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Spherical and Cylindrical Ion Acoustic Solitary Waves in Electron-Positron-Ion Plasmas with Non-Maxwellian Electrons and Positrons

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Abstract: *The propagation of cylindrical and spherical ion acoustic solitary waves in a plasma system consisting of ions, electrons and positrons are investigated. The electrons and positrons are assumed to be following the nonextensive distribution popularly known as Tsallis distribution. The standard nonlinear equation i.e. Korteweg de-Vries (KdV) equation has been solved numerically using suitable mathematical transformations. The effect of nonextensivity (q) and nonplanar geometry on the amplitudes and width of ion acoustic potential structures have been studied numerically. A transition from negative to positive potential structures have been observed for the planar as well as nonplanar geometries for lower values of q in the range* $-1 < q < 0$. *Soliton amplitude is maximum for spherical waves and is minimum and for planar waves while it lies in between the two for cylindrical waves. The present investigation may help us in understanding the study of cylindrical and spherical solitary waves in astrophysical plasmas.*
Keywords: *Cylindrical structures, Electron-positron-ion (e-p-i) plasmas, Reductive perturbation method, Spherical structures, Tsallis distribution.*

I. Introduction

The theoretical and experimental study of the dynamics of ion acoustic waves (IASWs) is of great interest for several decades. Many researchers and scholars have done much work on IASWs in different plasma systems. A solitary wave is a nonlinear wave having permanent shape and it maintains its shape during its propagation. The word acoustic is referred to as sound so these ion acoustic waves are also called as sound waves. It arises because of the balance between the effects of the nonlinearity and the dispersion. Ikezi *et al* [1] have done the first experimental observation of ion-acoustic solitons. After that many researchers [2-8] have reported these solitons with their theoretical work in different plasma systems. Much study of ion acoustic solitons (IASs) has been reported in electron-positron-ion (e-p-i) plasmas by a lot of researches during past few years [9−17]. Electron-positron-ion plasmas are composed of a small number of ions along with the electrons and positrons. An e-p-i plasma with positive ions is believed to be widespread in nature. Such a plasma can occur, e.g., in the inner region of accretion discs, in the vicinity of black holes [18-19], in magnetospheres of neutron stars [20-21], in active galactic cores [22], and even in solar flare plasma [23]. Recently, narrow-collimated extended relativistic jets of e-p-i plasma were observed in the vicinity of blazars and micro quasars [24-26]. There are many facts indicating that our Universe was a hot e-p-i plasma during the first minutes of its existence [27]. For laboratory plasmas, it is known that the propagation of a short relativistically strong laser pulse in matter can be accompanied by the formation of e-p-i plasma due to photoproduction of pairs during photons scattering by nuclei, etc. [28-30]. Tokamaks and other magnetic confinement systems are also another examples related to plasma. To determine the rate of transport of particle from the positron lifetime the annihilation line in the γ spectrum, charged positron or quasineutral electron-positron beams can be injected into such plasmas [31-32]. In this case, a small region occupied by e-p-i plasma can form in the bulk plasma. These mainly exist in astrophysical environments. So the study of linear and nonlinear dynamics in e-p-i plasmas are quite important and of great interest. With addition to these plasma systems a great attention has been paid to nonextensive statistic mechanics. It is derived from the main statistics i.e. Boltzmann-Gibbs-Shannon (BGS) statistics. As the Maxwell distribution is proved unsuitable for study of many plasma systems so Tsallis [33] introduced another generalization called the non-Maxwellian distribution in 1988. The main credit for this new generalization firstly goes to Renyi [34]. Nonlinearity plays an important role in plasma because it is based on nonlinear dynamics and having nonlinear equations. Therefore many researchers have been studied the spherical and cylindrical (nonplanar) modes of ion acoustic waves and compared the results of these geometries with the planar ones [35-38]. Stephen Maxon [38] studied the cylindrical and spherical waves and proposed that these are two different waveforms having different properties. The properties of these higher dimensional solitons are quite different from their planar counterpart. Javidan [39] has derived the cylindrical and spherical KdV equation for ion acoustic solitary waves in e-p-i plasmas using kappa distribution. Only few investigations have been reported on the study of nonplanar ion acoustic solitary waves in e-p-i plasmas [40-42]. Sahu and

Roychoudhury [42] investigated the exact solutions of cylindrical and spherical dust-ion acoustic waves. For this they derived the nonplanar KdV equation by using a suitable coordinate transformation which reduces the cylindrical KdV equation into the ordinary KdV equation which can be solved analytically. Eslami and Mottaghizadeh [43] used the standard reductive perturbation technique, a three-dimensional cylindrical Kadomtsev-Petviashvili equation (CKPE), which governs the dynamics of ion acoustic solitary waves (IASWs), is derived for small but finite amplitude ion-acoustic waves in cylindrical geometry in a collisionless unmagnetized plasma with kappa distributed electrons, thermal positrons, and cold ions. Electron-positron (e-p) pairs exist in the plasmas emanating both from the pulsars and from the inner region of the accretion discs surrounding the central black hole in active galactic nuclei [44-46]. Such pairs are also present in the Van Allen radiation belt and near the polar cap of fast rotation neutron stars [47-50], in semiconductor plasma [51], in intense laser fields [28], in tokamaks [52] and in the solar atmosphere [53]. It has been found that the electron positron (e-p) plasmas behave differently from typical electron-ion (e-i) plasmas [54]. In astrophysical environments always there exist a small number of ions with electrons and positrons, it is then important to study the linear and nonlinear dynamics of plasma waves in electron-positron-ion (e-p-i) plasmas. A number of works has been carried out to study the linear and nonlinear behaviors of waves in e-p and e-p-i plasmas over the last few years [55-59]. Based on Maxwellian assumption, many authors have been studied the propagation of ion acoustic waves in electron-positron-ion plasmas [60-63].

In this paper, we investigate the propagation of cylindrical and spherical ion acoustic solitary waves in a plasma system consisting of ions, electrons and positrons. The electrons and positrons are assuming to be following the nonextensive distribution popularly known as Tsallis distribution. The standard nonlinear equation i.e. Korteweg de-Vries (KdV) equation has been solved numerically using suitable mathematical transformations. The effect of nonextensivity (q) and nonplanar geometry on the amplitudes and width of ion acoustic potential structures have been studied numerically.

II. Governing Equations And Derivation of KdV Equation

We focus on the cylindrical and spherical ion acoustic solitary waves in plasmas containing cold ion fluid, nonextensively distributed electrons and positrons. In equilibrium, the charge neutrality condition is $n_{e0} = n_{i0} + n_{p0}$. Where n_{i0}, n_{p0} and n_{e0} are the unperturbed number densities of the ion, positron and electron respectively. The nonlinear dynamics of IAWs in cylindrical and spherical geometries is governed by the following set of nonlinear equations

$$\frac{\partial n}{\partial t} + \frac{1}{r^m}\frac{\partial(r^m nu)}{\partial r} = 0 \qquad (1)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} = -\frac{\partial \phi}{\partial r} \qquad (2)$$

$$\frac{1}{r^m}\frac{\partial}{\partial r}\left(r^m \frac{\partial \phi}{\partial r}\right) = n_e - n_p - n_i \qquad (3)$$

where m = 0 stands for one-dimensional flat geometry and m = 1,2 for cylindrical and spherical geometries respectively. The ion density (n_i) and the ion velocity (u_i) are normalized by the ion equilibrium density n_{i0} and the ion-acoustic speed $C_s = \sqrt{\frac{T_e}{m_i}}$ respectively, where m_i is the ion mass. ϕ is electrostatic potential which is normalized by $\frac{T_e}{e}$ where T_e is the electron temperature and "e" is the absolute value of electron charge. In the equations (1), (2) and (3), the densities of the plasma species are normalized by the unperturbed electron density n_{e0}, space variable is normalized by the electron Debye length $\lambda_D = \sqrt{\frac{T_e}{4\pi n_{e0} e^2}}$ and time variable is normalized by the electron plasma period $T = \sqrt{\frac{m_e}{4\pi n_{e0} e^2}}$. The number density of electron and positron fluid, with nonextensive distribution is given by,

$$n_e = \frac{1}{1-p}[1 + (q-1)\phi]^{\frac{q+1}{2(q-1)}} \qquad (4)$$

$$n_p = \frac{p}{1-p}[1 - (q-1)\sigma\phi]^{\frac{q+1}{2(q-1)}} \qquad (5)$$

Here $p = \frac{n_{p0}}{n_{e0}}$ is the ratio of unperturbed positron density to unperturbed electron density and σ is the temperature ration of electrons to that of positron given by $\sigma = \frac{T_e}{T_p}$. Further the nonextensivity of electrons and positrons are assumed to be same. We use reductive perturbation technique (RPT) to derive the standard KdV equation. we introduce the stretched coordinates ξ and τ as follows [64]

$$\tau = \epsilon^{\frac{3}{2}t}, \;\; \xi = -\epsilon^{\frac{1}{2}}(r + \lambda t) \tag{6}$$

Here ε is a small parameter and λ is the wave phase velocity. The dependent variables expended as

$$n_i = 1 + \varepsilon n_i^{(1)} + \varepsilon^2 n_i^{(2)} + \varepsilon^3 n_i^{(3)} + \cdots$$
$$u_i = \varepsilon u_i^{(1)} + \varepsilon^2 u_i^{(2)} + \varepsilon^3 u_i^{(3)} + \cdots \tag{7}$$
$$\phi = \varepsilon \phi^{(1)} + \varepsilon^2 \phi^{(2)} + \varepsilon^3 \phi^{(3)} + \cdots$$

Substituting the stretching coordinates (6) and the expansions (7) into the basic equations (1), (2), (3) and (4-5) and comparing the coefficients of ε after solving the equation, we have

$$u_i^{(1)} = -\frac{\phi^{(1)}}{\lambda} \quad \text{and} \quad n_i^{(1)} = \frac{\phi^{(1)}}{\lambda^2} \tag{8}$$

To the next higher order, we get the phase velocity as

$$\lambda = \sqrt{\frac{2(1-p)}{(1+p\sigma)(q+1)}} \tag{9}$$

Finally after a tedious and long algebraic manipulation, we obtain a modified KdV equation

$$\frac{\partial \phi^{(1)}}{\partial \tau} + \frac{m\phi^{(1)}}{2\tau} + A\phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} + B\frac{\partial^3 \phi^{(1)}}{\partial \xi^3} = 0 \tag{10}$$

Where A is non-linear constant and B is dispersion constant given by

$$A = \frac{1}{2}\left[\frac{3}{\lambda} - \frac{(q+1)(3-q)(1+p\sigma^2)}{8(1-p)}\right] \quad \text{and} \quad B = \frac{\lambda^3}{2} \tag{11}$$

Equation (10) is the cylindrical/spherical KdV equation describing the nonlinear propagation of the ion acoustic solitary waves in plasmas with nonextensive electrons. The coefficients A and B measures the nonlinearity and dispersive effects and their balance leads to the formation of localized wave known as soliton. Now we have to solve the final equation for its solutions for different values of m.

For one dimensional flat geometry, m = 0. Equation (10) reduces to

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A\phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} + B\frac{\partial^3 \phi^{(1)}}{\partial \xi^3} = 0 \tag{12}$$

Here we use the transformation $\chi = \xi - u\tau$ and equation reduces to

$$-u\frac{\partial \phi^{(1)}}{\partial \chi} + A\phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \chi} + B\frac{\partial^3 \phi^{(1)}}{\partial \chi^3} = 0$$

After solving, we have the solution of this equation is

$$\phi^{(1)} = \phi^{(0)} Sech^2\left(\frac{\chi}{\omega}\right) \tag{13}$$

Where $\phi^{(0)} = \frac{3u}{A}$ and $\omega = \sqrt{\frac{4B}{u}}$

For nonplanar cylindrical geometry, m = 1

$$\frac{\partial \phi^{(1)}}{\partial \tau} + \frac{\phi^{(1)}}{2\tau} + A\phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} + B\frac{\partial^3 \phi^{(1)}}{\partial \xi^3} = 0 \tag{14}$$

To obtain an analytic solution of cylindrical KdV equation, let us first use Hirota's transformation [65] given by

$$\phi = \frac{v}{\tau} + \frac{\xi}{2A\tau} \tag{15}$$

Under this transformation equation (14) transforms to

$$\frac{\partial v}{\partial \tau} + \frac{Av}{\tau}\frac{\partial v}{\partial \xi} + \frac{\xi}{2\tau}\frac{\partial v}{\partial \xi} + B\frac{\partial^3 v}{\partial \xi^3} = 0 \tag{16}$$

Again we use a transformation given by

$$\tau' = -2\tau^{-1/2}, \quad \xi' = \xi\tau^{-1/2}$$

Equation (16) reduces to

$$\frac{\partial v}{\partial \tau'} + Av\frac{\partial v}{\partial \xi'} + B\frac{\partial^3 v}{\partial \xi'^3} = 0 \tag{17}$$

Above equation is the usual KdV equation. So the solution of this equation is given by $v = \phi^{(0)} Sech^2\left(\frac{\chi}{\omega}\right)$ and the exact solution of equation (14) is given by

$$\phi_2 = \frac{1}{\tau}\left\{\frac{\xi}{2A} + \left(\frac{3u}{A}\right) Sech^2\left[\sqrt{\frac{u}{4B\tau}}(\xi + 2u)\right]\right\} \qquad (18)$$

This solution is valid for $\tau \neq 0$.
For nonplanar spherical geometry, m = 2, the equation (10) reduces to

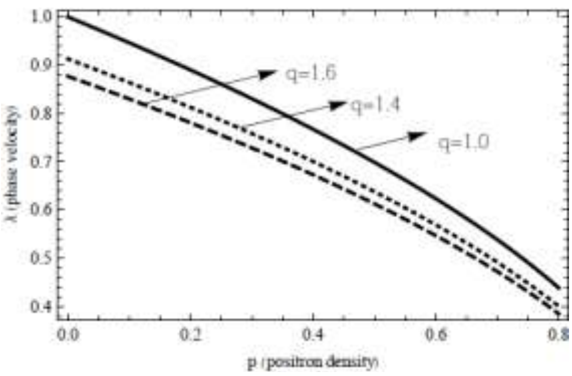
$$\frac{\partial\phi^{(1)}}{\partial\tau} + \frac{\phi^{(1)}}{\tau} + A\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial\xi} + B\frac{\partial^3\phi^{(1)}}{\partial\xi^3} = 0$$

solution is given by

$$\phi_3 = \frac{1}{\tau}\left[\frac{\xi}{Aln\tau} + \frac{c}{ln\tau}\right] \qquad (19)$$

III. Results And Discussion

When the geometrical effect is taken into account (i.e. for m≠0), an exact analytical solution of equation (10) is not possible without any transformation. Therefore, we have solved the equation (10) numerically and have studied the effects of nonextensive electrons and positrons on the propagation of ion acoustic solitary waves. The wave phase velocity λ given by equation (9) is found numerically using equation (8). In order to investigate the effect of nonextensivity on the wave phase velocity we have plotted λ as a function of positron density (p) in Fig. 1(a) for three different values of nonextensive parameters q i.e. q=1.0, q=1.4 and q=1.6 with $\sigma = 0.05$ *and* $u = 0.1$. It is observed that phase velocity λ decreases with positron density and nonextensivity for the range $q > 1$. Similar behavior has been observed for other two ranges of nonextensivity i.e. $-1 < q < 0$ and $0 < q < 1$ (not shown). As q=1 corresponds to usual Maxwellian nature of the particles. So any deviation from Maxwellian behaviour leads to decrease in phase velocity of the soliton. This also becomes clear from the mathematical expression for phase velocity given by equation (9) where λ is inversely related to q.



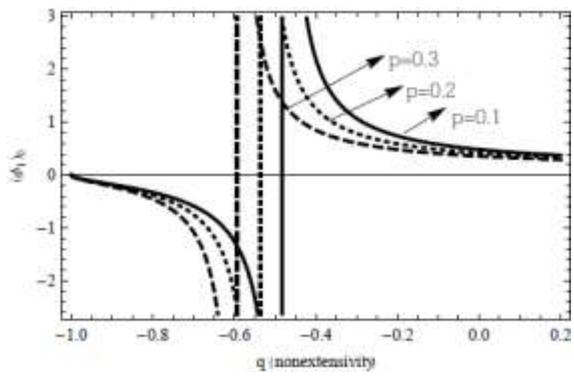


Fig. 1(a): Variation of wave phase velocity λ with positron density p for $q = 1.0$ (solid line), $q = 1.4$ (dotted line) and $q = 1.6$ (dashed line) with $\sigma = 0.05$ and $u = 0.1$.
Fig. 1(b): Variation of peak amplitude of planar solitary structures $(\phi_1)_0$ with nonextensive parameter q for $p = 0.1$ (solid line), $p = 0.2$ (dotted line) and $p = 0.3$ (dashed line) with $\sigma = 0.05$ and u = 0.1

For planar structures i.e. m=0, Fig. 1(b) shows the variation of peak amplitude of solitary structures $(\phi_1)_0$ as a function of nonextensive parameter q for three different values of positron density p. Here we have taken three values of positron density, p=0.1 (solid line), p=0.2 (dotted line) and p=0.3 (dashed line) for the region$-1 < q < 0$. The other parameters are taken as $\sigma = 0.05, u = 0.1$ etc. It is observed that nonextensivity leads to transition from negative to positive potential solitary structures. It is further mentioned that this transition occurs earlier for larger values of positron density. Similar transition behaviour is also clear from the plot of soliton solution of planar solitary structures as a function of variable χ for the range $-1 < q < 0$ as is given in Fig. 2(a) for three different values of q. While only positive potential structures are obtained for the regions $0 < q < 1$ and $q > 1$. It is mentioned that for all the ranges of q, the peak amplitude decreases with increase in q. This behaviour is depicted in Fig. 2(b), where a plot of planar soliton potential $\phi_1$as a function of variable χ is given for the range $0 < q < 1$ for three different values of nonextensive parameters.

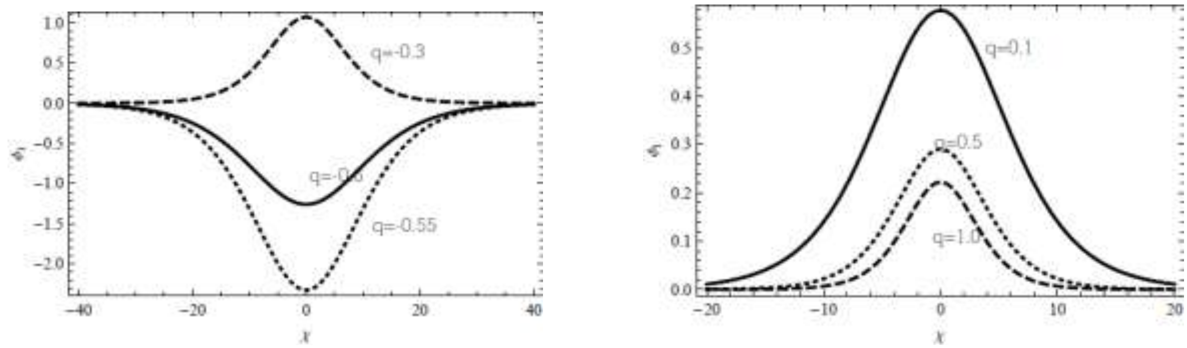


Fig. 2(a): For the range $-1 < q < 0$, variation of soliton solution ϕ_1 as a function of variable χ with $\sigma = 0.05$ and u = 0.1.
Fig. 2(b): Variation of soliton solution ϕ_1 as a function of variable χ for the range $q > 0$ with $\sigma = 0.05$ and u = 0.1.

In equation (10), $\frac{m}{2\tau}\phi$ is a geometrical term and singular at $\tau \to 0$. For sufficiently large values of $|\tau|$, the spherical and cylindrical solitary wave reduces to planar waves. This is due to the fact that large $|\tau|$ makes the term $\frac{m}{2\tau}\phi$ non-dominent. However for lower value of τ, the geometrical term will be dominant and both spherical and cylindrical waves and will be different from the one dimensional solitary waves. In Fig. 3(a), the variation of the soliton solution of cylindrical ion acoustic solitary waves ϕ_2 with respect to space coordinate ξ for three different values of nonextensive parameter q has been given for the region $0 < q < 1$. Here solid line is for q=0.1, dotted line id for q=0.2 and dashed line is for q=0.3. We observe that the cylindrical soliton amplitude goes on decreasing with increase in q. This variation remains similar for the region $q > 1$ (not shown here)

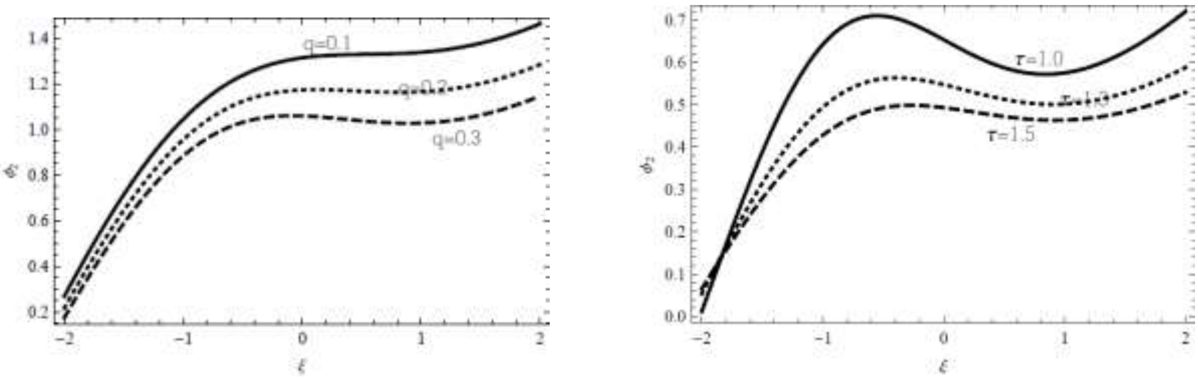


Fig. 3(a): Variation of cylindrical soliton solution ϕ_2 as a function of space coordinate ξ for three different values of q with $\sigma = 0.1$, u = 0.5, $p = 0.3$.
Figure 3(b): Variation of cylindrical soliton solution ϕ_2 as a function of space coordinate ξ for three different values of τ with $\sigma = 0.1$, u = 0.5, $p = 0.3$ and q=0.9.

The solution ϕ_2 for cylindrical geometry is given by equation (18) is valid for $\tau > 0$. In Fig. 3(b), we take the variation of the soliton solution ϕ_2 with respect to space coordinate ξ for different values of time coordinate τ $i.e.$ $\tau = 1.0, \tau = 1.3$ and $\tau = 1.5$ for the region $0 < q < 1$. The other parameters are with $\sigma = 0.1, p = 0.3, m = 1, q = 0.9$ and $u = 0.5$. It is mentioned that their behaviour is similar to the one obtained by Sahu and Roychoudhury [42]. We also observed that the soliton amplitude goes on decreasing with increase in τ. The observed solitary structures are of compressive nature or they are positive potential structures. It may be noted that this variation remains same for the region $q > 1$.

The 3D profile for the variation of soliton solution of cylindrical geometry with as a function of $\xi$and q is given in Fig. 4(a). The other parameters are taken as $\sigma = 0.1, p = 0.3, u = 0.5$ and $\chi = 2$ for $q > 1$. Here we observed the compressive solitons for the nonplanar geometry in case of m=1. If we compare these structures with the 3D profiles of planar structures, we have seen that the peaks of the nonplanar solitons are slightly moved away from the origin. While in planar cases the peaks are formed at the origin. For the range $q < 0$, the three dimensional view of the soliton profile is given in Fig. 4(b).

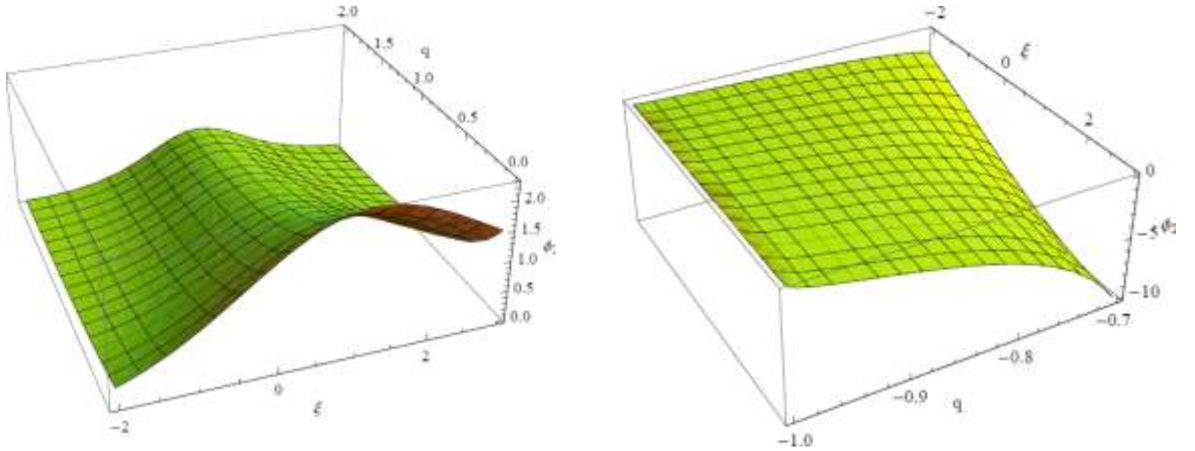


Fig. 4(a): For m=1 and q>0, 3D plot of variation of cylindrical soliton solution ϕ_2 as a function of ξ and q with $\sigma = 0.1,\ p = 0.3,\ \tau = 1\ and\ u = 0.5$.
Fig. 4(b): For m=1 and lower values of q, 3D plot of variation of soliton solution ϕ_2 as a function of ξ and q with $\sigma = 0.1, p = 0.3, m = 1\ and\ u = 0.5$.

In last we give the comparison between the planar and nonplanar geometries. This comparison consists of the variation of the peak amplitudes of the planar as well as nonplanar geometries with respect to the nonextensive parameter q for the ranges $-1 < q < 0$ and $q > 1$ and is shown in figures 5(a) and 5(b) respectively.

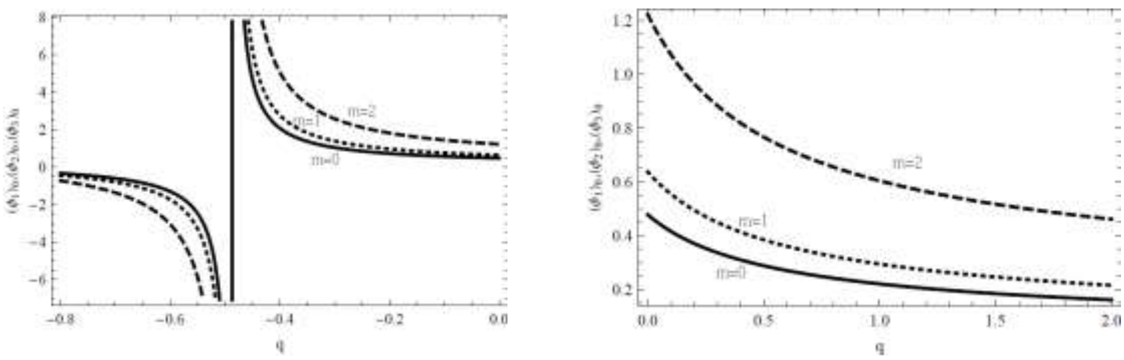


Fig. 5(a): Variation of peak amplitudes of flat geometry$(\phi_1)_0$ (m=0), cylindrical geometry$(\phi_2)_0$ (m=1) and spherical geometry$(\phi_3)_0$ (m=2) structures with q for $-1 < q < 0$ with $\sigma = 0.1,\ p = 0.1,\ \tau = 2$ and $u = 0.1$.
Fig. 5(b): Variation of peak amplitudes of flat geometry $(\phi_1)_0$, cylindrical geometry $(\phi_2)_0$ and spherical geometry $(\phi_3)_0$ structures with q (nonextensive parameter) for $q > 1$ having $\sigma = 0.1, p = 0.1, \tau = 2$and $u = 0.1$.

The set of parameters taken are mentioned in respective captions. For both the ranges of q, spherical solitary waves have maximum amplitude and planar waves has minimum. While the amplitude for cylindrical solitary waves lies between these two. This behaviour is similar to the one observed by Javidan [39] and Sahu and Roychoudhury [42]. However, for large values of τ the amplitude of cylindrical and spherical solitary waves become similar to the planar case. As mentioned earlier, this is because the nonplanar geometrical effects are no longer dominant for larger values of τ. The presented investigation may helpful in better understanding of study of cylindrical and spherical solitary waves in astrophysical plasmas. The comparison of the obtained results with other particle distributions in spherical and cylindrical geometries can help is to find better knowledge in plasma physics.

IV. Conclusion

The properties of spherical and cylindrical ion acoustic solitary waves (IASWs) have been studied in e-p-i plasma consisting of ions, nonextensive electrons and positrons by considering Tsallis distributed electrons and positrons. Here we have observed that wave phase velocity (λ), decreases with both positron density (p) and nonextensivity (q) of particles. We found that a transition from negative (rarefactive) to positive (compressive) potential structures have been observed for all the three geometries for which peak amplitudes decreases with q for range $-1 < q < 0$, while only positive potential structures are observed for the ranges $0 < q < 1$ and $q > 1$. For the same set of parameters, peak amplitude of spherical waves is maximum and planer waves are minimum while the peak amplitude of cylindrical waves lies between the two. It may be mentioned that the obtained results are similar to the one obtained by Javidan [39] and Sahu and Roychoudhury [42]. The presented investigation may helpful in better understanding of study of cylindrical and spherical solitary waves in astrophysical plasmas. The comparison of the obtained results with other particle distributions in spherical and cylindrical geometries can help is to find better knowledge in plasma physics.

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