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Dissipative Soliton Resonance Curve under Influence of Nonlinear Gain

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Abstract: *We investigate the effect of the cubic nonlinear gain on the resonance curve in the frame of the dissipative soliton resonance (DSR) in a complex cubic-quintic Ginzburg-Landaumodel. The DSR occurs when the energy of a soliton in the system increases without limit at certain values of the system parameters. We remind analytical expressionfor the resonance curve,thanks to the collective variable approach.We show that the nonlinear gain has a linear response for the spectral filtering and the saturation of the Kerr nonlinearity,however, has a nonlinear response for the linear loss and the saturation of the nonlinear gain. Thus the gain and its saturation in the system can be suitably chosen for generation of high-energy pulses.These results can be helpful to design laser systems orto optimize the active parameters of cavities that generate solitons with the highest possible energy.*

Keywords: *Cubic nonlinear gain, Dissipative soliton resonance, High-energy pulses, Mode-locked laser, Resonance curve*

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I. Introduction

Dissipative solitons refer to solitary waves in nonlinear systems with nonlinear gain or loss [1, 2]. The formation of integrable conventional soliton, in optics, results from the single balance between nonlinearity and dispersion/diffraction. In non-integrable and non-conservative systems (dissipative systems), the single balance is replaced by the balance between gain and loss on the one hand and between dispersive and nonlinear conservative effects on the other hand. The existence and stability of dissipative solitons (DS) depend deeply on the energy balance [3]. Their dynamic and self-organization are governed by the perpetual and bidirectional exchange of energy with their environment. More specifically, this energy has to be dissipated in the medium where solitons are found. The dynamic and stability, more specifically, the amplitude, chirp, width and all other parameters of dissipative soliton and its energy are predetermined by the system parameters, rather by the initial conditions. According to system parameters, we can obtain a wide range of dissipative solitons, which show a huge quantity of different behaviors. In this way, the redistribution of energybetween various parts of the dissipative soliton can lead stationary soliton to pulsating or chaotic soliton dynamics [4, 5].However, for a given set of parameters;the profile of the DS is indeed fixed. These characteristics of dissipative solitons help for a wide range of applications, such as the generation of stable trains of laserpulses by mode-locked cavities, or the in-line regeneration of opticaldata streams [3].

The laser cavities can be considered as an ideal experimental frame for the exploration of dissipative soliton dynamics. The mode-locked lasers with nonlinearity, saturable absorber and which allows the generation of ultra-short optical pulsescan be regarded as perfect surroundings for the concept of dissipativesolitons.Each laser model depends on thedissipative soliton dynamics, that way, agiven laser system can be modeledby varying several components. In addition, an exact model should involve consecutive sets of propagation equationsthat incorporate the mainphysical ingredients at play in mode-locked lasers. It has been shown that the complex cubic–quintic Ginzburg–Landau equation (CGLE) is an accurate model to approach the mode-locking [6] and use to describe a widerange of nonlinear optical systems, such as passively mode-lockedlasers with fast saturable absorbers, parametric oscillators, wideaperturelasers and nonlinear optical transmission lines [7]. The CGLEis very important in nonlinear optic due to the clear physical meaning of all its terms in any particular application. Under certain conditions, it is possible to relate all the terms of the CGLE to the physical parameters of the laser cavity [8].

For a given set of the system parameters, it has been demonstrated that the dissipative soliton energy can increase indefinitely and so the process resembles the resonance phenomenon in the theory of oscillators. In such cases, the solitons increase their width indefinitely while keeping their amplitude constant. This behavior of dissipative soliton was named "dissipative soliton resonance".Likewise, in the frame of CGLE [9, 10] Chang et al have foreseen a novel soliton formation, dissipative-soliton-resonance,recently. Using technique called the method of moments (trial function technique); they found the resonance approximately [9].It knowsthat finding analytical solutions of the CGLE is critical task procedure except for specific values of the complex cubic–quintic Ginzburg–Landau equation parameters [5]. To the best of our knowledge,it is also an impossible task to findthe dissipative-soliton-resonance (DSR) parameters using direct analytic technique. For a given set of parameters and a given initial conditions, the complex cubic–quintic Ginzburg–Landau equation has several configuration parameters, so it is tough or even impossible to identify its solutions or the region of parameters where dissipative soliton resonance exists. This will requires an enormous number of numerical simulations and is an extremely lengthy and costly procedure. To overcome this task, it appears necessaryto acquire theoretical tools that help to perceive CGLE soliton solutions and to find the set of parameters, which predicts the domains where resonances can be found. This task can be simplified in the case of DSRmore efficiently.

Recently, using the method of collective variable approach, we have foundthe resonance curve and described the influence of dissipative terms of CGLE on this curve. Accurately we have demonstrated how the linear loss, the gain and its saturation in the system and the spectral response of the cavity can be suitably chosen for generation of high-energy pulses.In the present study, we remind in section 2 the resonance curve found by collective variable approach and focus on specific values on nonlinear gain in order to evaluate their actions on the resonance curve. The section 3 is devoted to the influence of the cubic nonlinear gain and how this parameter affects the spectral response, the linear loss coefficient, the saturation of the nonlinear gain and the saturation of Kerr nonlinearity.Finally in the section 4, we conclude after the discussion of the results.

II. Materials and method

The dynamic of dissipative soliton in nonlinear cavity can be modeledby the complex cubic-quintic Ginzburg-Landau equation. The normalized propagation equation, in the (1+1) dimensional case, reads:

$$\psi_z - i\frac{D}{2}\psi_{tt} - i\gamma|\psi|^2\psi - i\nu|\psi|^4\psi = \delta\psi + \varepsilon|\psi|^2\psi + \beta\psi_{tt} + \mu|\psi|^4\psi \quad (1)$$

Where$z$ is the propagation distance or the cavity round-trip number and t is the retarded time in the frame moving with the pulse. The normalized optical envelope$\psi = \psi(t,z)$, is a complex function of two real variables. In the context of the dimensionless CGLE, the parameters$D$,γ,ν, μ, ε, δ and β havetheir standard meaning.$D$the group velocity, accounts for the dispersion, being positive (negative), in the anomalous (normal) dispersion regime and$\gamma$is the Kerr nonlinearity coefficient.The dispersion is responsible for the net dispersion in the cavity while $\nu$the saturation coefficient of the Kerr nonlinearity, describes the active part of the reactive nonlinearity. ε represents the cubic nonlinear gain while the term with μ represents, if negative, the saturation of the nonlinear gain ε. δ is here the linear loss coefficient and accounts for spectral filtering in the cavity. The coefficients μ, ε and $\delta$are mainly determined by the gain in the system, cavity losses, and transmission characteristics of the mode-locking device. βdescribes the spectral responseof the cavity.Higher-order dissipative terms are responsible for the nonlinear transmission characteristics of the cavity, which allows, for example, passive mode locking.

The CGLE equation (1) describes very well experimental observations of high-energy pulses from passively mode-locked lasers, both fiber and solid-state ones [11, 12], and its coefficients can refer to the physical parameters of laser cavities.Likewise, the equation (1)can be used to design laser systems for the generation of high-energy pulses, as the dissipative resonance effects are predictive features. The main and tedious problem is that each particularlaser requires specific modeling and numerical simulations [13, 14]. To address this critical constraint, one option, is to use the master equation approach, to a certain extent to overcome this problem. This procedurehelps to describe any particular laser model, and to find the critical parameters of the system that will generate the pulse with the highest possible energy.We have highlighted in our previous work [5] the stationary and pulsating solutions of the CGLE using the collective variable method and have demonstrated the relevance of that procedure. Using the same approach we have analytically found the resonance curve - see details in the reference [15] - leading to a dramatic reduction of the computation time.

As in these works [5, 15], we are dealing with the collective variable theory [16], with the same way, we decompose the field $\psi(t,z)$ in the following way:

$$\psi(t,z) = f(X_1, X_2, \ldots, X_N, t) + q(t,z) \quad (2)$$

Where f the trial function, is a function of X_N(the collective variable) and the other excitations(radiation, dressing field, noise, etc.) in the system are represented by q. Thereafter, we can consider that the exact pulse field $\psi = \psi(t,z)$ is completely characterized by the trial function$f$. The bare approximation

[16] is named to describe this approximation of neglecting the field q. When approximations are made, the precise form of the ansatz function that introduces the collective variables in the theory is rather crucial. This choice is especiallyimportant for the success of the technique.
Like this,weconsider the following higher-order Gaussian function to the success of our approach:

$$f(t,z) = A\, exp\left(-\frac{t^2}{w^2} - \frac{t^4}{w^4} + ict^2 + ip\right) \quad (3)$$

In this way$A$represents the soliton amplitude, $w$and$c$its width and chirp respectively.p is the global phase that evolves along with propagation. These parameters A, w, c, p(collective variables)are variable that evolve along the propagation direction z. They control the dynamic of the dissipative soliton.The total energy$Q$ is the natural control parameter to some extent. Here, in the collective variables approachone of the key benefit is that the total energy can also expressed as function of the trial function parameters. Here it is interesting to gain insight from this simple and useful quantity, which is defined as:

$$Q = \int_{-\infty}^{\infty} |\psi|^2\, dt$$

$$Q = 1.051A^2 w \quad (4)$$

It is clearly function to the soliton amplitude and its width. At a later stage, when the choice of the trial function is made, one can carry out variational analysis by setting the residual field to zero ($q(t,z) = 0$) to the CGLE. Hence substituting the pulse field (ψ)by a given trial function (f) and projecting the resulting equations in the direction [16] of

$$\frac{\partial f^*}{\partial X}\ (X = A, w, c, p) \quad (5)$$

we obtain easily the collective variables evolve according to the set of coupled ordinary differential equations, which can be expressed as a function of the total energy Q as following :

$$Q_z = Q\left(2\delta - 1.158\, c^2 w^2 \beta + 1.433\frac{Q\varepsilon}{w} + 1.146\frac{Q^2}{w^2}\mu\right)$$

$$w_z = w\left(-0.815\, c^2 w^2 \beta - 0.252\frac{Q\varepsilon}{w} - 0.269\frac{Q^2}{w^2}\mu\right) \quad (6)$$

$$c_z = \frac{1}{w^2}\left(-2\, c^2 w^2 D - 1.327\frac{\gamma Q}{w} - 1.491\frac{\nu\, Q^2}{w^2}\right)$$

We observe that the CGLE, equation (1) is reduced to an ordinary differential equation given by the soliton energy Q and width w. From the equation (6) one can easily determine the resonance curve; see details in [15]. It is known that near the resonance curve,the zeros of the right-hand side of the equation (6) define the fixed points.As well as the values Q/wandcwtend to approach constants with c being negative. As you can see in [15],thus we obtain with a certain ease the expression for the resonance curve in terms of the system parameters:

$$D = k\frac{\beta(0.240k\nu - 0.533\gamma\mu)}{\mu(0.062k\varepsilon - 0.863\delta\mu)} \quad (7)$$

where

$$k = 1.461\varepsilon + \sqrt{2.135\,\varepsilon^2 - 8.125\delta\mu} \quad (8)$$

All the details of this calculation are referred in our previous work in [15]. So the method of collective variable approach helps to found approximatively the relation between the parameters of the normalized complex cubic-quintic Ginzburg-Landau equation and simple analytical expression for the resonance curve. For the chosen values of the CGLE (see inside the Fig. 1) we plot in the plane of parameters D and ε,the resonance curve obtained by the collective variable approach.This curve is reasonably good qualitative agreement with those found by the method of moments [9]. The curve is atypical, for small values on the dispersion D and nonlinear gain ε it tends to a relative constant value. Conversely, when this parameters increase, the resonance curve rises quickly and its slope tends almost to zero. The nonlinear gain plays a key role in the dynamic of dissipative soliton and as well,near these curve the soliton energy and width increase to infinitely large values. If we carefully examine the curve (Fig. 1), we focused on four values of nonlinear gain $\varepsilon = 0.17$, $\varepsilon = 0.27$, $\varepsilon = 0.5$, and $\varepsilon = 2$ respectively in blue, red, black and green on the Fig. 1 that we explore further in the next section.

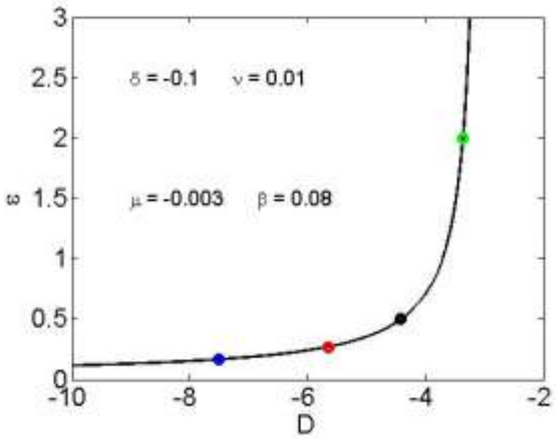

Figure 1. Resonance curve in the (D, ε) planefound by the collective variable approach. Other CGLE parameters appear inside the Figure. We highlight on four values of nonlinear gain $\varepsilon = 0.17$, $\varepsilon = 0.27$, $\varepsilon = 0.5$ and $\varepsilon = 2$ respectively in blue, red, black and green.

III. Results And Discussion

The nonlinear gain controls the evolution of the dissipative soliton. According its value, the amplitude, the width, the shape of the soliton can have different characteristics. In the reference [11] the authors have precisely shown that when weget closer to the resonance, the pulse profile of the solitons and their spectra change quickly. That way, when the cubic nonlinear gain ε increases, the pulses can become narrower and of higher intensity.εincreases is mainly due tothe increase in the pulse width. Likewise,Niang et al investigated in [18] the effect of the gain dynamics on the collectivebehavior of solitons and shown that the gain dynamics modifies the soliton velocity and their interactions.In addition, as ε determine the gain in the system, it can be suitably chosen for generation of high-energy dissipative solitons.This is due to the fact that the dissipative soliton resonance effect of CGLE ispredictive. As well, it is widely known that the physical meaning of each particular laser depends on the real problem which must be examined and requires numerous modeling and numerical simulations [13, 14]. Now in this paper it seems important to consider the influence of the cubic nonlinear gain εtermson the resonance curveand how it affects the spectral responseof the cavity, the linear loss coefficient, the saturation of the nonlinear gain and of the Kerr nonlinearity.

Firstly we plot in the Fig. 2 the evolution of the resonance curve in the (D, β) plane for a given set of parameters inside of the figure. For a specific value of the cubic nonlinear gain $\varepsilon$the resonance curve has a linear evolution. Thereby for $\varepsilon = 0.17$, $\varepsilon = 0.27$, $\varepsilon = 0.5$, and $\varepsilon = 2$ respectively in blue, red, black and green we have three straight lines that evolve separately with the same origin. They have the commonpoint (0.0) and negative slopes. We note that when the nonlinear gain increases, the resonance curve moves to the right. This is accomplished in practice by a rapid increase of the soliton energy. It can also be noted that when the spectral responseof the cavityβ increases the gap between the different resonance curves also increases.This characterizes different profiles (amplitude, wide, shape) and behaviors (stable, non-stable) of the dissipative structures.It can be seen here that whatever the value of the cubic nonlinear gain ε,the resonance curve has the same qualitative dynamics.This behavior can help in the design of lasers with special characteristics.

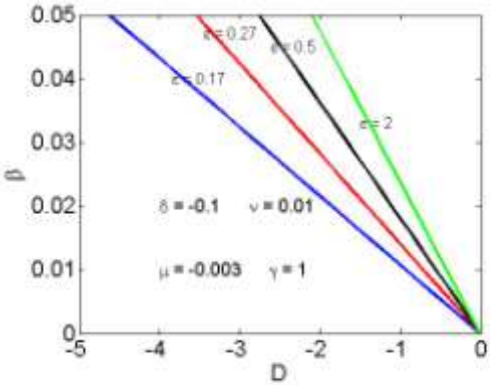

Figure 2. Resonance curves in the (D, β) planefound by the collective variable approach for different values of nonlinear gain $\varepsilon = 0.17$ (blue), $\varepsilon = 0.27$ (red),$\varepsilon = 0.5$ (black) and $\varepsilon = 2$ (green). Other CGLE parameters appear inside the Figure.

As shown in the following Fig. 3, we plot the evolution on the resonance curve in the(D, δ) plane varying the linear loss coefficientδ and the dispersion parameter$D$. We notice that the resonance curve varies accordingto the nonlinear gain. For nonlinear gain values close to 2, the curve tends to a straight line, on the other hand for low values close to 0.17 the pace of the curve is totally different.It is no longer linear and becomesdistant from the other three. For this specific value of nonlinear gain ($\varepsilon = 0.17$) in blue, we are seeing that the resonance occurs over a wide range for the dispersion from -15 to -7.15. In this instance, in spite of the losses in the cavity, it is possible to have pulses with highest energies, thereby benefiting for the conception and design of the generators with record-high energies.

Likewise we draw the resonance curves by plotting the dispersion against the saturation of the nonlinear gain.Reasonably close to zero, the curves are close each other or overlap. For$\varepsilon = 0.27$ (in red), $\varepsilon = 0.5$ (in black) and $\varepsilon = 2$ (in green), see Fig. 4, the resulting resonance curves follow the same evolution. Thisthreecurves (red, black and green) are similar and have qualitatively the same behaviours and characteristics. A more thorough inspection reveals that the resonance curves in the (D, ε) plane and in the (D, μ) plane – the nonlinear gain (ε) and its saturation (μ)– have the inversely proportional features and evolutions. On the other hand, the curve for $\varepsilon = 0.17$, in blue, is clearly differentiated from the others. It follows approximately an arc of circle, like that, each dispersion value between -13.5 and -6 refers to two points on the resonance curve. The selected nonlinear gain significantly alters the dissipative soliton dynamic, changing $\varepsilon = 0.27$ to $\varepsilon = 0.17$ the resonance curve is modified considerably.

The saturation coefficient of the Kerr nonlinearity (ν) action on the resonance curve is summarize on the Fig. 5 for different values of nonlinear gain $\varepsilon = 0.17$ (blue), $\varepsilon = 0.27$ (red), $\varepsilon = 0.5$ (black) and $\varepsilon = 2$ (green). These curves depict straight lines which are parallel to each other. The line segments (for $\varepsilon = 0.17$,$\varepsilon = 0.27$, $\varepsilon = 0.5$ and $\varepsilon = 2$) with negative slope represent uniformly decreasing resonance.

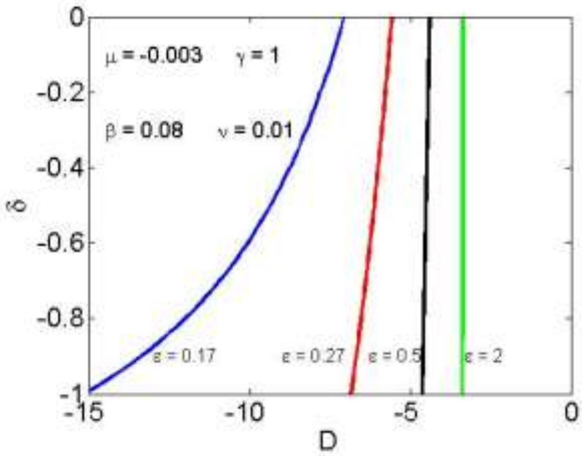

Figure 3. Resonance curves in the (D, δ) planefound by the collective variable approach for different values of nonlinear gain $\varepsilon = 0.17$ (blue), $\varepsilon = 0.27$ (red), $\varepsilon = 0.5$ (black) and $\varepsilon = 2$ (green). Other CGLE parameters appear inside the Figure.

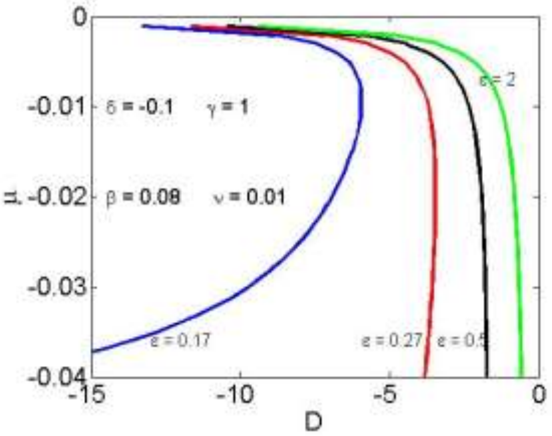

Figure 4. Resonance curves in the (D, μ) planefound by the collective variable approachfor different values of nonlinear gain $\varepsilon = 0.17$ (blue), $\varepsilon = 0.27$ (red), $\varepsilon = 0.5$ (black) and $\varepsilon = 2$ (green). Other CGLE parameters appear inside the Figure.

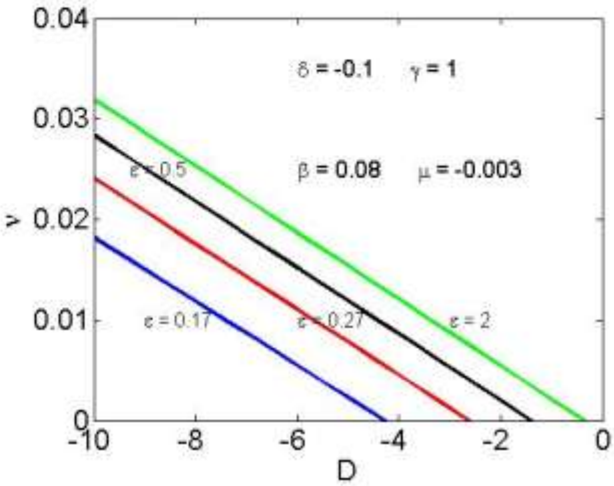

Figure 5. Resonance curves in the (D, ν) planefound by the collective variable approachfor different values of nonlinear gain $\varepsilon = 0.17$ (blue), $\varepsilon = 0.27$ (red), $\varepsilon = 0.5$ (black) and $\varepsilon = 2$ (green). Other CGLE parameters appear inside the Figure.

IV. Conclusion

In this present study, we reiterated collective variable approach for the phenomenon of dissipative soliton resonance. Thanks to this approach, we have found an approximate relation between the parameters of the normalized complex cubic-quintic Ginzburg-Landau equation and simple analytic expression for the resonance curve. The coefficients for the CGLE are in practice complex function or very complicated specific design factors, and must be clarified for each specific laser. As the master equation that describes laser systems has many parameters, it's so difficult to study the influence of all of them on the soliton properties. Here we have investigated the influence of the cubic nonlinear gain εtermof CGLE on the resonance curve and how it affects the spectral response of the cavity, the linear loss coefficient, the saturation of the nonlinear gain and the saturation of Kerr nonlinearity. This parameter (nonlinear gain ε term) is essential in the modelling of high energylaser systems and tunable. In this work, we clearly showed it could have a linear response for the spectral response of the cavity and the saturation of the Kerr nonlinearity on the one hand. On the other hand the same nonlinear gain ε term has a nonlinear response forthe linear loss and the saturation of the nonlinear gain.In this latter case, the curve is much more impacted by the low values of nonlinear gain. These facts provide useful hints that can be further analyzed and interpreted. Theseresults bring particular advantagesforgeneration of high-energy pulses and to enhance the gain termsinthe mode-locked laser systems.Obviously this work can be useful for the configuration of the laser systems that generate record-high energy short pulses, without the need for additional amplifiers.

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