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Grand Unification: Modified Value of CG coefficients For SU (5) Model Using Vacuum Polarization

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Abstract: *The grand unification basically provides unification of strong, hypercharge and weak couplings by the gauge group of standard model, SU(3)⊗SU(2)⊗U(1) having coupling constants for each group and Weinberg angle. In this paper the convergence of coupling constants is achieved using SU(5) model at GeV energy level, although earlier it was turned out that in the SU(5) model, the coupling constants do not come together at one point at energies of GeV scale. The value of Clebsch-Gordon coefficient is modified taking into account the vacuum polarization function; as the running coupling constants of the gauge group SU(3)⊗SU(2)⊗U(1) can be calculated only from vacuum polarization. The obtained value for* $sin^2\theta_W$ *is*

$$sin^2\theta_W = 0.233 + 0.00276/-0.00243$$

and accordingly θ_W *varies from 28.717° to 29.068°. The calculated value of coupling constant is*

$$\alpha_{un} = 0.034016$$

Keywords: *grand unification, vacuum polarization, clebsch-gordon coefficients, weinberg angle, coupling constant after convergence*

I. Introduction

The Standard Model(SM) comprises of the Glashow-Weinberg-Salam (GWS) model and the Quantum Chromodynamics (QCD) thus combining the SU(3) gauge group of QCD with SU(2)⊗U(1) gauge group of electroweak theory. Thus the gauge group of standard model is SU(3)⊗SU(2)⊗U(1). This gauge group is a product of three disconnected groups having running coupling constant for each group. The unification could be realized by the merging of the running coupling constants at energies M_x = 1.5×10^{16} GeV after the convergence of coupling constants. Earlier it was turned out that in the SU(5) model, the convergence of coupling constants can't be achieved at energies of GeV scale. However it can be achieved if supersymmetry is realized at energies above 1 TeV. But here in this paper, the convergence is achieved at GeV scale obtaining the value of Weinberg angle and the coupling constant after unification upto some agreement to its predicted value.

The two major lessons of SM are – symmetry breaking through field condensation (responsible for Higgs mechanism) and running couplings (asymptotic freedom). Asymptotic freedom is not developed from gauge bosons and the only contribution to vacuum polarization comes from fermions. Also, the coupling constant is absent for emergent gauge fields of fermionic quantum vacuum as because gauge bosons cannot exist as free fields which simply means without having fermions around to make the quantum vacuum. Thus entire coupling constant comes from vacuum polarization.

Unification predicts the relationship among the strong, weak and hypercharge couplings. To unify the couplings, vacuum polarization must be included which would give the spectra of virtual fermions. Thus the value of strong coupling constant used here, obtained from vacuum polarization function in three-flavor lattice QCD, is $\alpha_3(M_Z)$ = 0.1181 at Z boson mass scale at renormalization scale set to μ = 2 GeV. Further for the calculation of α_{un}, the QCD scale parameter used is Λ=0.247 GeV. The value of strong coupling constant and thus all others parameters accordingly, used here are based on recent lattice QCD results using vacuum polarization functions (Shintani, E. et al. 2014)

II. Calculation of Clebsch-Gordan Coefficient And Weinberg Angle

If G be the grand unified group having SU(3), SU(2), U(1) as its subgroups , i.e.,

G ⊃ SU(3)⊗SU(2)⊗U(1) (1)

And let g_G be the single coupling constant corresponding to the group G. And g_1 is coupling constant for SU(2) and $g_2$is for U(1) and $g_3$for SU(3). Thus we can denote g_i(Q) as the coupling constant for SU(2) (i=1), U(1) (i=2) and SU(3) (i=3) gauge groups, as they are function of energy Q. The coupling constants in natural system of units are

$g_1 = \frac{e}{\sin \theta_W}, g_2 = \frac{e}{\cos \theta_W}$ (2)

$\alpha = \frac{e^2}{4\pi} \approx \frac{1}{137}, \ \alpha_1 = \frac{{g_1}^2}{4\pi} \approx \frac{1}{30}$ (3)

$\alpha_2 = \frac{{g_2}^2}{4\pi} \approx \frac{1}{100}, \ \alpha_3 = \frac{{g_3}^2}{4\pi} = 0.1184$ (4)

The rationalized coupling constants α, α_1, α_2 and α_3 belong to gauge groups $U_{em}(1)$, $SU_L(2)$, $U_Y(1)$ and $SU_C(3)$ respectively. For variation of α_i(Q) with Q,

$\alpha_i(Q) = \frac{\alpha_i(\mu)}{1+\alpha_i(\mu) b_i \log \frac{Q^2}{\mu^2}}$ (5)

where the coefficients b_i of the logarithm term are different for three groups [SU(2), U(1) and SU(3)]. The characteristic values of b_i are

$b_i = \frac{(11 n_b - 4 n_f)}{12\pi}$ (6)

where n_b is the number of bosons and n_f is the number of fermion generations, which make contributions to one loop vacuum polarization. n_b is 0 for U(1), 2 for SU(2) and 3 for SU(3) and $n_f = 3$. Thus the values of b_i are

b_1= b [SU(2)]= $+\frac{5}{6\pi}$ (7)

b_2= b [U(1)]= $-\frac{1}{\pi}$ (8)

b_3= b [SU(3)]= $+\frac{7}{4\pi}$ (9)

where each b_i has magnitude less than one. Due to negative b_2 and slowly changing logarithm, the α_2[U(1)] increases very slowly and b_i being positive with different magnitudes, α_1[SU (2)] decreases slowly compared to α_3[SU(3)], which decreases more rapidly. The coupling constants α, α_1, α_2 at low energies are different but at high energies, at $M_x \approx 1.5 \times 10^{16}$ GeV, all coupling constants would merge into one i.e. only one coupling constant α_{un} would exist that is all the three interactions have been merged into one grand unified interaction.

For SU(5) group, value of Weinberg angle θ_W can be calculated by using C^2, where C is the Clebsch-Gordan coefficient. g_1(Q), g_2(Q) and g_3(Q) are the coupling constants associated with SU(2), U(1) and SU(3) groups respectively. For group G with Clebsch-Gordan coefficient , we introduce new coupling constants k_1, k_2, k_3 which can be defined in a way

$k_1(Q) = g_1(Q)$ (A)

$k_2(Q) = C g_2(Q)$ (B)

$k_3(Q) = g_3(Q)$ (C)

Now rationalized coupling constants are $\alpha_i = \frac{{k_i}^2}{4\pi}$

Equation (5) can be modified as

$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(Q)} - 2 b_i \log \frac{Q}{\mu}$ (10)

where b_i are given by equations (7),(8) and (9). For Q ≥ M_x, k_i(Q)=g_G i.e. convergence of coupling constants which physically means grand unification of interactions

From equation (10)

$\frac{1}{{k_i}^2(\mu)} = \frac{1}{{g_G}^2} - \frac{2 b_i}{4\pi} \log \frac{M_x}{\mu}$ (11)

Using equation (11) we can construct a linear combination

$\frac{1}{{k_1}^2} + \frac{C^2}{{k_2}^2} - \frac{1+C^2}{{k_3}^2} = -\frac{2}{4\pi}[b_1 + C^2 b_2 - (1+C^2) b_2 - (1+C^2) b_3] \log \frac{M_x}{\mu}$ (12)

where $\frac{1}{{g_G}^2}$ terms have canceled.

Now defining $\cos \theta_W = \frac{g_1}{\sqrt{{g_1}^2+{g_2}^2}}$ and $\sin \theta_W = \frac{g_2}{\sqrt{{g_1}^2+{g_2}^2}}$

From equations (A), (B) and (C); by using the values of g_1 and g_2 from equation (2) and along with using the definition of $\cos \theta_W$ and $\sin \theta_W$ obtained from the orthogonality relation, finally result is

$\frac{1}{{k_1}^2} + \frac{C^2}{{k_2}^2} = \frac{1}{{g_1}^2} + \frac{1}{{g_2}^2} = (\frac{1}{g_2 \cos \theta_W})^2 = \frac{1}{e^2}$ (13)

Now substituting equations (7), (8), (9) in R.H.S. of equation (12) and the equation (13) in L.H.S. of equation (12), we may write

$\frac{1}{e^2} - \frac{(1+C^2)}{{k_3}^2} = \frac{11(1+3C^2)}{(6\pi)(4\pi)} \log \frac{M_x}{\mu}$ (14)

from which we get

$\log \frac{M_x}{\mu} = \frac{6\pi}{11(1+C^2)} \left[\frac{1}{\alpha} - \frac{(1+C^2)}{\alpha_3} \right]$ (15)

Using here limits of $M_x \approx 1.5 \times 10^{16}$ GeV, μ= 2 GeV, α = $\frac{1}{137}$, α_3= 0.1181 in equation (15), we will get C^2= 3.287 for group G.

Thus at $M_x \approx 1.5 \times 10^{16}$ GeV, strong and electroweak interactions would merge into a single interaction and which can called as grand unified interaction. Here from the value of C^2, value of Weinberg angle can be calculated by

$sin^2\theta_W = \frac{1}{1+C^2}$

Using here the value C^2= 3.287 , the obtained value of $sin^2\theta_W$ is

$$sin^2\theta_W = 0.233$$

III. Coupling Constant At Grand Unification

The coupling constants g_1 and g_3 are for non-Abelian gauge groups whereas gauge bosons carry charge. W and Z bosons carry weak charges and gluons carry color charges. Due to these charges, self-interaction of gauge bosons takes place which rise to anti screening effects as a result coupling constants corresponding to both SU(2) and SU(3) decreases with increasing Q^2. The decrease in both cases is not same. The decrease in coupling constant for SU(3) is much more rapid whereas coupling constant for SU(2) decreases slowly. At Q=M_x, all the three coupling constants merge into one α_{un} which is given by

$\alpha = \frac{1}{b_n \log \frac{Q^2}{\Lambda^2}}$

where Λ is QCD scale parameter and $b_n = \frac{33-2n}{12\pi}$ where n is number of quark flavors. Using Λ=0.247 GeV and $b_n = \frac{33}{12\pi}$ as n=0 above grand unification and an approximate value of Q=1.5×10^{16} GeV, the obtained value of coupling constant after unification α_{un} is

$$\alpha_{un} = 0.034016$$

IV. Errors And Final Result

A. Error in weinberg angle

1. Fundamental constants

First of all the error could be due to fundamental constants. These natural constants have some abnormal features to an extent may be due to their time dependence. Recent studies suggest that the fine structure constant which has fundamental importance in describing the electromagnetic interaction, was in earlier times somewhat smaller than today. A research group in Australia studied 150 quasars some of them about 11 billion light years away and the redshift of the objects varied between 0.5 and 3.5 corresponding to ages varying between 23% and 87% of the age of the universe. Using many multiplet method, they found the value of fine structure constant close to $1/137.037$, as opposed to near $1/137.036$ which is observed today. Although the deviation is small but it could have drastic consequences for some theories. Thus the uncertainty in $sin^2\theta_W$ due to fine structure constant is estimated to be –0.000054.

2. Strong coupling constant

The value of strong coupling constant used here is 0.1181. Uncertainty in α_3 varies from 0.0014 to -0.0012 due to which the systematic error in $sin^2\theta_W$ is +0.000642 and – 0.000559.

B. Errors in α_{un}

The typical value of α_{un} is 0.03853. The deviation here in calculated value of α_{un} is of +0.00458 may be due to time dependent variation in QCD scale parameter or due to renormalization scale μ. The relative time dependencies are related by: $\delta\Lambda/\Lambda = \delta\alpha_S/\alpha_S \ln(\text{?}/\Lambda)$, although the relative changes in α_S cannot be uniform. The typical value of scale parameter is 0.213 +0.038/-0.035 GeV. Like fine structure constant, Λ also varies with time. The uncertainty in α_{un} due to QCD scale parameter is 0.004499.

Somewhat uncertainties can be due to value of M_x as its absolute value is not known and just an approximate value is used everywhere using only an approach that unification takes place at higher limits upto GeVs.

C. Final result

After the estimate of systematic errors from all the possible sources of error, using SU(5) model, the calculated value of $sin^2\theta_W$ is

$$sin^2\theta_W = 0.233 +0.00276/-0.00243$$

Thus accordingly θ_W varies from 28.717° to 29.068°.
And the calculated value of coupling constant α_{un} after grand unification is α_{un} = 0.034016

V. Conclusion

From calculations, it is clear that as energy approaches to $M_x = 1.5 \times 10^{16}$GeV, we would get Grand Unified Interaction as α_S for weak, electromagnetic and strong are same above M_x. The evidences for QCD, SU(3) color gauge theory is provided by virtual electron-positron pair which would be produced by vacuum polarization. This explains the concept of provision of charge to particles at the time of symmetry breaking → SU(3)⊗SU(2)⊗U(1). Due to vacuum quantization, the charge gained by the particles is an integral multiple of elementary charge. And most importantly SU(5) model can be used to find out the convergence of coupling constants at GeV energy scale.

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