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Inflationary Universe Scenario in Bianchi Type VI_0 Space Time with Flat Potential and Bulk Viscosity in General Relativity

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Abstract: *In the present study, we have investigated inflationary cosmological model with flat potential and bulk viscosity taking Bianchi Type VI_0 space-time as a source. To get the deterministic model of universe, we have also assumed that shear (σ) is proportional to expansion (θ)(Thorne [31]) and $\zeta\theta$ = constant as considered by Zimdahal [29] where ζ is the coefficient of bulk viscosity. We find that the observations of inflationary cosmology i.e. slow role parameters (ε,δ), third slow parameter (S) and anisotropic parameter ($\tilde{A}_m$) are in excellent agreement with the Planck (2013) results [39]. We also find that spatial volume increases exponentially representing inflationary scenario of the universe. The model in general represents anisotropic space time but isotropizes when n = 1. The model also represents accelerating and decelerating phases of universe which matches with latest observations of universe. The rate of Higgs field is initially large but decreases with time and vanishes for large value of time. The model has Point Type singularity at T = 0 (MacCallum [30]).*

Keywords: *Inflationary; Bianchi VI_0; Flat potential, Bulk viscosity*

I. Introduction

The present day universe is satisfactorily described by homogeneous and isotropic models given by FRW (Friedmann-Robertson-Walker) line-elements. The universe in smaller scale is neither homogeneous nor isotropic nor do we expect the universe to have these properties in its early stages. Patridge and Wilkinson [1] have pointed out that FRW models are unstable near the singularity. Also in the late eighties, Astronomical observations revealed that the predictions of FRW models do not always meet our requirements as was believed earlier (Smoot et al. [2]). Therefore, spatially homogeneous and anisotropic Bianchi models (I-IX) are undertaken to study the universe in its early stages of evolution. Among these, Bianchi Type I space time is the simplest one and is anisotropic generalization of zero curvature of FRW models. Bianchi Type VI_0 space-times are of particular interest because these are simple generalization of Bianchi Type I space-time. Barrow [3] in his investigation has pointed out that Bianchi Type VI_0 universes give a better explanation of some of the cosmological problems like primordial helium abundance and these can be isotropized in special case. Seeing the importance of these models, various authors viz. Ellis and MacCallum [4], Collins [5], Dunn and Tupper [6], Roy and Singh [7], Tikekar and Patel [8], Bali et al. [9], Ram and Singh [10] have studied Bianchi Type VI_0 cosmological models in different contexts. The introduction of viscosity in the cosmic fluid content, has been found very useful in explaining many significant physical aspects of the dynamics of homogeneous cosmological models as per investigations by many authors viz. Ribeiro and Sanyal [11], Patel and Koppar [12], Bali et al. [13,14], Verma and Shri Ram [15] in Bianchi Type VI_0 models for viscous fluid distribution in different contexts.

The primordial acceleration in which the universe undergoes rapid exponential expansion is known as inflation. There is a great interest in the inflationary universe scenario since this scenario solves different problems of modern cosmology like homogeneity, the isotropy, flatness of observed universe and primordial monopole problems. Guth [16] suggested that rapid expansion is due to false vacuum energy and after inflation, the universe is filled with bubbles. This inflationary scenario is also confirmed by CMB (Cosmic Microwave Background) observations. Historically a model closely related to the inflationary universe was first suggested by Starobinsky [17] but the inflationary cosmological models became popular after an important paper of Guth [16]. The most prevailing inflationary models are investigated through the scalar field which acts as a source of inflation and generates cosmic acceleration (Sato [18], Linde [19]). It also explains the distribution of large scale structure and origin of observed anisotropy of CMB radiation in the inflationary era (Gold et al. [20]). Inflationary scenario for homogeneous and isotropic models (FRW models) has been studied by many authors viz. Linde [21], Wald [22], Barrow [23], La and Steinhardt [24], Rothman & Ellis [25] have pointed out that we can have solution for isotropic problem if we work with anisotropic space-time that isotropizes in special case. Keeping in view of these observations Bali and Jain [26], Bali [27] investigated inflationary scenario in LRS Bianchi Type I and Bianchi Type I space-time respectively.

II. Metric and Field Equations

We consider Bianchi Type VI_0 metric in the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{2x} dy^2 + C^2 e^{-2x} dz^2 \tag{1}$$

where A,B,C are metric potentials and functions of t-alone.
We assume the co-ordinates to be comoving so that

$$v^1 = 0 = v^2 = v^3, v^4 = 1.$$

The Lagrangian is that of gravity minimally coupled to a scalar field (ϕ) with effective potential $V(\phi)$, we have (as given by Stein-Schabes [28])

$$S = \int \sqrt{-g}\left\{R - \frac{1}{2} g^i_j \partial_i \phi \partial^j \phi - V(\phi)\right\} d^4x \tag{2}$$

The Einstein Field equations (in gravitational units G = c = 1) in case of massless scalar field ϕ with potential $V(\phi)$ are given by

$$R^j_i - \frac{1}{2} R g^j_i = -8\pi\, T^j_i \tag{3}$$

with

$$T^j_i = \partial_i \phi \partial^j \phi - \left[\frac{1}{2}\partial_\rho \phi \partial^\rho \phi + V(\phi)\right] g^j_i - \zeta\theta(g^j_i + v_i v^j) \tag{4}$$

The conservation relation $T^j_{i;j} = 0$ leads to

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g}\partial^\mu \phi) = -\frac{dV}{d\phi} \tag{5}$$

The Einstein's field equations (3) for the metric (1) leads to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} + \frac{1}{A^2} = -8\pi\left[\frac{1}{2}\phi_4^2 - K - \alpha\right] \tag{6}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} - \frac{1}{A^2} = -8\pi\left[\frac{1}{2}\phi_4^2 - K - \alpha\right] \tag{7}$$

$$\frac{A_{44}}{A} + \frac{A_4 B_4}{AB} + \frac{B_{44}}{B} - \frac{1}{A^2} = -8\pi\left[\frac{1}{2}\phi_4^2 - K - \alpha)\right] \tag{8}$$

$$\frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC} + \frac{B_4 C_4}{BC} - \frac{1}{A^2} = 8\pi\left[\frac{1}{2}\phi_4^2 + K\right] \tag{9}$$

$$\frac{B_4}{B} - \frac{C_4}{C} = 0 \tag{10}$$

III. Solution of Field Equations

We have assumed flat region to get inflationary scenario. Thus $V(\phi)$ = constant = K and $\zeta\theta = \alpha$ as considered by Zimdahl [29].

The isotropization of the cosmic fluid induced by viscosity is an important physical effect as discussed by Brevik and Peterson [35,36]. Bamba et al. [37] have investigated that bulk viscous fluid model can explain the recent Planck results of the observations for inflationary universe. We use the ansatz $\zeta\theta$ = constant because it has significant role to connect with occurrence of Little Rip (LR) cosmology using FRW models as given by Brevik et al. [38].

Equation (10) leads to

$$B = mC \tag{11}$$

where m is constant of integration.

The equation (5) for the scalar field (ϕ) leads to

$$\phi_{44} + \left(\frac{A_4}{A} + \frac{2B_4}{B}\right)\phi_4 = \frac{dV}{d\phi} \tag{12}$$

where suffix '4' indicates ordinary partial derivatives with respect to t. Using the condition of flat region i.e. $V(\phi) = K$ (constant) in equation (12), we have

$$\phi_{44} + \left(\frac{A_4}{A} + \frac{2B_4}{B}\right)\phi_4 = 0 \tag{13}$$

From equation (13), we have

$$\phi_4 = \frac{\ell}{AB^2} \tag{14}$$

where ℓ is constant of integration.
The scale factor R for line-element (1) is given by

$$R^3 = ABC = AB^2 \tag{15}$$

To get the deterministic solution in terms of cosmic time t, we assume that shear (σ) is proportional to expansion (θ) as considered by Thorne [31]. Thus, we have

$$A = B^n \tag{16}$$

where A and B are metric potentials, n is a constant and

$$\sigma = \frac{1}{\sqrt{3}}\left|\frac{A_4}{A} - \frac{B_4}{B}\right| \tag{16(a)}$$

and

$$\theta = \frac{A_4}{A} + \frac{2B_4}{B} \tag{16(b)}$$

The motive for assuming the condition $\sigma \,\alpha\, \theta$ is explained as: Referring to Thorne [31], the observations of the velocity – redshift relation for extra galactic sources suggest that the Hubble expansion of the universe is isotropic within 30 percent (Kantowski and Sachs [32], Kristian and Sachs [33]). More precisely, the red shift studies place the limit $\frac{\sigma}{H} \le 0.30$ where σ is shear and H the Hubble constant. Also Collins et al. [34] have pointed out that for spatially homogeneous metric, the normal congruence to the homogeneous hypersurface satisfies the condition $\frac{\sigma}{\theta}$ = constant.

Equations (6) and (9) after using (10) lead to

$$\frac{2B_{44}}{B} + \frac{2B_4^2}{B^2} + \frac{2A_4B_4}{AB} = 8\pi(2K + \alpha) \tag{17}$$

Using equation (16), we have

$$2B_{44} + 2(n+1)\frac{B_4^2}{B} = 8\pi(2K + \alpha)B \tag{18}$$

To find the solution of equation (18), we assume that

$B_4 = f(B)$. Thus equation (18) leads to

$$\frac{df^2}{dB} + \frac{2(n+1)}{B}f^2 = 8\pi(2K + \alpha)B \tag{19}$$

From equation (19), we have

$$f^2 = \frac{8\pi(2K + \alpha)}{(2n+4)}B^2 + \lambda B^{-2(n+1)} \tag{20}$$

where λ is constant of integration. Equation (20) leads to

$$\frac{d\xi}{\sqrt{\xi^2+\beta^2}}=\gamma\, dt \tag{21}$$

where $\frac{4\pi(2K+\alpha)}{(n+2)}=m, \frac{\lambda}{m}= \beta^2, \sqrt{m}(n+2)=\gamma$

and $B^{(n+2)}=\xi$

Thus, we have

$$\xi=B^{n+2}=\beta \sinh\,(\gamma t+\delta) \tag{22}$$

where δ is constant of integration. Now, we have

$$B=\beta^{\frac{1}{n+2}}\sinh^{\frac{1}{n+2}}T \tag{23}$$

$$A=B^n=\beta^{\frac{n}{n+2}}\sinh^{\frac{n}{n+2}}T \tag{24}$$

$$C=\frac{B}{m}=\frac{\beta^{\frac{1}{n+2}}}{m}\sinh^{\frac{1}{n+2}}T \tag{25}$$

where $\gamma t+\delta=T.$

After suitable transformation of coordinates, the metric (1) leads to the form

$$ds^2=-\frac{1}{\gamma^2}dT^2+\sinh^{\frac{2n}{n+2}}T\,dX^2+\sinh^{\frac{2}{n+2}}T\left\{e^{\frac{2X}{\beta^{n/n+2}}}dY^2+e^{-\frac{2X}{\beta^{n/n+2}}}dZ^2\right\} \tag{26}$$

where

$$\beta^{\frac{n}{n+2}}x=X$$

$$\beta^{\frac{1}{n+2}}y=Y$$

$$\frac{\beta^{\frac{1}{n+2}}}{\sqrt{m}}z=Z$$

IV. Physical and Geometrical Aspects

The rate of Higgs fields (ϕ) is given by equation (14) as

$$\phi_4=\frac{\ell}{AB^2}=\frac{\ell}{\beta\sinh T} \tag{27}$$

which leads to

$$\phi=\frac{\ell}{\gamma\beta}\log\,\tanh\frac{T}{2}+N \tag{28}$$

where N is constant of integration.

The spatial volume (R^3) for the model (26) is given by

$$R^3=ABC=\frac{B^{n+2}}{m}=\frac{\beta}{m}\sinh T=\frac{\beta}{2m}\left(e^T-e^{-T}\right) \tag{29}$$

Spatial volume increases exponentially, hence represents inflationary universe.

$$\text{shear}\,(\sigma)=\frac{1}{\sqrt{3}}\left|\frac{A_4}{A}-\frac{B_4}{B}\right|=\frac{(n-1)}{\sqrt{3}}\left|\frac{B_4}{B}\right|=\frac{(n-1)}{\sqrt{3}}\coth T \tag{30}$$

The expansion $(\theta)=\frac{A_4}{A}+\frac{2B_4}{B}=\gamma\coth T$ (31)

$$\frac{\sigma}{\theta}=\frac{(n-1)}{\sqrt{3}\gamma} = \text{constant} \tag{32}$$

Deceleration Parameter $(q)=-\frac{R_{44}/R}{R_4{}^2/R^2}=-3\tanh^2 T+2$ (33)

$q < 0$ leads to $\tanh^2 T > 2/3$ and $q > 0$ leads to $\tanh^2 T < 2/3$
Thus the model (26) represents accelerating and decelerating phases of universe.

The Hubble Parameter $(H)=\frac{R_4}{R}=\frac{\gamma}{3}\coth T$ (34)

V. Inflationary Parameters

We calculate the inflationary parameters i.e. slow role parameters $(\in,\delta)$, third slow parameter (S) and anisotropic parameter ($\widehat{A}_m$) for the model (26) to examine whether these parameters are in excellent agreement with the Planck (2013) results [39] for canonical scalar field.

The scalar factor (R) for the model (26) to the first approximation is given by

$$R=T^{\frac{1}{3}} \tag{35}$$

where $\gamma t+\delta=T$ (36)

The Hubble parameter (H) is given by

$$H=\frac{R_4}{R}=\frac{\alpha}{T} \tag{37}$$

where $\alpha=\frac{\gamma}{3}$ (38)

The slow roll parameters $\in$ and δ is defined by Unnikrishnan and Sahni [40] as

$$\in=-\frac{H_4}{H^2}=\frac{1}{\alpha} \tag{39}$$

and

$$\delta=\in-\frac{\dot{\in}}{2H\in} \tag{40}$$

$$\in=\frac{1}{\alpha} \tag{41}$$

Thus slow role PLI (Power Law Inflation) corresponds to $\in << 1$ which occurs when $\alpha >> 1$.

We also discuss a new Power Law Inflation model in which inflation is driven by canonical scalar field with the Lagrangian

$$L(\phi,X)=X-V(\phi) \tag{42}$$

where

$$X=\frac{\phi_4^2}{2} \tag{43}$$

where ϕ is scalar field. For a generic $L(\phi,X)$, it is convenient to introduce a third role parameter S as given by Hu [41]

$$S = \frac{\frac{\partial C_S}{\partial T}}{H\, C_S}$$

where C_S is the speed of sound of the scalar field as given by Garriga & Mukhanov [42]

$$C_S^2 = \frac{\frac{\partial L}{\partial X}}{\frac{\partial L}{\partial X} + 2X\left(\frac{\partial^2 L}{\partial X^2}\right)} = 1 \tag{44}$$

Thus S = 0 flow roll inflation requires not only $\in << 1$ and $|\delta| << 1$ but also $|S| << 1$. For a canonical scalar field, the value of S is identically zero and this is also the case for kinematically driven as well as the non-canonical model (Unnikrishnan et al. [43])

If $\widehat{A}$ is an anisotropy parameter and H_1, H_2, H_3 are Hubble parameters in x, y, z directions then anisotropy parameter $\widehat{A}$ is defined as

$$\widehat{A} = \frac{1}{3}\left\{\left(\frac{H_1}{H} - 1\right)^2 + \left(\frac{H_2}{H} - 1\right)^2 + \left(\frac{H_3}{H} - 1\right)^2\right\} \tag{45}$$

$$= \frac{2(n-1)^2}{(n+2)^2} \tag{46}$$

VI. Discussion and Conclusion

We find that spatial volume increases exponentially representing inflationary scenario of the universe. The model in general represents anisotropic space time but isotropizes when n = 1. The model also represents accelerating and decelerating phases of universe which matches with the recent astronomical observation because the deceleration parameter $q < 0$ and $q > 0$ respectively. The rate of Higgs field is initially too large but decreases with time and vanishes for large values of T. The model (26) has Point Type singularity at T = 0 (MacCallum [30]). The rate of Higgs field evolves slowly but the universe expands. The Hubble parameter is initially large but leads to finite quantity for large values of time. At the time of evolution of universe, the anisotropy is constant and during the inflation, it is still homogeneous and anisotropic but isotropizes in special case. We find that the observations of inflationary cosmology i.e. slow role parameters (ε,δ), third slow parameter (S) and anisotropic parameter $(\tilde{A}_m)$ are in excellent agreement with the Planck (2013) results [39].
The results obtained in the manuscript matches with the result of cosmic no-hair theorem that any anisotropic metric is diluted and leads to de-sitter space time asymptotically.

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