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Time Dependence of Density Parameters for a Variable EoS Parameter in Brans-Dicke Framework

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Abstract: *The time dependence of the equation-of-state (EoS) parameter has been determined in the framework of the Brans-Dicke (BD) theory, using BD field equations and the wave equation for the scalar field, for a homogeneous, isotropic and spatially flat universe. This study is based on a model where a linear combination of BD field equations has been taken. The generalized expression of the EoS parameter is a function of the ratio of the constant coefficients used for this linear combination. Expressions of the densities of matter and dark energy and also the density parameters for matter and dark energy have been derived in terms of the EoS parameter. The value of the EoS parameter, at the present epoch, is found to depend, almost solely, on a constant that determines the time dependence of the scalar field. This parameter and the ratio of constant coefficients have been varied, to show graphically, the time dependence of the matter density and the density parameters for matter and dark energy. For certain combinations of these parameters, the matter density is found to decrease with time after initially rising to a peak value, indicating generation of matter, probably from dark energy, during an era in the past.*

Keywords: *Equation-of-State Parameter, Brans-Dicke Theory, Scalar Field, Brans-Dicke Parameter, Density parameters, Dark Energy, Cosmology.*

I. Introduction

The universe is found to be highly homogeneous and isotropic on large scales, as obtained from observations on large scale structure and cosmic microwave background radiation [1, 2]. On the basis of several observations the idea of an accelerating universe has emerged in the recent years [3]. One of the major aims of research in this regard is to find the true nature of a strange type of repulsive force, causing the accelerated expansion, which is said to be generated by an entity named dark energy (DE). One does not have a concrete understanding about the true nature of DE which is known to have a constant or a slightly changing energy density as the universe expands [4]. Dark energy has been conventionally characterized by the equation of state (EoS) parameter ($\gamma \equiv P/\rho$), which should not be regarded as a constant. We find $-1.67 < \gamma < -0.62$ from observational results obtained from SN Ia data [5]. One should not essentially treat γ as a constant. On account of insufficient observational evidence to estimate the time variation of γ, the EoS parameter has been regarded as a constant in many theoretical studies, with values -1, 0, 1/3 and +1 for vacuum fluid, dust fluid, radiation and stiff fluid dominated universe respectively [6]. In general, the EoS parameter is a function of time or redshift [7]. Recent years have witnessed the emergence of various models on the time dependence of γ [8]. Ray *et al* has recently studied variable EoS parameter on the basis of generalized dark energy models [9, 10].

The accelerated expansion of the universe has been convincingly proved by the analysis of data obtained from high precision astrophysical observations [11]. This accelerated expansion is said to be caused by a strange entity, named dark energy (DE). The exact nature of this DE is yet to be determined. A parameter, known as cosmological constant (Λ) in General Relativity (GR), has very often been used to represent dark energy in theoretical calculations. It has its own shortcomings, although it has explained several experimental observations satisfactorily [4]. To account for gravitational observations, various alternative theoretical models have been proposed. One may find ample information regarding the strengths and weaknesses of these models in scientific literature [12]. Among the non-minimally coupled scalar field theories, the Brans-Dicke (BD) theory of gravity has been found to be highly useful in explaining the observations of accelerated expansion [13]. In the prediction of observational findings, the dimensionless parameter ω in BD theory plays a very significant role [6, 14]. BD theory has several models where a small value of ω, typically of the order of unity, leads to accelerated expansion [6, 13, 14]. The Brans-Dicke scalar field alone has been found to predict an accelerated expansion, in the present matter dominated era of the universe, without taking into account the presence of any quintessence matter or any interaction between the dark matter and the BD field [15]. A generalized version of Brans-Dicke theory was proposed by Bergman and Wagoner [16]. Nordtvedt proposed a more useful form of this theory which can predict these observations [17, 18]. In the generalized form of BD theory, the dimensionless BD parameter (ω) is regarded as a function of the scalar field (φ), and thus it should be regarded as a function of time [18].

From the field equations of the Brans-Dicke theory of gravity for flat space and also the wave equation for the scalar field (φ), the time dependence of the equation of state parameter (γ) has been determined in the present study. For this purpose, empirical expressions for the scale factor, scalar field and the BD parameter have been used. The scale factor has been chosen in a manner such that it generates a time dependent deceleration parameter that changes sign with time, from positive to negative, indicating a transition of the expanding universe from a phase of deceleration to acceleration, as per several recent observations [13, 18]. As a step towards generalization, a linear combination of the two field equations has been taken and the ratio of the corresponding constant coefficients has been varied to see its effect on the time dependence of the EoS parameter. The value of EoS parameter at the present epoch (γ_0) has been found to depend on a constant parameter which controls the variation of the scalar field as a function of scale factor. Expressions of matter and dark energy densities, and the density parameters for matter and dark energy, have been formulated in terms of the time dependent EoS parameter. The time dependence of matter density and the density parameters have been shown graphically. The density of matter is found to increase with time initially for certain combinations of the empirical parameters in the present model. Unlike other recent studies [9, 10], a novel aspect of the present study is that, the entire formulation is in the BD framework and without assuming anisotropy of space.

II. Solution Of Field Equations

The field equations of generalized Brans-Dicke theory, obtained from FRW space-time, for a space of curvature k, are expressed as,

$$3\frac{\dot{a}^2+k}{a^2}+3\frac{\dot{a}}{a}\frac{\dot{\varphi}}{\varphi}-\frac{\omega(\varphi)}{2}\frac{\dot{\varphi}^2}{\varphi^2}=\frac{\rho}{\varphi} \qquad (01)$$

$$2\frac{\ddot{a}}{a}+\frac{\dot{a}^2+k}{a^2}+\frac{\omega(\varphi)}{2}\frac{\dot{\varphi}^2}{\varphi^2}+2\frac{\dot{a}}{a}\frac{\dot{\varphi}}{\varphi}+\frac{\ddot{\varphi}}{\varphi}=-\frac{\gamma\rho}{\varphi} \qquad (02)$$

The wave equation for the scalar field (φ) is expressed as,

$$\ddot{\varphi}+3\frac{\dot{a}\dot{\varphi}}{a}=\frac{\rho-3P}{2\omega+3}-\frac{\dot{\omega}\dot{\varphi}}{2\omega+3} \qquad (03)$$

Combining equations (1), (2) and (3), one obtains,

$$\dot{\rho}+3\frac{\dot{a}}{a}(\rho+P)=0 \qquad (04)$$

The equation of state of the cosmic fluid is $P=\gamma\rho$, where P and ρ are the pressure and the density of matter (dark + baryonic) respectively and γ is the equation of state (EoS) parameter. Solution of equation (4), considering a constant value of γ, is given by,

$$\rho=\rho_0 a^{-3(1+\gamma)} \qquad (05)$$

A new differential equation can be formed by taking a linear combination of the equations (1) and (2). Adding these two equations, after multiplying them by two constants, *x* and *y* respectively, one gets the following expression of the EoS parameter (γ).

$$\gamma=\frac{1}{y}\left[x-\frac{\varphi}{\rho}\left\{2y\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}(3x+y)+\frac{\omega\dot{\varphi}^2}{2\varphi^2}(y-x)+\frac{\dot{a}\dot{\varphi}}{a\varphi}(3x+2y)+y\frac{\ddot{\varphi}}{\varphi}\right\}\right] \qquad (06)$$

Using the relation $P=\gamma\rho$ in equation (3) one gets,

$$\frac{\varphi}{\rho}=\frac{1-3\gamma}{2\omega+3}\left(\frac{\ddot{\varphi}}{\varphi}+3\frac{\dot{a}\dot{\varphi}}{a\varphi}+\frac{\dot{\omega}}{2\omega+3}\frac{\dot{\varphi}}{\varphi}\right)^{-1} \qquad (07)$$

Substituting for $\frac{\varphi}{\rho}$ in equation (6) from equation (7) one gets,

$$\gamma=\frac{\frac{x}{y}-\frac{1}{y(2\omega+3)}\frac{f_1}{f_2}}{1-\frac{3}{y(2\omega+3)}\frac{f_1}{f_2}} \qquad (08)$$

where,

$f_1 = 2y\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}(3x+y) + \frac{\omega\dot{\varphi}^2}{2\varphi^2}(y-x) + \frac{\dot{a}\dot{\varphi}}{a\varphi}(3x+2y) + y\frac{\ddot{\varphi}}{\varphi}$

$f_2 = \frac{\ddot{\varphi}}{\varphi} + 3\frac{\dot{a}\dot{\varphi}}{a\varphi} + \frac{\dot{\omega}}{2\omega+3}\frac{\dot{\varphi}}{\varphi}$

Equation (8) may be regarded as the general expression of the EoS parameter (γ) in BD theory. To determine its time dependence, following empirical expressions have been used.

$$a = a_0\, Exp\left[\alpha\left(t^{\beta} - {t_0}^{\beta}\right)\right] \tag{09}$$

$$\varphi = \varphi_0\left(\frac{a}{a_0}\right)^n \tag{10}$$

$$\omega = \omega_0\left(\frac{\varphi}{\varphi_0}\right)^m \tag{11}$$

The scale factor (eqn. 9) has been so chosen that the deceleration parameter changes sign from positive to negative, as per recent studies, indicating a change of phase from decelerated expansion to accelerated expansion [13]. We have taken $a_0 = 1$ for all calculations.
Here α, β should have the same sign to ensure an increase of the scale factor with time. Using equation (9), the Hubble parameter (H) and the deceleration parameter (q) are obtained as,

$$H = \frac{\dot{a}}{a} = \alpha\,\beta t^{(\beta-1)} \tag{12}$$

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = -1 + \frac{1-\beta}{\alpha\,\beta}\; t^{-\beta} \tag{13}$$

For $0 < \beta < 1$ and $\alpha > 0$ one finds that, $q > 0$ at $t = 0$ and $q \to -1$ as $t \to \infty$, showing a signature flip of q. Taking $H = H_0$ and $q = q_0$, at $t = t_0$, one gets,

$$\alpha = \frac{H_0}{1 - H_0 t_0(1+q_0)}\, {t_0}^{\{H_0 t_0(1+q_0)\}} \tag{14}$$

$$\beta = 1 - H_0 t_0(1+q_0) \tag{15}$$

The scalar field (eqn. 10) has been chosen on the basis of some previous studies on Brans-Dicke theory of gravitation [13, 18]. The empirical expression of BD parameter (eqn. 11) has been chosen according to the generalized Brans-Dicke theory, where ω is regarded as a function of the scalar field (φ) [13]. The values of ω_0 and m have to be determined from the field equations.
Using equation (10) along with the relation $G = 1/\varphi$ one obtains,

$$n = -\frac{1}{H_0}\left(\frac{\dot{G}}{G}\right)_{t=t_0} \tag{16}$$

With the help of experimental observations regarding H_0 and $\left(\frac{\dot{G}}{G}\right)_{t=t_0}$, the parameter n can be determined from equation (16). Experimental observations regarding $\left(\frac{\dot{G}}{G}\right)_{t=t_0}$, as obtained by many researchers, have been found to be both positive and negative [19].
According to S. Weinberg, we must have, $\left|\left(\frac{\dot{G}}{G}\right)_{t=t_0}\right| \leq 4\times10^{-10}Yr^{-1}$ [20]. Using equation (16), this requirement can be expressed as,

$$|n| \leq \frac{4\times10^{-10}\,Yr^{-1}}{H_0} \text{ or, } |n| \leq 5.44 \qquad (\text{taking } H_0 = 7.348\times10^{-11}Yr^{-1}) \tag{17}$$

An expression of ω_0, determined from equation (1), taking $k = 0$, is given by,

$$\omega_0 = \frac{6}{n^2}\left\{1 + n - \frac{\rho_0}{3\varphi_0 {H_0}^2}\right\} \tag{18}$$

Combining equations (2), (3), (10) and (11) and writing all parameter values for $t = t_0$ one gets,

$$m = \frac{\rho_0/\varphi_0 + {H_0}^2\{q_0(2n\omega_0 - 6) + 3(1-n) - 0.5\omega_0 n^2 - 4n\omega_0\}}{\omega_0 n^2 {H_0}^2} \tag{19}$$

The values of n and ω_0 in equation (19) can be obtained from equations (16) and (18).
Combining equations (9), (10) and (11) the time dependence of BD parameter is obtained as,

$$\omega = \omega_0\, Exp\left[mn\alpha\left(t^{\beta} - {t_0}^{\beta}\right)\right] \qquad (20)$$

The values of n, ω_0 and m in equation (20) can be obtained from equations (16), (18) and (19) respectively.
Combining equations (9) and (10), the scalar field (φ) can be expressed as,

$$\varphi = \varphi_0\, Exp\left[n\alpha\left(t^{\beta} - {t_0}^{\beta}\right)\right] \qquad (21)$$

In equations (20) and (21), the values of α and β are obtained from equations (14), (15) respectively.
Using equations (10) and (11) in the generalized γ expression (eqn. 8), one obtains,

$$\gamma = \frac{r - \frac{f}{2\omega+3}}{1 - \frac{3f}{2\omega+3}} \qquad (22)$$

where, $r = \frac{x}{y}$ and $f = \frac{-2q+3r+1+0.5\omega n^2(1-r)+n(3r+2)+n(n-q-1)}{n^2-nq+2n+\frac{m\omega n^2}{2\omega+3}}$

In equation (22), the values of q, n, m and ω should be taken from equations (13), (16), (19) and (20) respectively. Thus, in the present model, the time dependence of the EoS parameter (γ) can be studied from equation (22). The value of γ at $t = t_0$, from equation (22), is,

$$\gamma_0 = \frac{r - \frac{f_0}{2\omega_0+3}}{1 - \frac{3f_0}{2\omega_0+3}} \qquad (23)$$

where, $r = \frac{x}{y}$ and $f_0 = \frac{-2q_0+3r+1+0.5\omega_0 n^2(1-r)+n(3r+2)+n(n-q_0-1)}{n^2-nq_0+2n+\frac{m\omega_0 n^2}{2\omega_0+3}}$

The values of different cosmological parameters used for the present study are given below.

$$H_0 = \frac{72\frac{Km}{s}}{Mpc} = 2.33 \times 10^{-18}\, sec^{-1}, \quad q_0 = -0.55, \quad \rho_0 = 2.83 \times 10^{-27} Kg\, m^{-3}$$
$$\varphi_0 = \frac{1}{G_0} = 1.498 \times 10^{10} Kg^2\, m^{-2} N^{-1}, \quad t_0 = 4.36 \times 10^{17}\, s$$

Using equation (19) in (23) and using the above values, the expression of γ_0 becomes

$$\gamma_0 = \frac{-1.6884 + 5.0652\, r + n\{-1.51667 + 4.55\, r + n(r - 1/3)\}}{0.0115996 - 0.0347987\, r} \quad \text{where, } r = \frac{x}{y} \qquad (24)$$

It is found from equation (24) that γ_0 depends much more on the value of n than r. However, the time variation of γ, obtained from equation (22) is controlled by both n and r.
Combining equation (10) with equation (6), one gets the following expression of the density of matter.

$$\rho = \frac{\varphi H^2}{r-\gamma}\left[-2q + 3r + 1 + 0.5\omega n^2(1-r) + n(3r + 2 + n - q - 1)\right] \qquad (25)$$

In equation (25), the values of H, q, n, ω, φ and γ can be obtained from equations (12), (13), (16), (20), (21) and (22) respectively.
The density parameters (Ω) for matter (*dark matter* + *baryonic matter*) and dark energy of the universe are given by,

$$\Omega_m = \frac{\rho}{\rho_c} = \frac{\varphi H^2}{\rho_c(r-\gamma)}\left[-2q + 3r + 1 + 0.5\omega n^2(1-r) + n(3r + 2 + n - q - 1)\right] \qquad (26)$$

and $\Omega_d = \frac{\rho_d}{\rho_c} \approx 1 - \Omega_m$ (near the present epoch)

or, $\Omega_d \approx 1 - \frac{\varphi H^2}{\rho_c(r-\gamma)}\left[-2q + 3r + 1 + 0.5\omega n^2(1-r) + n(3r + 2 + n - q - 1)\right]$ (27)

Here, ρ and ρ_d denote the densities of matter and dark energy of the universe respectively.
From the above equations, the density of dark energy (ρ_d) can be expressed as,

$$\rho_d = \rho_c \Omega_d \approx \rho_c - \frac{\varphi H^2}{r-\gamma}[-2q + 3r + 1 + 0.5\omega n^2(1-r) + n(3r + 2 + n - q - 1)] \qquad (28)$$

Equations (26) and (27) give us approximate estimates of relative proportions of matter and dark energy respectively, with respect to the total content of matter-energy of the universe.

III. Results

One obtains from equation (24) that, $\gamma_0 = 1.58 \times 10^{-9}$ for $n = -1.942701081$. This value of γ_0 can be effectively taken to be zero. The value of γ_0, for the matter dominated universe, has been taken to be zero in several studies [6, 14]. Here we have $\gamma_0 \approx 0$, -0.5, -1.0 and -1.5 for $n = -1.943$, -1.917, -1.894 and -1.872 respectively. Figures 1 and 2 have ρ versus time plots for $r = 2, 3$ respectively, for these four values of n. For the first two values of n, ρ is found to decrease with time monotonically. For the less negative values of n, ρ initially rises to a peak and decreases thereafter. In each of these figures, more negative values of n cause a faster fall of ρ. In figure 1, the curves for $n = -1.894$ and -1.872, attain the peaks earlier than those in figure 2. We have plotted the density parameters for matter and dark energy (Ω_m, Ω_d), as functions of time, for $r = 2, 3$ respectively in figures 3 and 4. In each of these figures, Ω has been plotted for $n = -1.943$, and -1.917 (with $\gamma_0 \approx 0$ and -0.5). Here, Ω_m and Ω_d are found to decrease and increase with time in all plots, showing their present values to be approximately 0.3 and 0.7 respectively. In each of the figures 3 and 4, more negative value of n causes faster change in both Ω_m and Ω_d. It is evident from these two figures that for a larger value of r, the effect of change of n is greater upon the time variation of Ω. It is found from equations (18) and (23) that, positive values of n produces positive values of ω_0 and large negative values of γ_0, contrary to advanced theoretical studies and observations [9, 10, 15]. For this reason we have chosen only negative values of n for the present study.

IV. Conclusions

A linear combination of equations (1) and (2) has been taken in the present study, with x and y as the constant coefficients, leading to equation (6) which may be treated as a generalized field equation. It has the same mathematical validity like the equations (1), (2) and (3), for any choice of values for x and y, in the sense that it is equally expected to be satisfied by the scale factor (a), scalar field (φ) and their time derivatives. This linear combination of two field equations of BD theory may be regarded as a first step towards a generalization to form an effective field equation. This method makes it difficult for us to determine the correct values of x and y to predict the time dependence of the cosmological parameters accurately. These variables have appeared in the expressions of γ and ρ as a ratio, denoted by r ($\equiv \frac{x}{y}$). So, we don't have to vary them separately. Only the ratio needs to be varied to see its effect on the time evolution of the EoS parameter, densities and density parameters for matter and dark energy. It has been found from this formulation that γ_0 depends, almost solely, on the parameter n which determines the dependence of the scalar field upon the scale factor, thereby governing the time dependence of the scalar field, although the nature of time dependence of γ is governed by both n and r. According to a study by Banerjee and Pavon, the approximate range of variation of ω_0 is $-1.5 < \omega_0 < 0$ [15]. It is found from equation (18) that ω_0 is always positive for positive values of n. Therefore, only the negative values of n are likely to predict the cosmic expansion characteristics correctly. We have to choose only those values of n for which γ_0 has a small negative value (close to -1), as per experimental observations [5]. Therefore, we must choose n such that $-1.942701081 \leq n < 0$. This range of n also satisfies the condition expressed by equation (17). One needs more astrophysical observations to determine the value of r. Figures 1 and 2 show that, for the less negative values of n, the density of matter (ρ) initially increases with time and it begins to fall after reaching a peak value prior to the present epoch ($t = t_0$). This behaviour may indicate that if we have $\gamma_0 \leq -1$ approximately, which means that if the present era is found to be dominated by vacuum fluid or phantom energy, there was certainly an era in the early universe when matter was generated from dark energy. One may think of an improvement over this model by taking a linear combination of equations (1), (2) and (3), involving three constant coefficients, which can be regarded as a generalization of the present formulation, with a greater difficulty of determining their correct combination of values. This theoretical model can also be improved by using a scale factor which is a solution of the field equations (eqns. 1 and 2), obtained from them by using the empirical expression of the scalar field (eqn. 10). The time dependence of matter density and density parameters have been determined from the time dependence of the EoS parameter, derived from Brans-Dicke theory, in the present formulation. An unique feature of the present study is that, unlike other recent studies [9, 10], the nature of time dependence of the EoS parameter has been determined, from Brans-Dicke field equations and the wave equation for the scalar field, assuming the space to be absolutely isotropic and homogeneous and without incorporating anything in the theoretical calculations that represents the role of dark energy which is responsible for the accelerated expansion of the universe.

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FIGURES

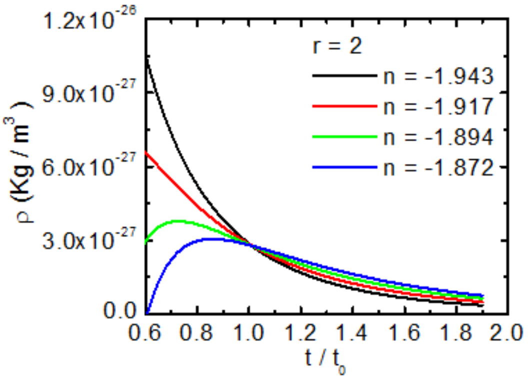

Figure 1: Plot of ρ versus time, for $r = 2$, and four values of n with $\gamma_0 = 0, -0.5, -1.0, -1.5$ respectively.

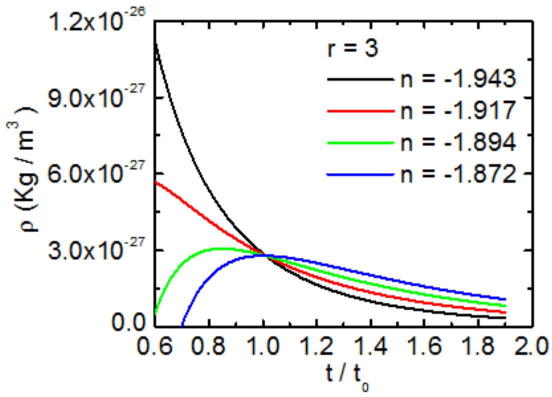

Figure 2: Plot of ρ versus time, for $r = 3$, and four values of n with $\gamma_0 = 0, -0.5, -1.0, -1.5$ respectively.

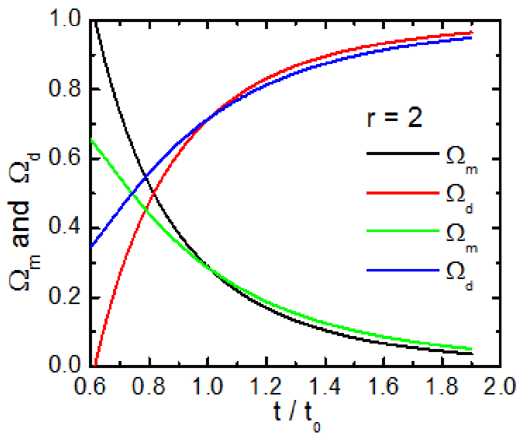

Figure 3: Plot of Ω_m, Ω_d versus time, for $r = 2$. Here $\gamma_0 = 0$ for the black and red curves, and $\gamma_0 = -0.5$ for the green and blue curves.

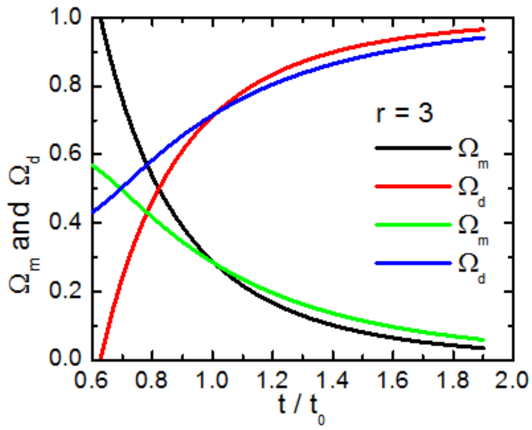

Figure 4: Plot of Ω_m, Ω_d versus time, for $r = 3$. Here $\gamma_0 = 0$ for the black and red curves, and $\gamma_0 = -0.5$ for the green and blue curves.

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