IOSR Journal of Applied Physics (IOSR-JAP)
e-ISSN: 2278-4861.Volume 8, Issue 6 Ver. III (Nov. - Dec. 2016), PP 113-122
www.iosrjournals.org

Even and Odd Non-linear Pair-Coherent States and Their Nonclassical Properties

H. H. Salah
Department of Physics, Faculty of Science (Girls Branch), Al-Azher University,Nasr City, Cairo, Egypt

Abstract: *In the present work a general formalism for constructing the even (odd) non-linear pair coherent states has been introduced which in special case lead to the standard coherent states. Since the construction of nonclassical states is a central topic of quantum optics, nonclassical features and quantum statistical properties of the introduced states have been investigated. The second-order correlation function has been used to discuss some nonclassical properties of the photon distribution of the state. The quadrature variances has been used to examine the behavior of the phenomenon of squeezing. The discussion is extended to include the quasiprobability distribution functions (W-Wigner and Q-functions). Also the Pegg - Barnett phase is considered.*

Keywords: *Nonclassical states, Squeezing phenomenon, phase distribution, Quasi-distribution functions*

I. Introduction

The generation of nonclassical fields of large quantum number is one of the most active fields in quantum optics. According to the fundamental principles of quantum optics many new types of nonclassical states have been designed [1]. Coherent states (CS's) of a simple harmonic oscillator with their variants and generalizations have been extensively studied over the last decades [2]. Coherent states [3, 4] are generally constructed by (i) using the displacement operator technique or defining them as (ii) minimum uncertainty states or (iii) annihilation operator eigenstates. A generalized class of the conventional coherent state, called the nonlinear coherent states (NLCS's) or the f-coherent state [5], has been constructed. These states, which correspond to nonlinear algebras rather than Lie algebras, are defined as the right eigenstates of an operator $\hat{a}f(\hat{n})$, which satisfy the eigenvalue equation $\hat{a}f(\hat{n})|\alpha, f\rangle = \alpha|\alpha, f\rangle$, where $f(\hat{n})$ is an operator-valued function of the number operator, $\hat{n} = \hat{a}^{+}\hat{a}$, and they are nonclassical [6, 7]. Recently a different type of finite-dimensional pair coherent states (FPCS) has been introduced and studied, see refs. [8 - 10].This new state is constructed from the eigenstate of the pair operators $\hat{a}^{+}\hat{b} + \frac{\zeta^{q+1}\left(\hat{a}\hat{b}^{+}\right)^{q}}{\left(q!\right)^{2}}$ and the sum of the photon number operators for two modes a and b. Another class of nonclassical states called the two mode nondegenerate entangled state. This state is constructed from the eigenstate of the pair operators $\left(\mu\hat{a}\hat{b} + \nu\hat{a}^{+}\hat{b}^{+} - \sqrt{\mu\nu}\left(\hat{a}\hat{a}^{+} + \hat{b}^{+}\hat{b}\right)\right)$ and the difference of the photon number operators $\left(\hat{n}_a - \hat{n}_b\right)$ for two modes [11].

On the other hand, two modes of a quantized electromagnetic field can become entangled showing many nonclassical effects compared to the decoupled modes. One of these two-mode optical cavities which involves strong entanglement is the pair-coherent states (PCS) [12]. Such states denoted by $|\xi, q\rangle$ are eigenstates of the operator pair $\left(\hat{a}\hat{b}\right)$ and the number difference $\left(\hat{n}_a - \hat{n}_b\right)$ where the parameter q is an integer , ξ may be a complex numbers, $\hat{a}$ and $\hat{b}$ are the annihilation operators of the field modes , $\hat{n}_a = \hat{a}^{+}\hat{a}$ and $\hat{n}_b = \hat{b}^{+}\hat{b}$ These states satisfy

$$\hat{a}\hat{b}|\xi, q\rangle = \xi|\xi, q\rangle and \left(\hat{n}_a - \hat{n}_b\right)|\xi, q\rangle = q|\xi, q\rangle \tag{1}$$

The experimental realization of such nonclassical states is of practical importance. Agarwal [12] suggested that the optical (PCS) can be generated via the competition of 4-wave mixing and two-photon absorption in a nonlinear medium. Another scheme the motion of a trapped ion in a two-dimensional trap has been used for generating vibrational pair coherent states [13]. Another type of correlated two-mode states is the finite-dimensional pair coherent state. This state has been introduced during the study of the statistical properties of a two-photon cavity mode in the presence of frequency converter [14 - 16]. Recently, a new type of a nonlinear entangled pair coherent state has been introduced.under a particular choice of the nonlinearity functions. The resulting recurrence relation has been solved, a feasible state was considered.

Depending on the aforementioned review, the aim of the present work is to introduce new even (odd) nonlinear pair coherent states (ENLPCSs and ONLPCSs). Some statistical properties of these new correlated two mode states are presented. This paper is organized as follows. In section 2 we introduce the definition of NLPCSs with its even and odd states are fully expressed. Certain statistical properties associated with these states are deduced, in section 3,such as the correlation function as well as the phenomenon of squeezing, In section 4, the quasi-probability distribution functions, namely the Wigner and Q-functions are discussed. Also the phase properties are considered, in section 5. Finally the conclusions are presented in section 6.

II. Even and Odd Nonlinear Pair Coherent States

The NLPCS can be defined as the eigenstate of a generalized pair annihilation operator $\hat{A} = \hat{a} f_a(\hat{n}_a)\hat{b} f_b(\hat{n}_b)$ for the two modes, and the photon number difference between the two modes

$$\hat{A}|\xi, q\rangle = \hat{a} f_a(\hat{n}_a)\hat{b} f_b(\hat{n}_b) \quad |\xi, q\rangle = \xi|\xi, q\rangle,$$

$$\left(\hat{a}^{+}\hat{a} - \hat{b}^{+}\hat{b}\right)|\xi, q\rangle = q|\xi, q\rangle \tag{2}$$

Where ξ is a complex parameter of the state while the q parameter is an integer number. The $f_i(\hat{n}_i)$, (i= a, b), are well behaved operator valued functions of the operators $\hat{n}_i$, the NLPCS takes the form,

$$|\xi, q\rangle = N_q \sum_{n=0}^{\infty} \frac{\xi^n}{\sqrt{n!(n+q)!}\, f_a(n+q)!\; f_b(n)!} |n+q, n\rangle \tag{3}$$

By choosing the non-linear function to take the form $f_i(\hat{n}_i) = \sqrt{n_i}$ the state is given by

$$|\xi, q\rangle = N_q \sum_{n=0}^{\infty} \frac{\xi^n}{n!(n+q)!} |n+q, n\rangle \tag{4}$$

in the Fock states of the two modes $|n_a, n_b\rangle$ and the normalization constant Nq is given by

$$N_q = \left[\sum_{n=0}^{\infty} \frac{|\xi|^{2n}}{\left[(n+q)!\right]^2 \left[n!\right]^2}\right]^{-1/2} \tag{5}$$

The PCS can be obtained as a special case from equation (3) when $f_i(n)! = 1$.The even and odd nonlinear pair coherent states can be constructed from equation (4), which are the symmetric and antisymmetric combination of the nonlinear pair coherent states. The general form of NLPCSs is given by

$$|\xi, q\rangle_j^k = N_j^k \sum_{n=0}^{\infty} \frac{|\xi^{kn+j}|}{(kn+j)!(kn+q+j)!} |kn+q+j, kn+j\rangle \tag{6}$$

Where

$$N_j^k = \left[\sum_{n=0}^{\infty} \frac{|\xi|^{2(kn+j)}}{\left[(kn+j+q)!\right]^2 \left[(kn+j)!\right]^2}\right]^{-1/2} \tag{7}$$

For $q = 2q_o$; q_o is integer , then the ENLPCSs are given when k=2, j=0 while theONLPCSs are given when k=2 , j=1.

III. Nonclassicality of the introduced states

Motivations to introduce generalized coherent states theoretically and to produce them in the laboratory are mainly due to their nonclassical properties, which their usefulness in sensitive measurements, is a well-known subject. The experimental feasibility of multi-mode models in a high-Q cavity has been considered by many authors [17]. To discuss the nonclassical properties of these states we shall consider two different phenomena. The first is Poissonian and sub-Poissonian behavior which can be measured using the Glauber second-order correlation function. The second is the squeezing phenomenon which can be quantified via the quadrature variances for the normal squeezing case. As is well known, the squeezing means reduction in the noise of an optical signal below the vacuum limit, in addition to the possibility of potential applications in optical detection in communications networks of gravitational waves [18 - 23]. Thus, in the following subsections we will investigate the influence of the controlling parameters q on the nonclassical behavior of the cavity field where, in particular, the sub-Poissonian distribution and the squeezing phenomenon are emphasized.

3.1. Auto - Correlation function

Practicaly it's well known that, the photon distribution can be measured by photon detectors based on photoelectric eff ect. The importance of the study comes up as a result of several applications, e.g. quantum non-demolition measurement, which can be generated in semiconductor lasers [18] and in the microwave region using masers operating in the microscopic regime [24]. Theoretically, for better understanding the nonclassical behavior of the system, examination of its second-order correlation function is needed. In fact the correlation function is usually used to discuss the sub-Poissonian and super-Poissonian behavior of the photon distribution from which we can distinguish between the classical and nonclassical behavior of the system. For this reason, behaviors of the correlation function for the present states under consideration are discussed. The sub-Poissonian behaviour is characterized by the fact that the variance of the photon number $\left\langle (\Delta\hat{n}_i)^2 \right\rangle$ is less than the average mean photon number $\left\langle \hat{a}_i^+ \hat{a}_i \right\rangle = \left\langle \hat{n}_i \right\rangle$. This can be expressed by means of the normalized second-order correlation function for the mode z in a quantum state $\left|\xi, q\right\rangle_j^k$ [25] as follows:

$$g_z^{(2)}(\xi) = \frac{{}_j^k\langle \xi, q|\hat{n}_z(\hat{n}_z - 1)|\xi, q\rangle_j^k}{{}_j^k\langle \xi, q|\hat{n}_z|\xi, q\rangle_j^{k\,2}}, \forall_z = a, b \tag{8}$$

Where

$${}_j^k\langle \xi, q|\hat{n}_z(\hat{n}_z - 1)|\xi, q\rangle_j^k = N_j^{k^2} \sum_{n=0}^{\infty} \frac{|\xi|^{2(kn+j)}(kn + z' + j)(kn + z' + j - 1)}{[(kn + q + j)!]^2[(kn + j)!]^2} \tag{9}$$

and

$${}_j^k\langle \xi, q|\hat{n}_z|\xi, q\rangle_j^k = N_j^{k^2} \sum_{n=0}^{\infty} \frac{|\xi|^{2(kn+j)}(kn + z' + j)}{[(kn + q + j)!]^2[(kn + j)!]^2} \tag{10}$$

For the first mode $z' = q$, while for the second mode $z' = 0$. For both modes the ENLPCSs are given when k=2, j=0 while the ONLPCSs are given when k=2, j=1.The function $g_z^{(2)}(\xi)$ given by (8) for the mode z serves as a measure of the deviation from the Poissonian distribution that corresponds to coherent states with $g_z^{(2)}(\xi) = 1$. If $g_z^{(2)}(\xi) < 1$ (> 1), the distribution is called sub (super)-Poissonian, if $g_z^{(2)}(\xi) = 2$ the distribution is called thermal and when $g_z^{(2)}(\xi) > 2$it is called super-thermal. In case of two-mode states (NLPCS), the physical content of the even (odd) NLPCS can be revealed by plotting $g_z^{(2)}(\xi)$ against $|\xi|$. At q=0 or 1 the function $g_z^{(2)}(\xi) = 0$, i.e., the NLPCS reduces to the standard PCS. This behavior may be attributed to the fact that, the states present are either vacuum or one photon and for both of them $g_z^{(2)}(\xi)$ is zero. For the first mode, to display the correlation function $g_z^{(2)}(\xi)$ in case of even (odd) NLPCS against the parameter ξ at different values of q =2, 5 and 10. It can be observed that, generally, the function $g_a^{(2)}(\xi)$ has similar sub-Poissonian behavior during all range of ξ whatever the value of the q -parameter, see Fig. 1 (a, b). However there is one main diff erence, at ξ = 0, the function starts depend on q -parameter, i.e., increasing q just changes the beginning of the function to higher values at ξ = 0. For both Even NLPCS and Odd NLPCS cases, Fig. 1 (a, b), the function starts at 0.5, 0.8 and 0.9 (0.66, 0.835 and 0.91) respectively. From the above analysis we conclude that for all values of the q -parameter the correlation function exhibits a sub-Poissonian behaviour for all range of ξ.

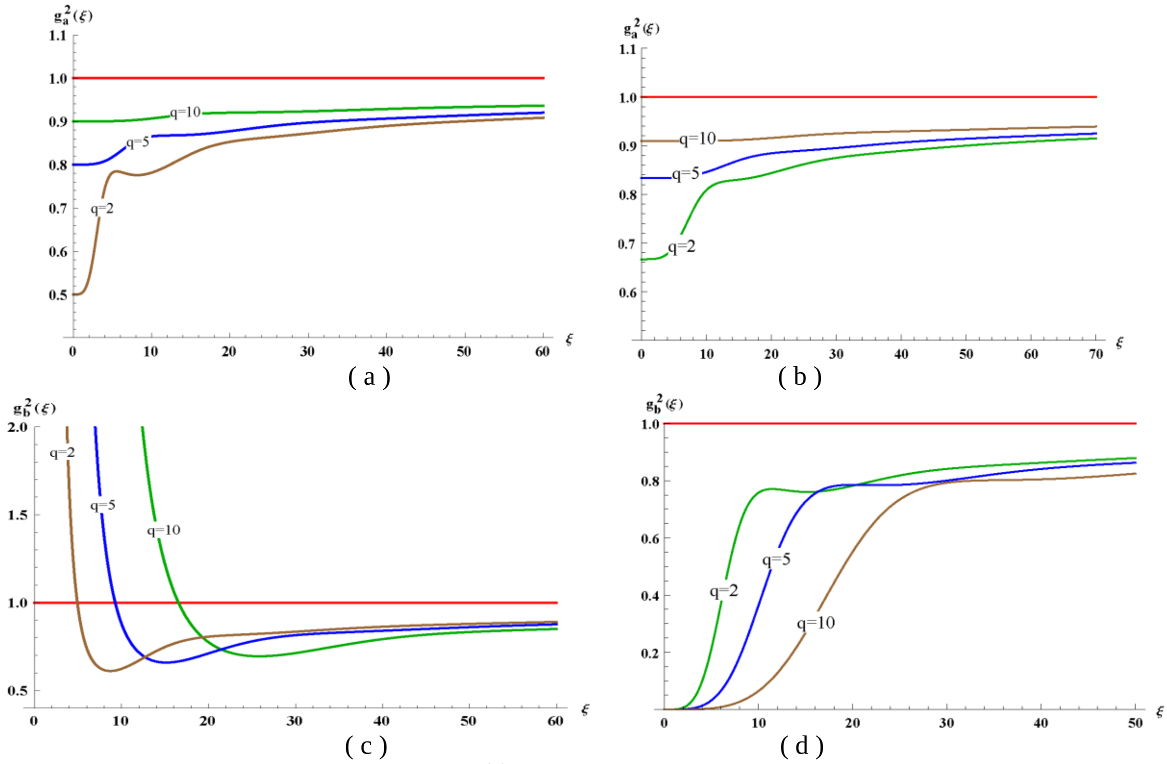


Fig. 1 Second - order correlation function $g^{(2)}(\xi)$ against $|\xi|$ for q = 2, 5 and 10 for even NLPCS and odd NLPCS in case of first mode, (a, b) and second mode (c, d), respectively.

On the other hand, the correlation function for even (odd) NLPCS behaves differently for the second mode case. From visual comparison, Fig. 1 (c), it can be found that in case of E NLPCS the function $g_b^{(2)}(\xi)$ has full super-poissonian behavior and reaches the thermal distribution whatever thevalue of the q parameter at certain range of ξ. With more increasing of ξ full sub-poissonian behavior occur for all q values. From the above analysis we conclude that for all values of the q-parameter the correlation function exhibits a sub-Poissonian behavior for a large values of ξ Regarding ONLPCS, Fig. 1(d), the function $g_b^{(2)}(\xi)$ has full sub-poissonian for all ξ range considered whatever the value of the q parameter. It can be noticed that thefunction starts to take different values at small range of ξ and its behavior improves with increasing the q – parameter.

3.2 The squeezing phenomenon

The phenomenon of squeezing has wide applications in optical communication networks [26], to interferometric techniques [27], and to optical waveguide trap [28]. Generation of squeezed light has been observed in many optical processes [29]. Squeezing means reduction in the noise of an optical signal below the vacuum limit. It is well known that squeezed light is a radiation field without a classical analogue where one of the quadratures of the electric field has less fluctuations than those for vacuum at the expense of increased fluctuations in the other quadrature. In such case the Heisenberg uncertainty relation should be fulfilled. Mathematically the squeezing can be measured by calculating the Hermitian quadrature variances $\hat{X}$ and $\hat{Y}$. In the present subsection, the two-mode frequency sum squeezing defined by the quadrature operators $\hat{X}$ and $\hat{Y}$ [30] will be discussed. Therefore the variances of $\hat{X}$ and $\hat{Y}$ takes the form

$$\hat{X} = \frac{\hat{a}\hat{b} + \hat{a}^{+}\hat{b}^{+}}{2}, \qquad \hat{Y} = \frac{\hat{a}\hat{b} - \hat{a}^{+}\hat{b}^{+}}{2i}, \tag{11}$$

Which satisfy the commutation relation

$$[\hat{X}, \hat{Y}] = i\hat{C} \qquad \text{with} \qquad \hat{C} = \frac{1}{2}(\hat{n}_a + \hat{n}_b + 1), \tag{12}$$

where the uncertainty relation

$$\left\langle\left(\Delta\hat{X}\right)^2\right\rangle\left\langle\left(\Delta\hat{Y}\right)^2\right\rangle \geq \frac{1}{4}\langle C\rangle^2 \tag{13}$$

should be satisfied. To calculate the quadrature variances $\left\langle\left(\Delta\hat{X}\right)^2\right\rangle$ and $\left\langle\left(\Delta\hat{Y}\right)^2\right\rangle$ the definition of the operators $\hat{X}$ and $\hat{Y}$ will be used. The variance is given in terms of annihilation and creation operators expectation values by

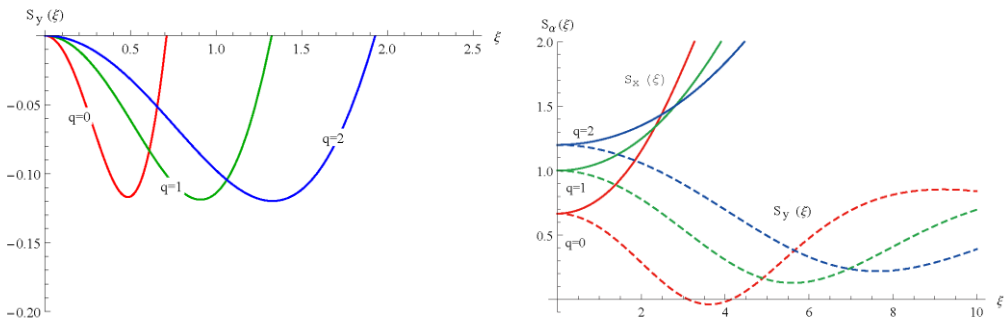
$$\left\langle\left(\Delta\hat{X}\right)^2\right\rangle = \frac{1}{4}+\frac{1}{4}[2\langle\hat{n}_a\hat{n}_b\rangle+\langle\hat{n}_a\rangle+\langle\hat{n}_b\rangle+2\Re e\left(\left\langle\hat{a}^2\hat{b}^2\right\rangle\right)]-\left(\Re e\left\langle\left(\hat{a}\hat{b}\right)\right\rangle\right)^2,$$

$$\left\langle\left(\Delta\hat{Y}\right)^2\right\rangle = \frac{1}{4}+\frac{1}{4}[2\langle\hat{n}_a\hat{n}_b\rangle+\langle\hat{n}_a\rangle+\langle\hat{n}_b\rangle-2\Re e\left(\left\langle\hat{a}^2\hat{b}^2\right\rangle\right)]+\left(Im\left\langle\left(\hat{a}\hat{b}\right)\right\rangle\right)^2. \tag{14}$$

The state would possess $\hat{X}$ -quadrature frequency sum squeezing if the S-factor defined by

$$S(\xi)=\frac{2\left\langle\left(\Delta\hat{X}\right)^2\right\rangle-\langle\hat{C}\rangle}{\langle\hat{C}\rangle} \tag{15}$$

Satisfies the inequality $-1 \leq S < 0$.A similar expression for the second quadrature with $\hat{X}$ replaced by $\hat{Y}$ .To exhibit the phenomenon of the squeezing and to illustrate the general features related to the present state, the function S (ξ) was plotted against ξ ,for deferent values of the parameter q , see Fig. 2. In case of ENLPCS, Fig. 2 (a), generally the state shows squeezing in the second quadrature for the considered q values 0, 1 and 2. The squeezing persists until ξ = 0.48, 0.93 and 1.35 corresponding to different q values 0, 1 and 2 respectively. Increasing the values of q more squeezing can be seen (even stronger) and the squeezing interval increases. This means that the squeezing for the present state is too sensitive to any variation in the value of the parameter q. In case of ONLPCS, Fig. 2 (b), the phenomenon of squeezing disappear for all the cases considered.



(a) (b)

Fig. 2 The squeezing function S (ξ) as a function of the parameter ξ for q = 0, 1 and 2, (a) For the second quadrature of ENLPCS and (b) For the two quadratures of ONLPCS.

IV. Quasiprobability Distribution Functions

The quasiprobability distribution functions (QDF) is perhaps one of the most well-known important functions in quantum optics [31 - 34]. These functions are important tools to shed more of light on the nonclassical features of the radiation fields. There are three known types of these functions: the Glauber–Sudarshan P function [35], the Wigner-Moyal W function [36] and the Husimi-Kano Q function [37]. These functions are corresponding to normally ordered, symmetric, and anti- symmetric. W-function can take on negative values for some states and this is regarded as an indication of the non-classical effects. Also, it is well known that Q function is positive definite at any point in the phase space for any quantum state.The s-parameterized characteristic function (CF) for the two-mode states is defined as follows

$$C_j^k(\lambda_a,\lambda_b,s)=\mathrm{Tr}[\hat{\rho}_j^k\,\hat{D}(\lambda_a)\hat{D}(\lambda_b)]\exp\{\frac{s}{2}\left(|\lambda_a|^2+|\lambda_b|^2\right)\}, \tag{16}$$

Where $\hat{\rho}_j^k$ is the density operator $\hat{\rho}_j^k = |\xi, q\rangle_j^k {}_j^k\langle q, \xi|$, $\hat{D}(\lambda)$ is the displacement operator given by $\hat{D}(\lambda) = \exp(\lambda \hat{a}^+ - \lambda^* \hat{a})$, and $\lambda = |\lambda| e^{i\theta}$. Here, s is ordering parameter where s = (−1)1 means (anti)normal ordering and s = 0 is symmetrical or Weyl ordering [38, 39].The s-parameterized QDF is the Fourier transformation of the s-parameterized CF [38, 40, 41].

$$F_j^k(\beta_a, \beta_b, s) = \left(\frac{1}{\pi^2}\right)^2 \iint C_j^k(\lambda_a, \lambda_b, s) \exp(\lambda_a^* \beta_a + \lambda_b^* \beta_b - \lambda_a \beta_a^* - \lambda_b \beta_b^*) d^2\lambda_a d^2\lambda_b. \tag{17}$$

Where the real parameter s defines the corresponding phase space distribution and associated with the ordering of the field bosonic operators. We consider a phase space QDF for the present states. To begin the state (6) will be written in the form

$$|\xi, q\rangle_j^k = \sum_{n=0}^{\infty} B_{nj}^k(\xi, q) |kn + q + j, kn + j\rangle, \tag{18}$$

Where

$$B_{nj}^k(\xi, q) = N_j^k \frac{|\xi|^{(kn+j)}}{(kn+j)!(kn+q+j)!} \tag{19}$$

It is clear that, the probability of finding $(kn+q+j)$ photons in the 1st mode, and $(kn+j)$ photons in the 2nd mode in the state $|\xi, q\rangle_j^k$ is given by

$$P_j^k(kn+q+j, kn+j) = \left|B_{nj}^k(\xi, q)\right|^2 \tag{20}$$

After obtaining the parameterized characteristic function by using minor algebra and evaluating the integral in equation (17) for s = 0 and s = −1, corresponding to the Wigner and the Q-function, respectively. Therefore the Wigner, and Q-function will be discussed in the next subsections.

4.1 The Wigner function

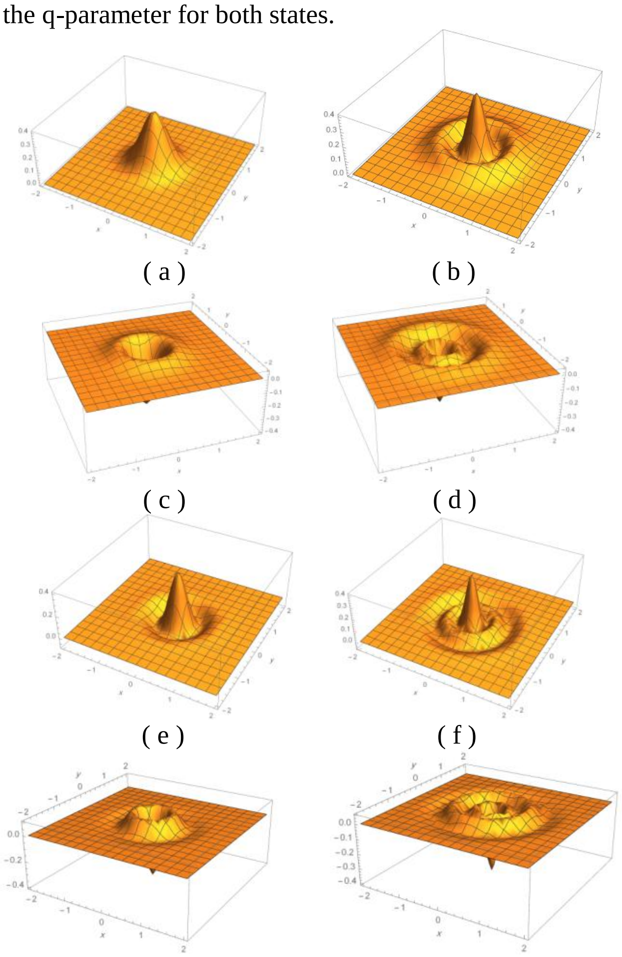
The Wigner function $W(\beta_a, \beta_b)$ can be obtained by inserting s = 0 in equation (17) as follows

$$W(\beta_a, \beta_b) = \frac{4}{\pi^2} \exp[-2(|\beta_a|^2 + |\beta_b|^2)] \sum_{n=0}^{\infty} \sum_{l=0}^{n+q} \sum_{m=0}^{n} \left|B_{nj}^k(\xi, q)\right|^2 \binom{n+q}{l} \binom{n}{m} \times (-2)^{l+m} L_l[2|\beta_a|^2] L_m[2|\beta_b|^2] \tag{21}$$

Where $L_n^q(x)$ are the associated Laguerre polynomials given by

$$L_n^q(x) = \sum_{r=0}^{n} \binom{n+q}{n-r} \frac{(-1)^r}{r!} x^r \tag{22}$$

To discuss the behavior of the Wigner function $W(\beta_a, \beta_b)$ for the even (odd) NLPCS, the function has been plotted in subspace $\beta_a = \beta_b = \alpha$ against $\Re e$ (α) and the Im (α) for different values of the q-parameter with keeping ξ parameter at constant value. For example, we have considered the cases in which q =0, 1, 2 and 3 with the parameter ξ = 0.75. For even (odd) NLPCS, in the absence of the q-parameter, i.e. q =0, the Wigner function shows Gaussian distribution with one positive peak for both states but it has wide shape at the center in case of even state (sharp peak with an interference ring around it at the base with spreading over the plane in case of odd state). Also the positive peak has a symmetrical shape around zero, while its maximum reaches nearly the value 0.4 for both states, see Fig. 3 (a, b).When the value of the q-parameter increases, for example q = 1, Fig. 3 (c, d), the Wigner function attained negative downward peak with a crater at its base which has symmetrical shape around zero for both states (in case of ONLPCS, the crater at the base has spreading over the plane.).This behavior for the Wigner function is a signature of the nonclassical effect. For q = 2 the function changes the direction of its peak again to become a positive peak upward at the center with attaining negative values at the base and the peak gets less broadening compared with the case in which q = 0 for both states even (odd) NLPCS. Furthermore, the maximum value of the function decreases for both states, in addition an interference ring around the peak starts to appear in case of ENLPCS (spreading of Wigner over the plane is clear as q increases, in case of odd state), as shown in Fig. 3 (e, f). This indicates that, for both states there is a reduction in the nonclassical effect and the oscillatory behavior starts to appear for even values of the q-parameter. Finally when we consider the case in which q = 3, the Wigner function shows similar behavior to that of q = 1 for both states, where the crater at the base of downward peak gets more clear with smaller diameter and an interference ring around it in case of ENLPCS. Also the oscillations increases around the center with spreading of Wigner over the plane is clear as q increases in case of ONLPCS, see Fig. 3 (g, h). Generally, from the above results it is clear that the Wigner function for the even (odd) NLPCS, attains positive values at q = 0 in addition to symmetrical behavior around the origin. Also the function shows a negative peak at the center for odd q and positive peak for even q. This means that the nonclassical effect behavior appears only for the odd numbers of the q-parameter while it disappears for even numbers. The function gets more sensitive to the variation in the q-parameter for both states.



(a) (b) (c) (d) (e) (f) (g) (h)

Fig. 3 The W-Wigner function against $\Re e$ (α) and $\mathrm{I}m$ (α) for fixed value of ξ = 0.75, for even and odd NLPCS (a) and (b) q = 0, (c) and (d) q = 1, (e) and (f) q = 2, (g) and (h) q = 3, respectively.

4.2. The Q-function

By setting s = -1 in equation (17), then the Q-function will has the form

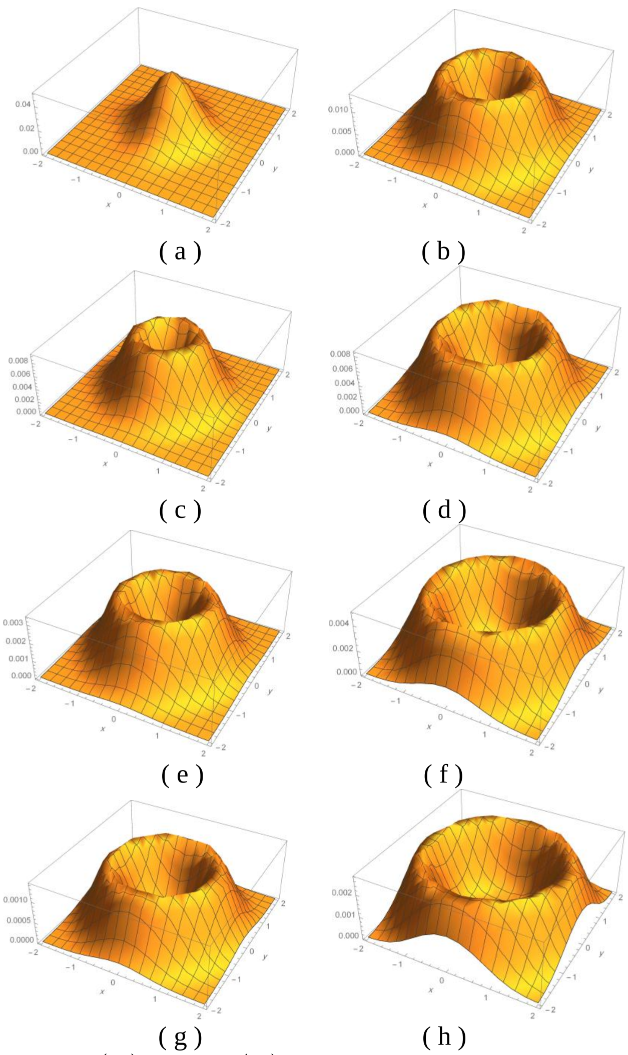
$$Q(\beta_a,\beta_b)=\frac{1}{\pi^2}\exp[-(|\beta_a|^2+|\beta_b|^2)]\sum_{n=0}^{\infty}\sum_{l=0}^{n+q}\sum_{m=0}^{n}\left|B_{n\,j}^{\,k}(\xi,q)\right|^2\binom{n+q}{l}\binom{n}{m} \times(-1)^{l+m}L_l[|\beta_a|^2]L_m[|\beta_b|^2] \tag{23}$$

Where $\beta_a, \beta_b \in C$, with $|\beta_a\rangle and |\beta_b\rangle$ being the usual coherent states. Since there are four variables associated with the real and imaginary parts of β_a and β_b. Therefore, by taking a subspace determined by, $\beta_a = \beta_b = \beta$ [42]. In this subspace the Q-function for the state (6) is expressed in the equivalent form

$$Q(x,y) = \frac{1}{\pi^2}\exp\left[-2\left(x^2+y^2\right)\right]\left|\sum_{n=0}^{\infty} B_{nj}^{k}(\xi,q)\frac{\beta^{2kn+2j+q}}{(kn+q+j)!(kn+j)!}\right|^2 \tag{24}$$

Where $\beta = x + iy$.

To display the behavior of the Q-function from equation. (23) for the even (odd) NLPCS, has been plotted against $\Re(\beta)$ and $\mathrm{Im}(\beta)$ using the same values of the parameters of the Wigner function. From Fig. 4 (a, b) and for q =0 it is easy to observe that the Q-function shows, in case of ENLPCS Fig. 4 (a), almost Gaussian shape with a single positive peak centered at the origin. The peak shows symmetry around zero. In case of ONLPCS, Fig. 4 (b), the function has crater at the origin. Increasing the value of the q-parameter, q = 1, both states even (odd) displays the crater shape with a reduction in their maximum value but the crater radius in the odd state is greater than that of the even case, see Fig. 4 (c, d). With more inceasing of q -parameter the crater radius increases for both states even (odd) NLPCS, see Fig. 4 (e, f, g and h). The obtained results reveal the non-classical nature of the even (odd) NLPCS.



(a) (b) (c) (d) (e) (f) (g) (h)

Fig. 4 The Q-function against $\Re(\beta)$ and $\mathrm{Im}(\beta)$ with the same parameters as in Fig. 3, for even and odd NLPCS (a) and (b) q = 0, (c) and (d) q = 1, (e) and (f) q = 2, (g) and (h) q = 3, respectively.

V. Phase properties

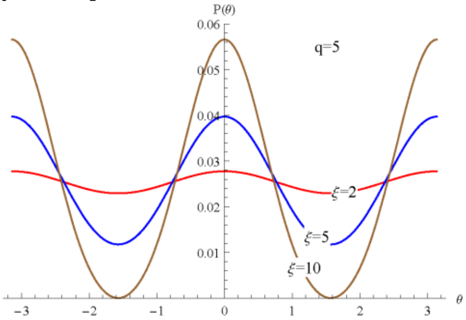
In this section the phase distribution for the present states has been discussed using the Pegg-Barnett formalism [43 - 45]. It is well known that the phase operator is defined as the projection operator on a particular phase state multiplied by the corresponding value of the phase. Therefore the Pegg-Barnett phases distribution function $P_{PB}(\theta_1,\theta_2)$ for the present state will take the form [44, 45]

$$P_{PB}(\theta_1,\theta_2)=\frac{1}{4\pi^2}\sum_{n,m=0}^{\infty}B_{nj}^{k}(\xi,q)B_{mj}^{k*}(\xi,q)\exp\{i(n-m)(\theta_1+\theta_2)\} \tag{25}$$

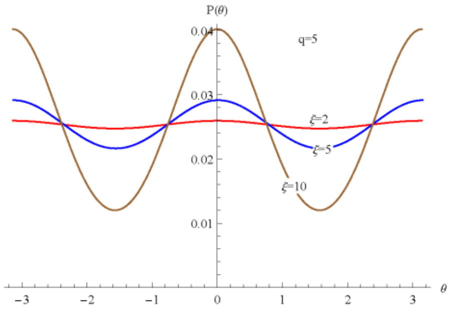
Where $\theta_1 and \theta_2$ are the phases related to the two modes a and b with the normalization condition

$$\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}P_{PB}(\theta_1,\theta_2)d\theta_1 d\theta_2=1$$

The phase distribution will depend on the sum of the phases of the two modes, due to the correlation between them. Therefore, we consider in equ. (25) $\theta=\theta_1+\theta_2$. The Pegg-Barnett phases distribution function $P_{PB}(\theta)$ is plotted against the angle θ for a fixed value q = 5 and different values of the parameter ξ = 2,5 and 10, for even (odd) NLPCS, see Fig. 5 (a). In case of ENLPCS, the phase distribution shows a center peak at $\theta=0$ with two symmetric wings around the peak center at $\theta=\pm\pi$. Also, there is a reduction in the maximum value of the peak and wings when the parameter ξ decreases from the value 10 to 5 and there is no phase distribution for more lower ξ. Fig. 5 (b) display the function behavior in case of ONLPCS. Generally the phase distribution function has the same behavior as the case for ENLPCS with decreasing the value of ξ. However, the function is shifted upward in the positive values with a reduction in the maximum value of the peak and the wings when the parameter ξ decreases. This means that the parameter ξ plays the crucial role of controlling the positivity of the phase distribution.



(a)



(b)

Fig. 5 The phase distribution $P_{PB}(\theta)$ against θ for a fixed value q = 5 and different values of the parameter ξ = 2,5 and 10, (a) For even NLPCS and (b) for odd NLPCS.

VI. Conclusion

Based on the well-known nonlinear pair coherent states approach in quantum optics filed, the present work proposed a formalism is to introduce new even (odd) nonlinear pair coherent states (ENLPCS and ONLPCS). Certain statistical properties associated with these states are deduced. Regarding the second-order correlation function, in the first mode, for both even (odd) NLPCS the system exhibits a sub-Poissonian behavior for all values of the q-parameter and ranges of ξ.However, for the second mode in case of ENLPCS the function has full super-poissonian behavior and reaches the thermal distribution whatever the value of the q parameter at certain range of ξ. For all values of the q-parameter the correlation function exhibits a sub-Poissonian behavior for a large values of ξ. Regarding ONLPCS, the function has full sub-poissonian for all ξ range considered whatever the value of the q parameter. Also the squeezing phenomenon has been investigated for which we found that ENLPCS shows squeezing in the second quadrature for the considered q values while for ONLPCS the phenomenon of squeezing disappearance of for all the cases considered. For both even and odd NLPCS, the Wigner function behavior reports nonclassical properties for the odd values of the q-parameter. That means the function gets more sensitive to the variation in the q-parameter for both states. In the meantime the Q-function displays Gaussian behavior in case of ENLPCS with a single positive peak centered at the origin. The peak shows symmetry around zero. In case of ONLPCS, the function has crater at the origin. Increasing the value of the q -parameter, q = 1, both states even (odd) displays the crater shape with a reduction in their maximum value but the crater radius in the odd state is greater than that of the even case. With more inceasing of q -parameter the crater radius increases for both states even (odd) NLPCS. The obtained results reveal the non-classical nature of the even (odd) NLPCS. Finally the properties of the present states have been examined in terms of the phase distribution function introduced by Barnett and Pegg. Generally the phase distribution function has the same behavior for both even (odd) NLPCS with decreasing the value of ζ. However, the odd function is shifted upward in the positive values with a reduction in the maximum value of the peak and the wings when the parameter ζ decreases. This means that the parameter ζ plays the crucial role of controlling the positivity of the phase distribution.We hope that the even (odd) nonlinear pair coherent states have been introduced in the present work may be also find their applications in various physical fields, as well as the standard pair coherent states.

Acknowledgments

The auther would like to thank Mrs. S. Sanad of the Mathemtics Depat-ment, Faculty of Science, Al-Azhar University (Girls Branch) for helpingin the programs for the figures.

References

[1]. H. Fu, Y. Feng, A.I. Solomon, *J. Phys. A 33*, (2000), 2231; A.V. Barranco, J. Roversi, Phys. Rev. A 50, (1994), 5233.
[2]. V.V. Dodonov, *J. Opt. B: Quantum Semiclass. Opt. 4, R1* (2002);
[3]. E. Schrödinger, *Naturwiss. 14*, (1926), 664.
[4]. R. J. Glauber, *Phys. Rev. 131*, (1963), 2766 .
[5]. R.L. de Matos Filho, W. Vogel, *Phys. Rev. A 54*, (1996), 4560; V.I. Manko, G. Marmo, E.C.G. Sudarshan, F. Zaccaria, *Phys. Scr.55*, (1997), 528.
[6]. M M Nieto and D R Truax, *Phys. Rev. Lett. 71*, (1993), 2843 .
[7]. L´ opez-Pe˜ na R, Man'ko V I, Marmo G, Sudarshan E C G and Zaccarria F *J. Russ. Laser Res. 21*, (2000), 305.
[8]. E. M. Khalil, *J. Phys. A: Math. Gen., 39*, (2006), 11053.
[9]. E. M. Khalil, M. S Abdalla. and A. S.-F Obada., *Ann. Phys. (N.Y.), 321*, (2006), 421 .
[10]. Fu Hong-Chen and Sasaki Ryu, *J. Phys. A: Math. Gen., 29*, (1996), 5637 .
[11]. M.S. Abdalla, A-S.F. Obada and E.M. Khalil, *IL Onuovo cement 125*, (2010), 1509-1528 .
[12]. G.S. Agarwal, *Phys. Rev. Lett. 57*, (1986), 827 ; G. S. Agarwal, *J. Opt. Soc. Am. B* **5**, , (1988), 1940.
[13]. S.-C. Gou, J. Steinbach and P.L. Knight, *Phys. Rev. A 54*, (1996), 4315.
[14]. A.-S.F. Obada and E.M. Khalil, *Optics. Comm. 260*, (2006), 19-24.
[15]. E .M. Khalil, *Inter. J. of Theoretical Physics, 46*, (2007), 2816-2828.
[16]. A.-S.F. Obada, E.M. Khalil, M.A. El-Deberky and S. Sand, *Quant. Inf. Rev. 3, No. 1*, (2015), 9-15.
[17]. P.R. Bermann (Ed.), Cavity *Quantum Electrodynamics Supplement 2 to Advances in Atomic, Molecular, and Optical physics*, Academic, San Diego, (1994); R.W. Boyd, Nonlinear Optics, Academic Press, Boston, MA, (1992); S.P. Tewari, G.S. Agarwal, *Phys. Rev. Lett. 56*, (1986), 1811; X.S. Li, C.D. Gong, *Phys. Rev. A 33*, (1986), 2801 .
[18]. J.H Shapiro, H.P Yuen, M.J.A Machado, *IEEE Trans. Inf. Theory IT 25*, (1979), 179.
[19]. M.J Collett, C.W Gardiner,. *Phys. Rev. A 30*, (1984), 1386.
[20]. M.J Collett, R Loudon, *J. Opt. Soc. Am. B 4*, (1987), 1525.
[21]. C.M Caves, B.L Schumaker, *Phys. Rev. A 31*, (1985), 3068.
[22]. B.L Schumaker, C.M Caves, *Phys. Rev. A 31*, (1985), 3093 .
[23]. C.M Caves, *Phys. Rev. D 23*, (1981), 1693.
[24]. G. Remoe, F. Schmidt-Kaler and H. Walther, *Phys. Rev. Lett. A 293*, (1990), 2783-2786 .
[25]. R. Loudon, *the quantum theory of light* (Clarendon Press, Oxford), (1983).
[26]. Yuen H. P. and Shapiro J. H., *IEEE Trans. Inform. Theory, 24*, (1978), 657 ; 26, (1980), 78; J. H Shapiro, H. P Yuen. and M.J.A Machado., *IEEE Trans. Inform. Theory, 25*, (1979), 179.
[27]. C.M. Caves, B.L. Schumaker, *Phys. Rev. A 31*, (1985), 3068-3092.
[28]. J. H Shapiro, *Opt. Lett., 5*, (1980), 351.
[29]. P Meystre and D F Walls (eds) *Nonclassical Effect in Quantum Optics* (New York: AIP), (1991).
[30]. M Hillery, *Phys. Rev. A, 40*, (1989), 3147.
[31]. W A unsche, *Acta Phys. Slov. 48*, (1998), 385 .
[32]. N Moya-Cessa and P L Knight, *Phys. Rev. A 48*, (1993), 2479.
[33]. Th Richter, *J. Mod. Opt,. 48*, (2001), 1881.
[34]. W A unsche, J. Opt. B: *Quantum Semiclass. Opt. 6*, (2004), 159.
[35]. R J Glauber, *Phys. Rev. 131*, (1963), 2766; E C G Sudarshan, *Phys. Rev. Lett. 10*, (1963), 277 .
[36]. E Wigner, *Phys. Rev. 40*, (1932), 749 ; E Wigner,Z. *Phys. Chem. B 19*, (1932), 203.
[37]. K Husimi, *Proc. Phys. Math. Soc. Japan 22*, (1940), 264; Y Kano *J. Math. Phys. 6*, (1965), 1913.
[38]. K E Cahill and R J Glauber, *Phys. Rev. 177*, (1969) ,1857; K E Cahill and R J Glauber, *Phys. Rev. 177*, (1969), 1882.
[39]. N Moya-Cessa and P L Knight, *Phys. Rev. A 48*, (1993), 2479.
[40]. W Vogel and D-G Welsch, *Quantum Optics* (Weinheim: Wiley-VCH) (2006).
[41]. S M Barnett and P M Radmore, *Methods in Theoretical Quantum Optics* (Clarendon: OxfordUniversity Press) (1997).
[42]. Hyo Seok Yi, B.A. Nguyen and J. Kim, J. *Phys. A: Math.Gen. 37*, (2004), 11017-11036.
[43]. B. A Nguyen. and M. D Truong M. D., *J. Opt. B.: Quantum Semi. Opt., 4*, (2002), 289.
[44]. D. T Pegg. and S. M Barnett., *Eur. Phys. Lett., 6*, (1988), 483 ; S. M Barnett. and D. T Pegg., *J. Mod. Opt., 36*, (1989), 7.
[45]. D. T Pegg. and S. M Barnett., *Quantum Opt., 2*, (1997), 225.