daughter nucleon number to a magic number, the smaller the half-life of the parent nucleus.

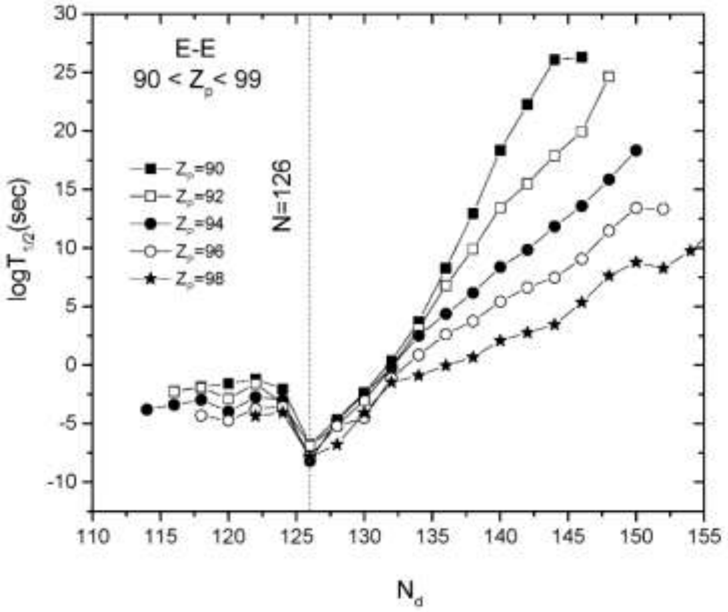

Fig.1 The calculated decimal logarithm of α-decay half-lives ($\log T_{1/2}$) for even-even nuclei within the range $90 \leq Z_P \leq 100$ as a function of neutron number of the daughter nuclei.

Figure 2 shows the same as fig.1 but for the range $100 \leq Z_p \leq 110$. It can be seen from fig. 2 that $\log T_{1/2}$ reaches a minimum value at N_d=152 corresponding to the parent neutron number N_p =154. So, we may conclude that N_d=152 is a daughter-neutron magic number as in Refs.[43-47]. In addition, another minima for $\log T_{1/2}$ is seen in fig. 2 for most Z_p curves at N_d=162 which can be considered as neutron magic number as in Refs. [43, 45-50]. As pointed earlier, the stability of the daughter nuclei is proportional to the depth of the minimum value in log $T_{1/2}$. This means that, the nucleus has a larger stability corresponds to a deeper minimum. Besides, it is obvious from fig. 2 that, the deepest minimum in log $T_{1/2}$ at N_d=152 and 162 corresponds to Z_d=108. Thus, Z_d=108 may be concluded as a proton magic number as in Refs.[43, 46-48].

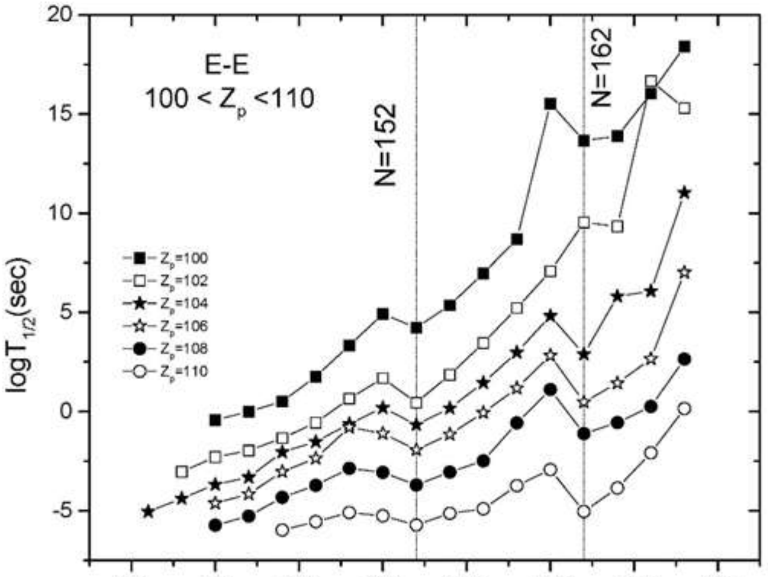

Fig.2 the same as fig. 1 but for the range $100 \leq ZP \leq 110$.

Figure 3 shows the same as fig.1 but for elements within the Z_p-range $112 \leq Z_p \leq 122$. Figure 3 enhances fig. 2 conclusion that N_d =162 is a neutron magic number because there are clear minima in $log T_{1/2}$ at it. Besides, few weak minima in $log T_{1/2}$ at N_d =178 and clear ones at N_d =184 are obtained in fig. 3. As a result, N_d =178 as in Ref. [51] and 184 as in Refs.[50, 52, 53] are neutron magic numbers. Due to the lowest minima at N_d =162, 178 and 184 corresponding to Z_d =114 (Z_p = 116), 118 (Z_p = 120) and 120 (Z_p = 122), respectively, these Z-values may be predicted as proton magic numbers. Besides Z_d =114 and 120 are concluded as proton magic numbers in Refs.[37, 54].

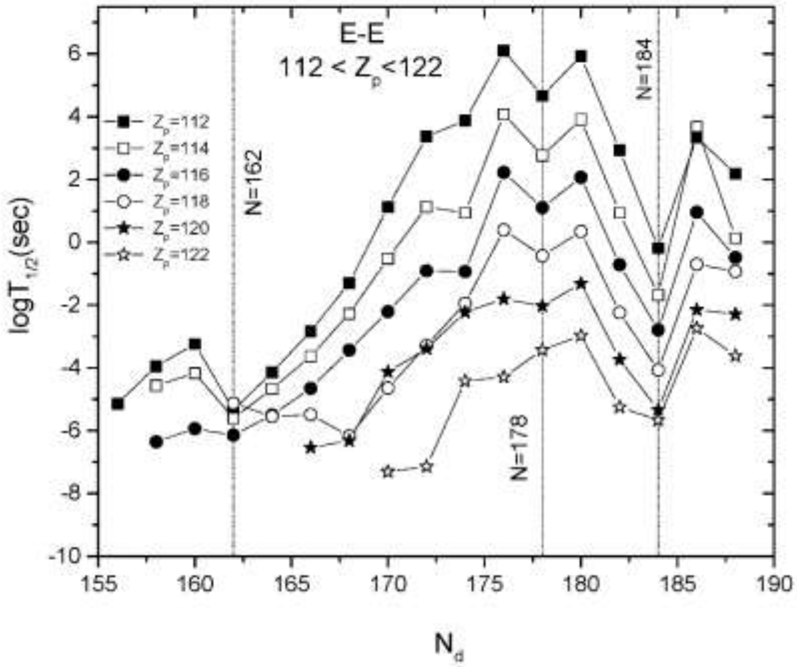

Fig.3 the same as fig. 1 but for the range $112 \leq Z_P \leq 122$.

It is noted from the figures discussion that it is possible to predict proton and neutron magic numbers for heavy and super heavy nuclei from studying the behavior of α-decay half-lives as a function of neutron number of the daughter nuclei using the present simple method.

V. Conclusion

In the framework of the preformed-cluster model, a simple method for calculating $T_{1/2}$ of heavy and super heavy nuclei is derived using the WKB approximation for the penetration of the Coulomb barrier with the Woods-Saxon potential for the nuclear part and the Langer modified centrifugal potential for the centrifugal part.Then, the variation of $log T_{1/2}$ with neutron numbersof the daughter nuclei was studied. Consequently, it is found by simple calculations for the alpha decay half-lives that N = 126, 152, 162, 178 and 184 are neutron magic numbers and Z = 108, 114, 118 and 120 are proton magic numbers.In fact, these magic numbers are in a good agreement with those predicted in other studies.

References

[1]. S. Hofmann, New elements - approaching Rep. Prog. Phys, 61 (1998) 639.

[2]. S. Hofmann, G. M¨unzenberg, The discovery of the heaviest elements, Rev. Mod. Phys, 72 (2000) 733.

[3]. H. Becquerel, Sur les radiations invisibles émises par les corps phosphorescents, C.R. Acad. Sci. Paris, 122 (1896) 420.

[4]. E. Rutherford, H. Geiger, An Electrical Method of Counting the Number of α -Particles from Radio-Active Substances, Proc. R. Soc, 81 (1909) 141.

[5]. G. Gamow, Zur quantentheorie des atomkernes, Z. Phys, 51 (1928) 204.

[6]. E.U. Condon, R.W. Gurney, Nature, 122 (1928) 439.

[7]. G. Wentzel, Eine Verallgemeinerung der Quantenbedingungen für die Zwecke der Wellenmechanik, Z. Phys, 38 (1926) 518.

[8]. G. Drukarev, N. Fr¨oman, P.O. Fr¨oman, The Jost function treated by the F-matrix phase integral method, J. Phys.A, 12 (1979) 171.

[9]. B. Buck, A.C. Merchant, S.M. Perez, α decay calculations with a realistic potential, Phys. Rev. C, 45 (1992) 2247.

[10]. B. Buck, A.C. Merchant, S.M. Perez, Half-lives of favored alpha decays from nuclear ground states, At. Data Nucl. Data Tables, 54 (1993) 53.

[11]. B. Buck, A.C. Merchant, S.M. Perez, Systematics of alpha-cluster states above double shell closures, Phys. Rev. C, 51 (1995) 559.

[12]. K.P. Santhosh, A. Joseph, Exotic decay in cerium isotopes, Pramana J. Phys, 58 (2002) 611-621.

[13]. G. Royer, R.K. Gupta, V.Y. Denisov, Cluster radioactivity and very asymmetric fission through compact and creviced shapes, Nucl. Phys. A, 632 (1998).

[14]. D.N. Poenaru, M. Ivascu, A. Sandulescu, W. Greiner, Atomic nuclei decay modes by spontaneous emission of heavy ions, Phys. Rev. C, 32 (1985) 572.

[15]. W.M. Seif, α decay as a probe of nuclear incompressibility, Phys. Rev. C, 74 (2006) 034302.

[16]. M. Ismail, A.Y. Ellithi, M.M. Botros, A. Adel, Systematics of α-decay half-lives around shell closures, Phys. Rev. C 81 (2010) 024602

[17]. D. Bucurescu, N.V. Zamfir, New nuclear structure features in transactinide nuclei, Phys. Rev. C, 87 (2013) 054324.

[18]. D. Seweryniak, K. Starosta, C.N. Davids, S. Gros, A.A. Hecht, N. Hoteling, T.L. Khoo, K. Lagergren, G. Lotay, D. Peterson, A. Robinson, C. Vaman, W. B.Walters, P. J.Woods, S. Zhu, α decay of 105 Te, Phys. Rev. C, 73 (2006) 061301(R).

[19]. D.N. Poenaru, W. Greiner, M. Ivascu, A. Sandulescu, Heavy cluster decay of trans-zirconium ‘‘stable’’ nuclides, Phys. Rev.C, 32 (1985) 2198.

[20]. A. Beiser, Concepts of Modern Physics, Tata McGraw-Hill Education 2003.

[21]. Royer, G, Alpha emission and spontaneous fission through quasi-molecular shapes, J. Phys. G: Nucl. Part. Phys., 26 (2000) 1149.

[22]. V.E.V. Jr., G.T. Seaborg, Nuclear systematics of the heavy elements—II Lifetimes for alpha, beta and spontaneous fission decay, Nucl. Chem. Chin, 28 (1966) 741.

[23]. A. Sobiczewski, Z. Patyk, S. Cwiok, Deformed superheavy nuclei, Phys. Lett. B 224 (1989) 1.

[24]. A. Sobiczewski, A. Parkhomenko, Description of structure and properties of superheavy nuclei, Prog. Part. Nucl. Phys, 58 (2007) 292.

[25]. A. Parkhomenko, A. Sobiczewski, Phenomenological Formula for alpha -Decay Half-Lives of Heaviest Nuclei, Acta Phys. Pol.B, 36 (2005) 3095.

[26]. Z.R. D. Ni, T. Dong, C. Xu, Phys. Rev. C, 78 (2008) 044310.

[27]. M. Horoi, B.A. Brown, A. Sandulescu, Scaling behaviour in cluster decay, J. Phys. G: Nucl.Part. Phys, 30 (2004) 945.

[28]. K.P. Santhosh, S. Sabina, R.K. Biju, Alpha radioactivity in heavy and super heavy elements, Nucl. Phys. A 825 (2009) 159.

[29]. C. Qi, F.R. Xu, R.J. Liotta, R. Wyss, Universal Decay Law in Charged-Particle Emission and Exotic Cluster Radioactivity, Phys. Rev. Lett., 103 (2009) 072501

[30]. G. Royer, Analytic expressions for alpha-decay half-lives and potential barriers, Nucl. Phys. A, 848 (2010) 279.

[31]. W.D. Myers, W.J. Swiatecki, Nuclear masses and deformations, Nucl. Phys. A, 81 (1966) 1.

[32]. A. Sobiczewski, F.A. Gareev, B.N. Kalinkin, Closed shells for Z > 82 and N > 126 in adiffuse potential well, Phys. Lett, 22 (1966) 500.

[33]. H. Meldner, Predictions of new magic regions and masses for superheavy nuclei from calculations with realistic shell model single particle Hamiltonians, Ark. Fys, 36 (1967) 593.

[34]. S.G. Nilsson, On the spontaneous fission of nuclei with Z near 114 and N near 184, Nucl. Phys. A, 115 (1968) 545.

[35]. J. Grumann, U. Mosel, B. Frink, W. Greiner, Investigation of the stability of superheavy nuclei around Z=114 and Z=164, Z. Phys, 228 (1969) 371.

[36]. S.G. Nilsson, C.F. Tsang, A. Sobiczewski, Z. Szymanski, S. Wyeech, C.Gustafsson, I.L. Lamm, P. Moller, B. Nilsson, On the nuclear structure and stability of heavy and superheavy elements, Nucl. Phys. A, 131 (1969) 1.

[37]. S.K. Patra, R.K. Gupta, W. Greiner, Structure of 294,302120 Nuclei Using the Relativistic Mean-Field Method, Mod. Phys. Lett. A, 12 (1997) 1727.

[38]. V.Y. Denisov, H. Ikezoe, α-nucleus potential for α-decay and sub-barrier fusion, Phys. Rev. C, 72 (2005) 064613.

[39]. Z.Z.R. C. Xu, Systematical calculation of α decay half-lives by density-dependent cluster model, Nucl. Phys. A, 753 (2005) 174.

[40]. R.E. Langer, On the Connection Formulas and the Solutions of the Wave Equation, Phys. Rev. C, 51 (1937) 669.

[41]. A. Zdeb, M. Warda, K. Pomorski, Half-lives for α and cluster radioactivity within a Gamow-like model, Phys. Rev. C, 87 (2013) 024308.

[42]. G. Audi, The Nubase2012 evaluation of nuclear properties, Chin. Phys. C, 36 (2012) 1157.

[43]. P. Moller, J.R. Nix, Stability of heavy and superheavy elements, J. Phys. G: Nucl.Part. Phys, 20 (1994) 1681.

[44]. D. Poenaru, R. Gherghescu, W. Greiner, Cluster decay of superheavy nuclei, Phys. Rev.C, 85 (2012) 034615.

[45]. W. Zhang, J. Meng, S. Zhang, L. Geng, H. Toki, Magic numbers for superheavy nuclei in relativistic continuum Hartree–Bogoliubov theory, Nucl. Phys. A, 753 (2005) 106-135.

[46]. J. Hamilton, S. Hofmann, Y. Oganessian, The importance of closed shell structures in the synthesis of super heavy elements, Journal of Physics: Conference Series, IOP Publishing, 2015, pp. 012019.

[47]. W. Greiner, R.K. Gupta, Heavy Elements and Related New Phenomena:(In 2 Volumes), World scientific1999.

[48]. S. Ćwiok, S. Hofmann, W. Nazarewicz, Shell structure of the heaviest elements, Nucl. Phys. A, 573 (1994) 356-394.

[49]. P. Greenlees, R.-D. Herzberg, S. Ketelhut, P. Butler, P. Chowdhury, T. Grahn, C. Gray-Jones, G. Jones, P. Jones, R. Julin, High-K structure in Fm 250 and the deformed shell gaps at N= 152 and Z= 100, Phys. Rev. C, 78 (2008) 021303.

[50]. M. Ismail, A. Adel, Shell corrections for heavy and superheavy nuclei, International Journal of Modern Physics E, 21 (2012) 1250062.

[51]. M. Ismail, A.Y. Ellithi, A. Adel, H. Anwer, On magic numbers for super- and ultraheavy systems and hypothetical spherical double-magic nuclei, J. Phys. G: Nucl. Part. Phys., 43 (2016) 015101.

[52]. M. Ismail, A. Ellithi, A. Adel, H. Anwer, On magic numbers for super-and ultraheavy systems and hypothetical spherical double-magic nuclei, J. Phys. G: Nucl. Part. Phys., 43 (2015) 015101.

[53]. K. Santhosh, R. Biju, Neutron and proton shell closure in the superheavy region via cluster radioactivity in 280–314116 isotopes, Pramana, 72 (2009) 689-707.

[54]. M. Ismail, W. M. Seif, A. Abdurrahman, Relative stability and magic numbers of nuclei deduced from behavior of cluster emission half-lives, Phys. Rev. C 94 (2016) 024316.