IOSR Journal of Applied Physics (IOSR-JAP)
e-ISSN: 2278-4861.Volume 9, Issue 5 Ver. I (Sep. - Oct. 2017), PP 21-29
www.iosrjournals.org

Triple Differential Cross Sections for Ionization of Metastable 3P-State Hydrogen Atoms by Electrons

S.Dhar[1*], S. Akter[2], A. Saha[3], N. Nahar[4]
[1,3,4]*Department of Mathematics, Chittagong University of Engineering and Technology, Chittagong, Bangladesh*
[2] *Department of Mathematics, Premier University, Chittagong, Bangladesh*
Corresponding author: S.Dhar

Abstract: *Triple differential cross sections (TDCS) for ionization of metastable 3P-state Hydrogen atoms by electrons have been calculated for various kinematic conditions in the asymmetric coplanar geometry using a multiple scattering theory of Das and Seal. The results are compared with the first Born results and existing hydrogenic ground state experimental data and those of other theoretical results. The present results show a good qualitative agreement with the compared results. Yet there is no available theoretical and experimental study for ionization of metastable 3P-state hydrogen atoms by electrons.* ***These*** *offers wider scope for the study of ionization problems of hydrogen atoms in their metastable states.*

Keywords: *Cross-Section, Electron, Ionization, Scattering, Transition matrix.*

Date of Submission: 24-07-2017 Date of acceptance: 05-09-2017

I. Introduction

For many atomic systems, Ionization by electron impact has been studied with a great interest theoretically and experimentally, which has important challenge in several fields of physics such as plasma physics, astrophysics and irradiation of living matter. Ionization of hydrogen atoms by electrons is the fundamental and simplest ionization problem in this study. In this point of view, the electron-electron coincidence experiments is called (e,2e) experiments, provide a useful description of the kinematics of the collision by giving information about the direction of the scattered and ejected electrons. The quantity measured in this kind of experiment is proportional to the TDCS, which represents the angular distribution of the ejected electron for selected incident and scattered electron momenta.

In the last five decades significant progress has been made in understanding the ionization process both in the ground state [1-10] and metastable states [11-20] of atomic hydrogen. The TDCS for the (e, 2e) techniques were widely studied for the ground state hydrogen atom both experimentally [21-25] and theoretically [26-30].

Jones and Madison [7-9] had studied ionization of atomic hydrogen by electron impact using asymptotically correct two-centre wave functions to describe the scattering system both initially and finally. The theoretical understanding of the (e,2e) reaction is very satisfactory in the case of the ionization from the ground state atomic hydrogen [10]. Recently the TDCS for the ionization of metastable 2P-state [18] and 2S-state [14] hydrogen atoms by electrons have been calculated following the multiple scattering theory of Das and Seal [4]. Here we use same theory for ionization of hydrogen atoms by electrons for describing the TDCS of metastable 3P-state hydrogen atom by electrons considering at intermediate and high energies.

II. Theory

The single ionization processes of atomic hydrogen by electron of the following type,

$$e^- + H(3P) \to H^+ + 2e^- \tag{1}$$

where the symbol 3P denotes the hydrogenic metastable state and has been obtained in the coplanar geometry by analyzing triple differential cross sections (TDCS) measured in (e,2e) coincidence experiments. The TDCS is a measure of the probability that in an (e,2e) reaction an incident electron of momentum $\bar{p}_i$ and energy E_i will produce on collision with the target two electrons having energies E_1 and E_2 and momenta $\bar{p}_1$ and $\bar{p}_2$ emitted respectively into the solid angles $d\Omega_1$ and $d\Omega_2$ centred about the directions (θ_1,ϕ_1) and (θ_2,ϕ_2) .

Triple differential cross section is denoted by $\dfrac{d^3\sigma}{d\Omega_1 d\Omega_2 dE_1}$.

The multiple scattering theory of ionization of hydrogen atoms by electrons is described in detail [4,18]. A brief discussion of the theory for the particular case of hydrogenic 3P-state at intermediate and high energies is given here.

The direct transition matrix element for ionization of hydrogen atoms by electrons [4], may be written as,

$$T_{fi} = \left\langle \Psi_f^{(-)} \middle| V_i \middle| \Phi_i \right\rangle \quad (2)$$

The perturbation potential V_i is given by

$$V_i = \frac{1}{r_{12}} - \frac{1}{r_2}$$

and the initial state hydrogenic wave function is given here.

$$\Phi_i = \frac{e^{i\bar{p}_i \cdot \bar{r}_2}}{(2\pi)^{3/2}} \phi_{3P}(\bar{r}_1) .$$

Where hydrogenic 3P-state wave function is

$$\phi_{3P}(\bar{r}_1) = \frac{\sqrt{2}}{81\sqrt{\pi}} \left(6r_1 - r_1^{\,2}\right) \cos\theta \, e^{-r_1/3} \quad (3)$$

Here $\lambda_1 = \frac{1}{3}$ and $\Psi_f^{(-)}$ is the final three-particle scattering state wave function [4] and co-ordinates of the ejected and scattered electrons are $\bar{r}_1$ and $\bar{r}_2$ respectively.

Here the approximate scattering state wave function $\Psi_f^{(-)}$ is given by

$$\Psi_f^{(-)} = N(\bar{p}_1, \bar{p}_2) \left[\phi_{\bar{p}_1}^{(-)}(\bar{r}_1) e^{i\bar{p}_2 \cdot \bar{r}_2} + \phi_{\bar{p}_2}^{(-)}(\bar{r}_2) e^{i\bar{p}_1 \cdot \bar{r}_1} + \phi_{\bar{p}}^{(-)}(\bar{r}) e^{i\bar{P} \cdot \bar{R}} - 2e^{i\bar{p}_1 \cdot \bar{r}_1 + i\bar{p}_2 \cdot \bar{r}_2} \right] / (2\pi)^3 \quad (4)$$

Here

$N(\bar{p}_1, \bar{p}_2)$ is normalization constant,

$$\bar{r} = \frac{\bar{r}_2 - \bar{r}_1}{2}, \; \bar{R} = \frac{\bar{r}_1 + \bar{r}_2}{2},$$

$$\bar{p} = (\bar{p}_2 - \bar{p}_1), \; \bar{P} = \bar{p}_2 + \bar{p}_1$$

and $\phi_q^{(-)}(\bar{r})$ is Coulomb wave function.

Now applying equations (3) and (4) in equation (2), we get

$$T_{fi} = T_B + T_{B'} + T_i - 2T_{PB} \quad (5)$$

where

$$T_{\mathbf{B}} = \left\langle \phi_{p_1}^{(-)}(\bar{r}_1) \, e^{i\bar{p}_2 \cdot \bar{r}_2} \middle| V_i \middle| \Phi_i \right\rangle \quad (6)$$

$$T_{B'} = \left\langle \phi_{p_2}^{(-)}(\bar{r}_2) \, e^{i\bar{p}_1 \cdot \bar{r}_1} \middle| V_i \middle| \Phi_i \right\rangle \quad (7)$$

$$T_i = \left\langle \phi_p^{(-)}(\bar{r}) \, e^{i\bar{P} \cdot \bar{R}} \middle| V_i \middle| \Phi_i \right\rangle \quad (8)$$

$$T_{PB} = \left\langle e^{i\bar{p}_1 \cdot \bar{r}_1 + i\bar{p}_2 \cdot \bar{r}_2} \middle| V_i \middle| \Phi_i \right\rangle \quad (9)$$

From equation (6), the first Born approximation T_B may be written as

$$T_{\mathbf{B}} = \frac{1}{162\pi^2}\int \phi_{p_1}^{(-)*}(\bar{r}_1) e^{-i\bar{p}_2.\bar{r}_2}\left(\frac{1}{r_{12}} - \frac{1}{r_2}\right) e^{i\bar{p}_i.\bar{r}_2}\left(6r_1 - r_1^{\,2}\right)\cos\theta\; e^{-\lambda_1 r_1} d^3r_1 d^3r_2$$

$$\begin{aligned}
T_B = {} & \frac{1}{162\pi^2}\int \phi_{p_1}^{(-)*}(\bar{r}_1) e^{-i\bar{p}_2.\bar{r}_2}\,\frac{1}{r_{12}} e^{i\bar{p}_i.\bar{r}_2}\,6r_1\cos\theta e^{-\lambda_1 r_1} d^3r_1 d^3r_2 \\
& - \frac{1}{162\pi^2}\int \phi_{p_1}^{(-)*}(\bar{r}_1) e^{-i\bar{p}_2.\bar{r}_2}\,\frac{1}{r_2} e^{i\bar{p}_i.\bar{r}_2}\,6r_1\cos\theta\, e^{-\lambda_1 r_1} d^3r_1 d^3r_2 \\
& - \frac{1}{162\pi^2}\int \phi_{p_1}^{(-)*}(\bar{r}_1) e^{-i\bar{p}_2.\bar{r}_2}\,\frac{1}{r_{12}} e^{i\bar{p}_i.\bar{r}_2} r_1^{\,2}\cos\theta\, e^{-\lambda_1 r_1} d^3r_1 d^3r_2 \\
& + \frac{1}{162\pi^2}\int \phi_{p_1}^{(-)*}(\bar{r}_1) e^{-i\bar{p}_2.\bar{r}_2}\,\frac{1}{r_2} e^{i\bar{p}_i.\bar{r}_2} r_1^{\,2}\cos\theta\, e^{-\lambda_1 r_1} d^3r_1 d^3r_2
\end{aligned}$$

(10)

For first Born approximation, we calculated the terms of equation (10).
Similarly the expression (7), (8) and (9) can be calculated as

$$T_{B'} = \frac{1}{162\pi^2}\int \phi_{\bar{p}_2}^{(-)*}(\bar{r}_2) e^{-i\bar{p}_1.\bar{r}_1}\left(\frac{1}{r_{12}} - \frac{1}{r_2}\right) e^{i\bar{p}_i.\bar{r}_2}(6r_1 - r_1^{\,2})\cos\theta e^{-\lambda_1 r_1} d^3r_1 d^3r_2$$

$$\begin{aligned}
T_{B'} = {} & \frac{1}{162\pi^2}\int \phi_{p_2}^{(-)*}(\bar{r}_2) e^{-i\bar{p}_1.\bar{r}_1}\,\frac{1}{r_{12}} 6r_1\cos\theta\, e^{i\bar{p}_i.\bar{r}_2} e^{-\lambda_1 r_1} d^3r_1 d^3r_2 \\
& - \frac{1}{162\pi^2}\int \phi_{p_2}^{(-)*}(\bar{r}_2) e^{-i\bar{p}_1.\bar{r}_1}\,\frac{1}{r_{12}} r_1^{\,2}\cos\theta\, e^{i\bar{p}_i.\bar{r}_2}\, e^{-\lambda_1 r_1} d^3r_1 d^3r_2 \\
& + \frac{1}{162\pi^2}\int \phi_{p_2}^{(-)*}(\bar{r}_2) e^{-i\bar{p}_1.\bar{r}_1}\,\frac{1}{r_2} r_1^{\,2}\cos\theta\, e^{i\bar{p}_i.\bar{r}_2}\, e^{-\lambda_1 r_1} d^3r_1 d^3r_2 \\
& - \frac{1}{162\pi^2}\int \phi_{p_2}^{(-)*}(\bar{r}_2) e^{-i\bar{p}_1.\bar{r}_1}\,\frac{1}{r_2} 6r_1\cos\theta\, e^{i\bar{p}_i.\bar{r}_2}\, e^{-\lambda_1 r_1} d^3r_1 d^3r_2
\end{aligned}$$

(11)

$$\begin{aligned}
T_i = {} & \frac{1}{162\pi^2}\int \phi_{\bar{p}}^{(-)*}(\bar{r}) e^{-i\bar{P}.\bar{R}}\left(\frac{1}{r_{12}} - \frac{1}{r_2}\right)(6r_1 - r_1^{\,2})\cos\theta e^{i\bar{p}_i.\bar{r}_2} e^{-\lambda_1 r_1} d^3r_1 d^3r_2 \\
= {} & \frac{1}{162\pi^2}\int \phi_{\bar{p}}^{(-)*}(\bar{r}) e^{-i\bar{P}.\bar{R}}\,\frac{1}{r_{12}} 6r_1\cos\theta e^{i\bar{p}_i.\bar{r}_2} e^{-\lambda_1 r_1} d^3r_1 d^3r_2 \\
& - \frac{1}{162\pi^2}\int \phi_{\bar{p}}^{(-)*}(\bar{r}) e^{-i\bar{P}.\bar{R}}\,\frac{1}{r_2} 6r_1\cos\theta e^{i\bar{p}_i.\bar{r}_2} e^{-\lambda_1 r_1} d^3r_1 d^3r_2 \\
& - \frac{1}{162\pi^2}\int \phi_{\bar{p}}^{(-)*}(\bar{r}) e^{-i\bar{P}.\bar{R}}\,\frac{1}{r_{12}} r_1^{\,2}\cos\theta e^{i\bar{p}_i.\bar{r}_2} e^{-\lambda_1 r_1} d^3r_1 d^3r_2 \\
& + \frac{1}{162\pi^2}\int \phi_{\bar{p}}^{(-)*}(\bar{r}) e^{-i\bar{P}.\bar{R}}\,\frac{1}{r_2} r_1^{\,2}\cos\theta e^{i\bar{p}_i.\bar{r}_2} e^{-\lambda_1 r_1} d^3r_1 d^3r_2
\end{aligned}$$

(12)

and

$$T_{PB} = \frac{1}{162\pi^2}\int e^{-i\bar{p}_1.\bar{r}_1} e^{-i\bar{p}_2.\bar{r}_2}\left(\frac{1}{r_{12}} - \frac{1}{r_2}\right)(6r_1 - r_1^{\,2})\cos\theta e^{i\bar{p}_i.\bar{r}_2} e^{-\lambda_1 r_1} d^3r_1 d^3r_2$$

$$T_{PB} = \frac{1}{162\pi^2}\int e^{-i\bar{p}_1.\bar{r}_1} e^{-i\bar{p}_2.\bar{r}_2}\,\frac{1}{r_{12}} 6r_1\cos\theta e^{i\bar{p}_i.\bar{r}_2} e^{-\lambda_1 r_1} d^3r_1 d^3r_2$$

$$-\frac{1}{162\pi^2}\int e^{-i\bar{p}_1.\bar{r}_1}e^{-i\bar{p}_2.\bar{r}_2}\frac{1}{r_2}6r_1\cos\theta e^{i\bar{p}_i.\bar{r}_2}e^{-\lambda_1 r_1}d^3r_1d^3r_2$$
$$-\frac{1}{162\pi^2}\int e^{-i\bar{p}_1.\bar{r}_1}e^{-i\bar{p}_2.\bar{r}_2}\frac{1}{r_{12}}r_1^2\cos\theta e^{i\bar{p}_i.\bar{r}_2}e^{-\lambda_1 r_1}d^3r_1d^3r_2$$
$$+\frac{1}{162\pi^2}\int e^{-i\bar{p}_1.\bar{r}_1}e^{-i\bar{p}_2.\bar{r}_2}\frac{1}{r_2}r_1^2\cos\theta e^{i\bar{p}_i.\bar{r}_2}e^{-\lambda_1 r_1}d^3r_1d^3r_2$$

(13)

The final transition matrix element [4] is written as

$$\frac{d^3\sigma}{d\Omega_1 d\Omega_2 dE_1}=\frac{p_1p_2}{p_i}\left|T_{fi}\right|^2 \tag{14}$$

where E_1 is the energy of the incident electron.

Using the Lewis integral [31], we have calculated T_{fi} analytically and then computed numerically for TDCS.

III. Results And Discussions

The triple differential cross sections (TDCS) for ionization of metastable 3P-state hydrogen atoms by electrons are presented here for scattering in a plane. Ionization of hydrogen atoms by electrons from the ground state theoretical results of Dal et al. [12], the BBK model of Brauner et al. [27] and the absolute data [24] are included here for comparison. Also the previous works on hydrogenic 2S-state [14] and 2P-state [18] ionization results is noted here for comparison. In the present calculation we have considered the TDCS for the ionization of metastable 3P-state hydrogen atoms by electrons for the incident electron energy E_i =250eV, the ejected electron energies E_1 =5eV and 50eV and the different scattering angles $\theta_2=3^0$ (Fig.1), $\theta_2=15^0$ (Fig.2), $\theta_2=25^0$ (Fig.3), $\theta_2=5^0$ (Fig.4), $\theta_2=7^0$ (Fig.5), $\theta_2=9^0$ (Fig.6), $\theta_2=11^0$ (Fig.7), $\theta_2=15^0$ (Fig.8), $\theta_2=20^0$ (Fig.9). In all figures the region for $\theta_1(0^0-150^0)$ and $\phi=0^0$, refers to the recoil region, while $\theta_1(150^0-360^0)$ and $\phi=180^0$ refers to the binary region. We have considered here θ varies from 0^0 to 360^0.

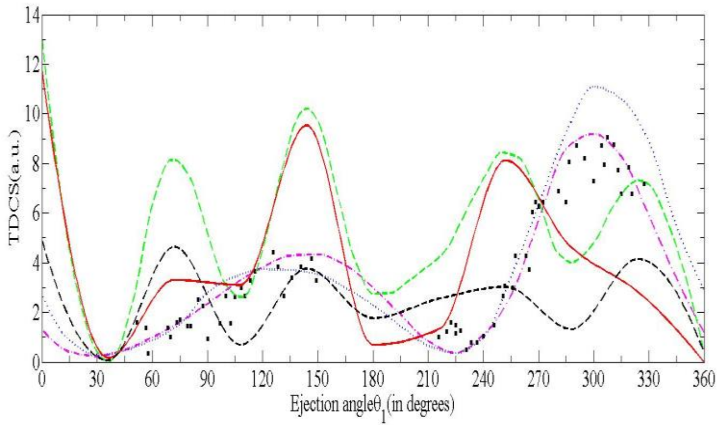

Fig. 1. Triple-differential cross sections (TDCS) for ionization of atomic hydrogen by 250 eV electron impact for $\theta_2=3^0$ vary against the ejected electron θ_1 relative to the incident electron direction. The ejected electron energy is $E_1=5eV$. Theory: full curve (red): present results; dash curve (black): present first Born results; dash curve (green): hydrogenic 2P-state results [18]; short dash curve (blue):hydrogenic ground state 2nd Born results [12]; dash-dotted curve (magenta): hydrogenic ground state BBK model [27] and square: hydrogenic ground state experiments [24] (multiplied by 0.88).

In **Fig. 1** a qualitative comparison among the present results with the hydrogenic ground state results of the BBK model [27], the earlier works on hydrogenic 2P-state result [18], the hydrogenic ground state experimental data [24] and first born results are shown. The peak values of the present results and first born results show good qualitative agreement with those of the compared results in the recoil region but show somewhat disagreement in the binary region. This may be happened because of change of the hydrogenic metastable states by electrons. Here in the recoil region the peak values of present and first born and 2P-state [18] results are about double results of the other compared results. The binary peak height of the present results shifted somewhat left from the other compared result.

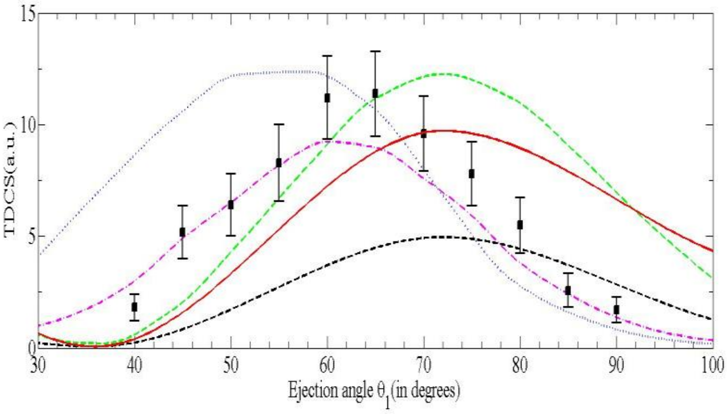

Fig. 2. Triple-differential cross sections (TDCS) for ionization of atomic hydrogen by 250 eV electron impact for $\theta_2 = 15^0$ vary against the ejected electron angle θ_1 relative to the incident electron direction. The ejected electron energy is $E_1 = 50eV$. Theory: full curve(red): present results; dash curve(black): present first Born results; dash curve(green): hydrogenic 2P-state results [18]; short dash curve(blue):hydrogenic ground state 2nd Born results [12]; dash-dotted curve(magenta): hydrogenic ground state BBK model [27] and square: hydrogenic ground state experiments [24] (multiplied by 0.00224).

In **Fig. 2** the peak value of present and first born results are lower than the hydrogenic ground state experimental results [24] and hydrogenic metastable 2P state [18]. Also the present peak values shifted slightly to the higher ejected angle near about θ_1=72° . The peak pattern of the present result shows exactly similar behavior as the BBK model [27] with slight shift.

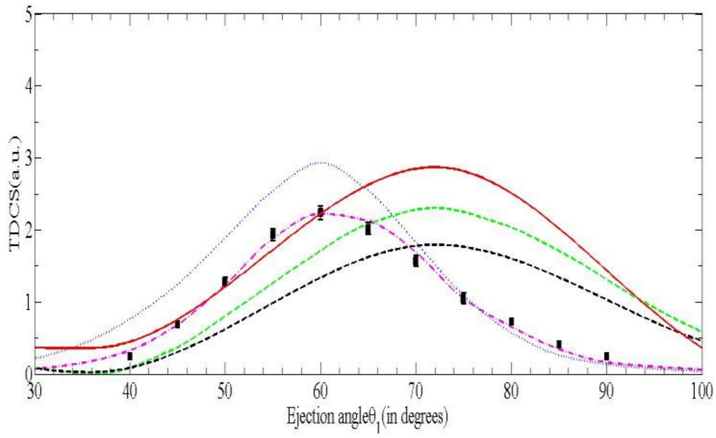

Fig. 3. Triple-differential cross sections (TDCS) for ionization of atomic hydrogen by 250eV electron impact for $\theta_2 = 25^0$ vary against the ejected electron angle θ_1 relative to the incident electron direction. The ejected electron energy is E_1 = 50eV. Theory: full curve(red): present results; dash curve(black): present first Born results; dash curve (green): hydrogenic 2P-state results [18]; short dash curve (blue):hydrogenic ground state 2nd Born results [12]; dash-dotted curve (magenta): hydrogenic ground state BBK model [27] and square: hydrogenic ground state experiments [24] (multiplied by 0.00224).

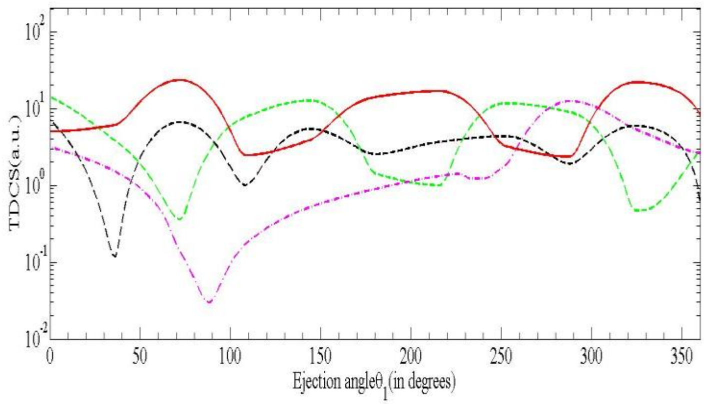

Fig. 4. Triple-differential cross sections (TDCS) for ionization of atomic hydrogen by 250 eV electron impact for $\theta_2 = 5^0$ vary against the ejected electron angle θ_1 relative to the incident electron direction. The ejected electron energy is E_1 = 5 eV. Theory: full curve (red): present results; dash curve (black): present first Born result; dash curve (green): hydrogenic 2P-state results [18]; dash dotted curve (magenta): hydrogenic 2S-state results [14].

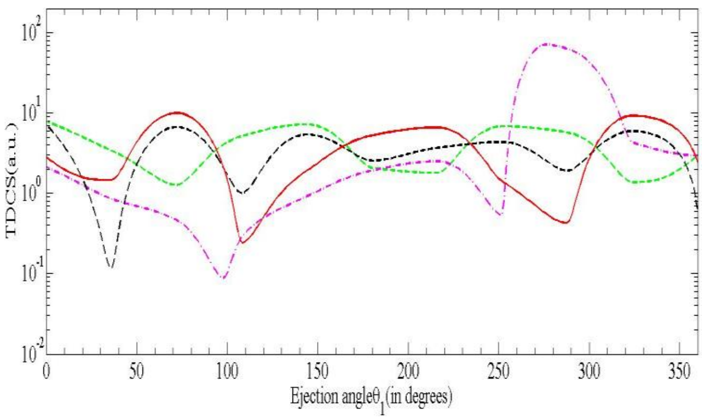

Fig. 5. Triple-differential cross sections (TDCS) for ionization of atomic hydrogen by 250 eV electron impact for $\theta_2 = 7^0$ vary against the ejected electron angle θ_1 relative to the incident electron direction. The ejected electron energy is E_1 = 5 eV. Theory: full curve (red): present results; dash curve (black): present first Born result; dash curve (green): hydrogenic 2P-state results [18]; dash dotted curve (magenta): hydrogenic 2S-state results [14].

In **Fig. 3** the present peak magnitude is the highest among all other compared results [12,18,24,27] representing almost similar peak position as Fig 2.

For $\theta_2 = 5^0$ and $\theta_2 = 7^0$ (Figs.4 and 5) the recoil peak values show qualitative agreement with 2S [14] and 2P-state [18] of hydrogen atoms from electron impact, whereas there arise somewhat opposite binary peaks.

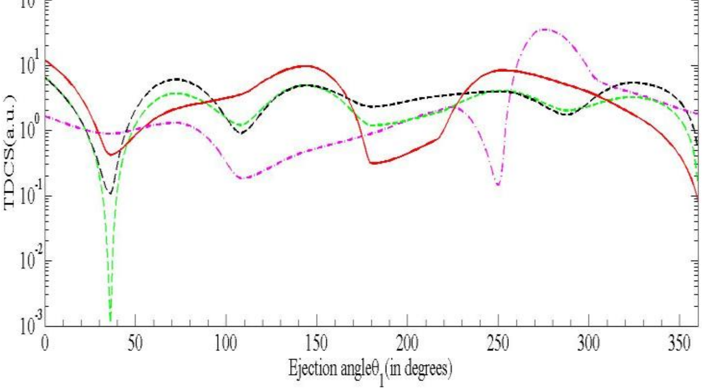

Fig. 6. Triple-differential cross sections (TDCS) for ionization of atomic hydrogen by 250eV electron impact for $\theta_2 = 9^0$ vary against the ejected electron angle θ_1 relative to the incident electron direction. The ejected electron energy is E_1 = 5eV. Theory: full curve (red): present results; dash curve (black): present first Born result; dash curve (green): hydrogenic 2P-state results [18]; dash dotted curve (magenta): hydrogenic 2S-state results [14].

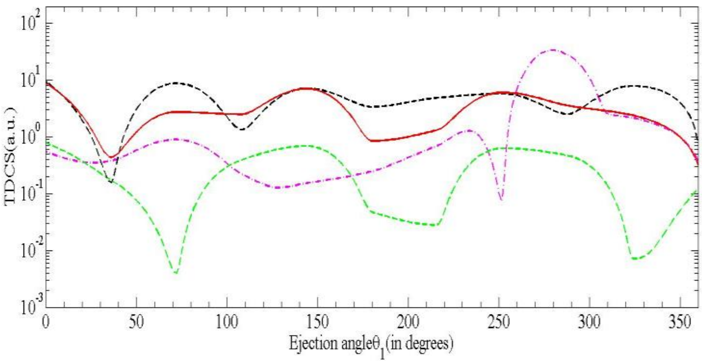

Fig. 7. Triple-differential cross sections (TDCS) for ionization of atomic hydrogen by 250eV electron impact for $\theta_2 = 11^0$ vary against the ejected electron angle θ_1 relative to the incident electron direction. The ejected electron energy is E_1 = 5eV.Theory: full curve (red): present results; dash curve (black): present first Born result; dash curve (green): hydrogenic 2P-state results [18]; dash dotted curve (magenta): hydrogenic 2S-state results [14].

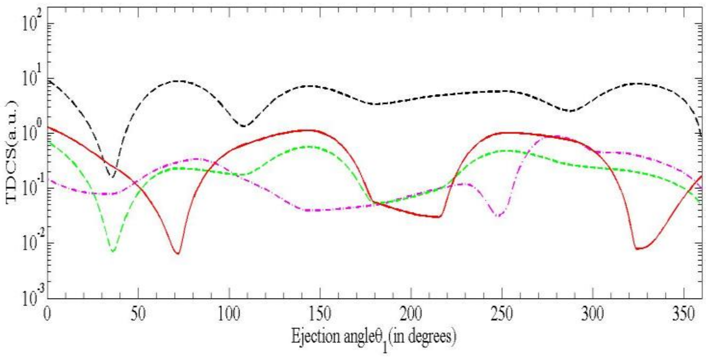

Fig. 8. Triple-differential cross sections (TDCS) for ionization of atomic hydrogen by 250eV electron impact for $\theta_2 = 15^0$ vary against the ejected electron angle θ_1 relative to the incident electron direction. The ejected electron energy is E_1 = 5eV. Theory: full curve (red): present results; dash curve (black): present first Born result; dash curve (green): hydrogenic 2P-state results [18]; dash dotted curve (magenta): hydrogenic 2S-state results [14].

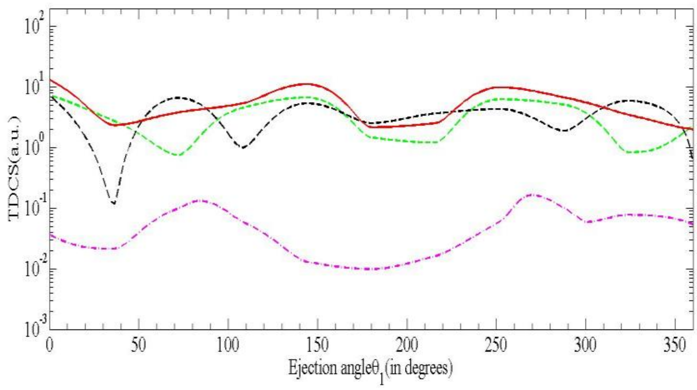

Fig. 9. Triple-differential cross sections (TDCS) for ionization of atomic hydrogen by 250eV electron impact for $\theta_2 = 20^0$ vary against the ejected electron angle θ_1 relative to the incident electron direction. The ejected electron energy is E_1 = 5eV. Theory: full curve (red): present results; dash curve (black): present first Born result; dash curve (green): hydrogenic 2P-state results [18]; dash dotted curve (magenta): hydrogenic 2S-state results [14].

Fig. 6 exhibits a deep lobed structure in recoil region almost at θ_1=36° whereas the present result form a short lobed structure in the same region with same ejection of the hydrogenic electron. But in the binary region, the present and first born results represent distinct peak pattern comparing with the 2S-state [14] and 2P- state [18] hydrogenic results.
We note that the present and present first born results (**Fig. 7**) appear with greater peak magnitude than the compared results [12,18,24,27], both in recoil and binary regions.
Fig. 8 shows that the peak values of the present result remain almost same in magnitude as the hydrogenic 2S-state[14] and 2P- state results[18] , whereas the first born result is increased simultaneously with the increase of the scattering angles (θ_2).
In **Fig. 9** the present and first born results provide exactly similar behavior as the 2P-state[18] results but show a gross difference with the results of 2S- state [14] both in recoil and binary region.
Finally a scattering mechanism for the ionization of metastable 3P-state collision of 250eV electron energy is given here. The first Born term of equation (4), i.e., $\phi^{(-)}_{\bar{p}_1}(\bar{r}_1)e^{i\bar{p}_2\cdot\bar{r}_2}$ are defined by a plane wave whereas the ejected electrons are defined by a Coulomb wave. For the second term, i.e., $\phi^{(-)}_{\bar{p}_2}(\bar{r}_2)e^{i\bar{p}_1\cdot\bar{r}_1}$, the scattered electrons are defined by a Coulomb wave while the ejected electrons are defined by a plane wave. In third term, i.e., $\phi^{(-)}_{\bar{p}}(\bar{r})e^{i\bar{P}\cdot\bar{R}}$, the projectile electron interaction shows almost similar behavior in the final channel in which the center of mass goes as a plane wave. The fourth term, i.e., $e^{i\bar{p}_1\cdot\bar{r}_1+i\bar{p}_2\cdot\bar{r}_2}$, represents two plane waves for the ejected and scattered particles. All these results offer good scope for the experimental investigation of these problems and offer a new test for different theories of ionization.
A table of comparison results for ionization of hydrogenic 2S-state, 2P-state and 3P-state atoms by electron is given here.

Table 1. Triple differential cross sections (TDCS) for ionization of atomic hydrogen atoms by electron impact at metastable 3P-state are obtained by using equation (14). The incident energy is 250eV, the scattering angle is $\theta_2 = 9^0$ and the ejected electron energy is $E_1 = 5eV$. In the given table we present 3P-state results and compared 2P-state and 2S-state results.

Ejected angle(θ_1)	2S	2P	3P
0	1.6501	6.3641	11.8679
36	0.9001	0.0010	0.4252
72	1.3875	3.7321	2.1485
108	0.1952	1.2291	3.5834
144	0.5401	4.9771	9.7624
180	0.8989	1.2029	0.3155
216	2.3450	1.8691	0.7248
252	0.3201	4.1191	8.4297
288	110.00	2.0473	5.0455
324	4.1567	3.2867	1.7845
360	1.7890	0.1475	0.0850

IV. Conclusion

In the present study are have calculated the triple differential cross sections (TDCS) for ionization of hydrogen atoms by 250eV energy in the metastable 3P state following a multiple scattering theory of Das and Seal [4] . We interestingly notice that the implementation of the final state wave function $\psi_f^{()}$ of Das and Seal yields good qualitative agreement with hydrogenic ground state as well as metastable 2S and 2P states [14, 18] result for qualitative enhancement, the present computational findings are encouraging for the future experiments which may play a vital role to provide more interesting and significant results in this area of research. There are wide scopes for improving the wave function of Das and Seal and for applying it to various ionization problems.

Acknowledgements

The computational work was performed in the Simulation Lab of the Department of Mathematics, Chittagong University of Engineering and Technology, Chittagong-4349, Bangladesh.

References

[1]. W F Byron, J C Joachain and B Piraux, Triple differential cross sections for the ionisation of atomic hydrogen by fast electrons: a second Born treatment. Journal of Physics B: Atomic, Molecular and Optical Physic, 13, 1980, L673
[2]. W F Byron, J C Joachain and B Piraux, Theory of coplanar asymmetric (e, 2e) reactions in helium, Journal of Physics B: Atomic, Molecular and Optical Physics, 19, 1986, 1201
[3]. J N Das, Momentum-Space Analysis of Scattering States with Possible Application to Atomic Ionization, Physical Review A, 42, 1990, 1376.
[4]. J N Das and S Seal, Electron-Hydrogen-Atom Ionization Collisions at Intermediate ($5I_0$-$20I_0$) and High ($\geq 20I_0$) Energies, Physical Review A, 47, 1993 2978.
[5]. J N Das and S Seal, Symmetric Scattering In $e^{\pm}$-H Ionization Collisions, Pramana, 40, 1993 253-258.
[6]. J N Das and S Dhar, Energy Spectrum of Ejected Electrons in Ionization of Hydrogen Atoms by Electrons, Pramana, 53, 1999, 869-875.
[7]. S Jones and D H Madison , Ionization of hydrogen atoms by fast electrons, Physical Review A, 62, 2000, 042701.
[8]. S Jones and D H Madison, Scaling behavior of the fully differential cross section for ionization of hydrogen atoms by the impact of fast elementary charged particles, Physical Review A, 65, 2002, 052727.
[9]. S Jones and D H Madison, Physical Review Lett. 81, 1998, 2886.
[10]. S Ghosh Deb, S Roy and C Sinha, Multiphoton (e, 2e) Process of Hydrogen Atom in Strong Laser Field, The European Physical Journal D, 55, 2009, 591-600.
[11]. S Vučič, RM Potvliege and C J Joachain, Second Born Triple-Differential Cross Sections for the Coplanar Asymmetric Ionization of H(2S) by Fast Electrons, Physical Review A, 35, 1987,1446.
[12]. C Dal Cappello, A Haddadou, F Menas, and A C Roy, The second Born approximation for the single and double ionization of atoms by electrons and positrons, Journal of Physics B: Atomic, Molecular and Optical Physics, 44, 2011, 015204.
[13]. J N Das and S Dhar, Symmetric Scattering in Electron and Positron Impact Ionization of Metastable 2S-State Hydrogen Atoms, Pramana, 47, 1996,263-269.
[14]. S Dhar, Electron Impact Ionisation of Metastable 2S-State Hydrogen Atoms, Australian Journal of Physics, 49, 1996, 937.
[15]. J N Das and S Dhar, Energy Spectrum of Scattered Electrons in K-Shell Ionization of Medium to Heavy Atoms by Relativistic Electrons, Journal of Physics B: Atomic, Molecular and Optical Physics, 31, 1998a , 2355.
[16]. J N Das and S Dhar, Energy spectrum of scattered electrons in K-shell ionization of medium to heavy atoms by relativistic electrons, Journal of Physics B: Atomic, Molecular and Optical Physics, 31, 1998, 2355.
[17]. Y Y Qi, L N Ning, J G Wang and Y Z Qu , Plasma effect on fast-electron-impact ionization from 2p state of hydrogen-like ions, Physics of Plasmas, 20, 2013,123301.
[18]. S Dhar and N Nahar , Electron impact ionization of metastable 2P-state hydrogen atoms in the coplanar geometry, Results in Physics, 5, 2015, 3-8.
[19]. S Dhar and N Nahar , Ionization of Metastable 2P-State Hydrogen Atoms by Electron Impact for Coplanar Asymmetric Geometry, Open Journal of Microphysics, 4, 2014,46-53.
[20]. S Ghosh, A Biswas and C Sinha, Multiphoton Ionization of a Metastable Hydrogen Atom by Electron and Positron Impact and Charge Asymmetry, Journal of Physics B: Atomic, Molecular and Optical Physics, 44, 2011, Article ID:215201
[21]. M Dörr et al, Higher-order contributions observed in three-dimensional (e, 2e) cross-section measurements at 1-keV impact energy, Physical Review A, 77, 2008, 032717.
[22]. X Ren, A Dorn and J Ullrich, Coulomb Four-Body Problem: Electron-Impact Double Ionization of Helium in the Threshold Regime, Physical Review Letters, 101, 2008, 093201.
[23]. M Dürr, B Najjari, M Schulz, A B Voitkiv and J Ullrich , Analysis of experimental data for ion-impact single ionization of helium with Monte Carlo event generators based on quantum theory. Physical Review A ,75, 2007, 062708.
[24]. H Ehrhardt, K Jung, G Knoth, and P Schlemmer, Differential Cross Sections of Direct Single Electron Impact Ionization, Zeitschrift für Physik D Atoms, Molecules and Clusters, 1, 1986, 3.
[25]. H Ehrhardt, G Knoth, P Schlemmer and K Jung, Absolute H(e, 2e)p cross section measurements: Comparison with first and second order theory, Physics Letters A **,** 110, 1985, 92-94.
[26]. M Brauner and J S Briggs, Ionisation to the projectile continuum by positron and electron collisions with neutral atoms, Journal of Physics B: Atomic, Molecular and Optical Physics, 19, 1986, L325.
[27]. M Brauner, J S Briggs and H Klar, Triply-differential cross sections for ionisation of Hydrogen atoms by electrons and positrons, Journal of Physics B: Atomic, Molecular and Optical Physics, 22, 1989, 2265.
[28]. M Brauner, J S Briggs and H Klar, Structures in differential cross sections for positron impact ionization of hydrogen, Journal of Physics B: Atomic, Molecular and Optical Physics, 24, 1991, 2227.
[29]. A Köver and G Laricchia , Triply Differential Study of Positron Impact Ionization of H2, Physical Review Letters, 80, 1998, 5309.
[30]. J Berakder and H Klar, Structures in triply and doubly differential ionization cross sections of atomic hydrogen, Journal of Physics B: Atomic, Molecular and Optical Physics. 26, 1993, 3891.
[31]. R R Lewis, Potential scattering of high-energy electrons in second born approximation, Physical Review, 102, 1956, 537.

S.Dhar. “Triple Differential Cross Sections for Ionization of Metastable 3P-State Hydrogen Atoms by Electrons.” IOSR Journal of Applied Physics (IOSR-JAP) , vol. 9, no. 5, 2017, pp. 21–29.