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Einstein's Theory on Gravitation and Exploration of Dark Matter

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Abstract: *In this paper we are exploring the roots from where the concept of dark matter came first. It is an interesting information that search of dark matter was first superfluous and as the problem of mass to light ratio and other observational problems like rotational curves of spiral galaxies start to let group of people of astrophysics to consider again about perception of dark matter. In the following paper we are analyzing those equations which first time bring in the concept like dark matter in the approach.*
Keywords: *Gravitation, Dark Matter*

I. Introduction

'Dark Matter' was for the first time identified by Fritz Zwicky of California Institute of Technology (Caltech) in 1933. He studied Coma cluster of galaxies and found that the gravity of visible galaxies in the cluster would be far too small for such fast orbits. He inferred that there must be some non-visible form of matter which would provide enough of the mass and gravity to hold the cluster together [1]. Decades of investigation confirmed his analysis. The work of O Chwolson who was an amateur observer of night sky during his sea navigation came first with the concept of gravitational lensing in 1924[2]. His work first impress Einstein and he also get some interesting paper in the field but he soon started to ignore the work as till that time enough positive results and observations were not confirmed the phenomenon of existence of dark matter [3]. We had produced the reason behind approach of discarding by Einstein in table 1. By the table 1 it is clear that the observational evidence of gravitational lensing is found heavier space objects like galaxy clusters and till that time the tools are not sufficient to find the positives results in this direction.

II. Einstein's Theory of Gravitation

By considering that the Universe in which we are leaving is not flat but is appearing flat locally he suggest that The inhomogeneous gravitational field near a massive body being equivalent to (a patchwork of flat frames describing) a curved space-time, the laws of nature (such as the law of gravitation) have to be described by generally covariant tensor equations. Thus the law of gravitation has to be a covariant relation between mass density and curvature. Because the relativistic field equations cannot be derived, Einstein searched for the simplest form such an equation may take. Deriving from Newton's law of gravitation he considered a body at distance r from a massive body of mass M and density ρ for which the force is calculated which is a vector quantity in three space :$\nabla\phi = -\boldsymbol{F}$. Integrating the flux of the force F through a spherical surface surrounding M and using Stokes's theorem, one can show that the potential ϕ obeys Poisson's equation $\nabla^2\phi = 4\pi G\rho$.
He next went with finding of relativistic equation of motion in the limit of week field. In the week field limit equation reduced to

$$\frac{d^2x^\mu}{d\tau^2} + c^2\Gamma^\mu_{00}(\frac{dt}{d\tau})^2$$

Where $\Gamma^\mu_{00} = -\frac{1}{2}g^{\mu\rho}\frac{\partial g_{00}}{\partial x^\rho}$ here g_{00} is derived as time- time component of $g_{\mu\nu}$ and sum over ρ is implied. For a flat space time it will be written as linear equation $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ where second term is small increment to the $g_{\mu\nu}$. Comparing this with the Newtonian equation of motion in the x^i direction we obtain the value of the time-time component of $h_{\mu\nu}$ as $h_{00} = 2\frac{\phi}{c^2}$ from which $g_{00} = 1 + 2\frac{\phi}{c^2} = 1 - \frac{2GM}{c^2r}$.

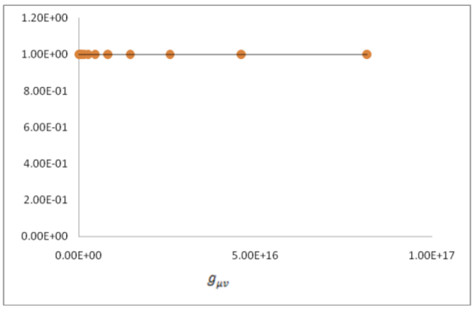

Fig.1: the value of $g_{\mu\nu}$ is observed under the Einstein's equation which shows the flatness of local universe

Stress–Energy Tensor: Let us now turn to the distribution of matter in the Universe. Suppose that matter on some scale can be considered to be continuously distributed as in an ideal fluid. The energy density, pressure and shear of a fluid of nonrelativistic matter are compactly described by the stress–energy tensor $T\mu\nu$. Taking $T\mu\nu$ to describe relativistic matter, one has to pay attention to its Lorentz transformation properties, which differ from the classical case. Under Lorentz transformations the different components of a tensor do not remain unchanged, but become dependent on each other. Thus the physics embodied by $T\mu\nu$ also differs: the gravitational field does not depend on mass densities alone, but also on pressure. All the components of $T\mu\nu$ are therefore responsible for warping the space-time.

Einstein's Equations: We can now put several things together: replacing ρ in the field equation by T_{00}/c^2 and substituting ϕ from value of g_{00} we obtain a field equation for weak static fields generated by nonrelativistic matter:

$$\nabla^2 g_{00} = \frac{8\pi G}{c^4} T_{00}.$$

Let us now assume with Einstein that the right-hand side could describe the source term of a relativistic field equation of gravitation if we made it generally covariant. This suggests replacing T_{00} with $T\mu\nu$. In a matter-dominated universe where the gravitational field is produced by massive stars, and where the pressure between stars is negligible, the only component of importance is then T_{00}. The left hand side of above equation is not covariant but is related to curvature. Now the major task was to replace $\nabla^2 g_{00}$ by a tensor matching the properties of right hand side. The tensor should be of rank two, should be related to the Riemann curvature tensor, should be symmetric in two indices and divergence free. It is called Einstein's Tensor which is covariant law of gravitation as: $G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$. The stress–energy tensor $T\mu\nu$ is the sum of the stress–energy tensors for the various components of energy, baryons, radiation, neutrinos, dark matter and possible other forms. Einstein's formula above expresses that the energy densities, pressures and shears embodied by the stress-energy tensor determine the geometry of space-time, which, in turn, determines the motion of matter[5]. Einstien Tensor get vanishes at weak stationary field this will help us to understand flatness of local universe as in this condition G_{00} reduced to $\nabla^2 g_{00}$. The calculation presented by Einstein in this work helps to overcome the short coming of Newtonian Gravitational field. It will said that space time can exist even in the absence of matter.

Table:1 Calculation of the Einstien ring radius for some standard value of masses of stars/galaxy/galaxy clusters [4].

mass of lensing object (solar masses)	distance (parsecs)	Einstein ring radius degrees	arc seconds
lensed by a star			
1	100	0.0000025	0.0089
	1,000	0.00000078	0.0028
	10,000	0.00000025	0.00089
lensed by a galaxy			
1 x 10^12	100 Mpc	0.0025	8.9
	1,000 Mpc	0.00078	2.9
	10,000 Mpc	0.00025	0.89
lensed by a galaxy cluster			
1 x 10^15	1,000 Mpc	0.025	89
	10,000 Mpc	0.0078	28
	100,000 Mpc	0.0025	8.9

III. Conclusion

The graph of figure.1 shows local flatness of universe what was expected in Einstein's gravitational equation. The concept fit well with the Newtonian gravitational formula. In future work we will investigate the effect of other parameters on structure of local universe.

Reference

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[2]. O. Chwolson (1924).Astronomische Nachrichten 221: 329. doi:10.1002/asna.19242212003.
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[4]. R.Sharma,G.K.Upadhyaya "Study of Dark Matter in Context of Recent Observations and
[5]. Experiments"Lambert Acad. Pub. (2011) see chapter 2 of book for detail calculation.
[6]. M.Roos "Introduction to Cosmology" 3rd Edition John Wiely & Sons ltd. Pub.

Author's Note:

[7]. Data Used in production of the graph of figure 1 are standard. The value of r is in kpc, M is solar mass, G is universal gravitational constant.