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Banerjee-Dzubur Dimensional Impairing Theorem: why we May Never Unify Gravity & Electromagnetism.

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Abstract: *For the past few decades, scientists are more desperate than ever to unify gravity and electromagnetism. It is the last missing piece in our picture of grand unification however, in this paper, I want to express my view about a mathematical possibility that why we may never be able to unify gravity and electromagnetism. As we will see, gravity and electromagnetism may actually be a same phenomenon, or rather, the consequence of a same phenomenon. They may unify, but in a higher dimension in which case, we may never be able to unify them while working in a 4 dimensional space time frame of reference.*

Keywords: *Banerjee-Dzubur Dimensional Impairing Theorem, gravity, electromagnetism, Perception of force fields, absoluteness of field properties.*

I. Introduction

There have been several attempts at creating a unified field theory beginning with the Riemannian geometry[1][2][3] of general relativity[4][5][6], followed by attempts of Einstein to incorporate electromagnetic fields into a more general geometry, since ordinary Riemannian geometry seemed incapable of expressing the properties of the electromagnetic field. However, Einstein was not alone in his attempts to unify electromagnetism and gravity; a large number of mathematicians and physicists, including Hermann Weyl [7][8], Arthur Eddington [9][10], Theodor Kaluza [11][12], and R. Bach also attempted to develop approaches that could unify these interactions. These scientists pursued several avenues of generalization, including extending the foundations of geometry and adding an extra spatial dimension. When the equivalent of Maxwell's equations for electromagnetism is formulated within the framework of Einstein's theory of general relativity, the electromagnetic field energy contributes to the stress tensor and thus to the curvature of space-time, which is the general-relativistic representation of the gravitational field; or putting it another way, certain configurations of curved space-time incorporate effects of an electromagnetic field. This suggests that a purely geometric theory ought to treat these two fields as different aspects of the same basic phenomenon. This is important as Banerjee-Dzubur Dimensional Impairing Theorem extends this notion and modifies it. Banerjee-Dzubur Dimensional Impairing Theorem states that if there is an ‘n’ dimensional force field F_n[13][14][15], imparting a displacement curve φ_n to the test particle, and if we try to study its effects from a frame of referencemade up of ‘m’ dimensions, (where m is any integer number smaller than ‘n’, i.e. m ϵ [0, n-1]) then, the result obtained will be consistent with an ‘m’ dimensional force field F_m, that imparts a displacement curve φ_m to the test particle. There is no experiment that we can perform, to indicate the fact that it is a higher dimensional phenomenon. Banerjee-Dzubur Dimensional Impairing Theorem also predicts that two or more same dimensional force fields of different nature may unify at higher dimensions.

II. Mathematical Operators Used

1. Let Δ_x^n represent $\frac{d}{dt}(\frac{d}{dt}(\frac{d}{dt}(\frac{d}{dt}(\dots\dots\dots\frac{d}{dt}(\frac{dx}{dt})$ n times i.e. nth differentiation of x with respect to time.
2. ∂_x^n represent $\frac{\partial}{\partial x}(\frac{\partial}{\partial x}(\frac{\partial}{\partial x}(\frac{\partial}{\partial x}(\frac{\partial}{\partial x}(\frac{\partial}{\partial x}(\dots\dots\frac{\partial}{\partial x}(\frac{\partial\varphi}{\partial x})$ n times i.e. nth partial differentiation of φ with respect to x.
3. $\partial_{x1,\, x2,\, x3,\, \dots\dots xn}{}^n$ represent $\frac{\partial}{\partial x1}(\frac{\partial}{\partial x2}(\frac{\partial}{\partial x3}(\frac{\partial}{\partial x4}(\frac{\partial}{\partial x5}(\frac{\partial}{\partial x6}(\dots\dots(\frac{\partial\varphi}{\partial xn})$ i.e. nth partial nth partial differentiation of φ with respect to x1, x2, x3, x4,….. xn.

Note: Neither Δ, nor ∂, follow the multiplicative equality that is,
$(\Delta_x^n) * (\Delta_x^n) \neq \Delta_x^{2n}$ but is $= (\Delta_x^n)^2$ which implies, $\Delta_x^{mn} \neq (\Delta_x^n)^m$
Similarly, $(\partial_x^n) * (\partial_x^n) \neq \partial_x^{2n}$ but is $= (\partial_x^n)^2$ which implies, $\partial_x^{mn} \neq (\partial_x^n)^m$

1: Analysis Of Effects Caused By Uni And Multi - Dimensional Forces.

This section deals with the first part of *“Banerjee-Dzubur Dimensional Impairing Theorem*” – analysis of how the effect of an ‘n’ dimensional force is perceived, when measuring from aframe of reference which is capable of supporting ‘n’ or more dimensions. We will solve for both cases, (a) when the force is constant and (b) a varying force [16], which will give the generalized equation for a force field of ‘i’ dimensions where, i = number of dimensions of the force which will be under consideration.

Note: During analysis of the forces and its effects, it is assumed that no other forces except for the one which is under study, is acting on the test particle.

1a:Analysis Of One-Dimensional Force And Its Effect.

Let there be aone-dimensional force, represented by the vector $\vec{F}_1$ that imparts a displacement curve (the curve traced out by the body during its motion under the influence of force $\vec{F}_1$) $\varphi_1$to the test particle. Let the observed effect be $\vec{E}_{1.}$. The subscripts indicate the number of dimensions of the force, under whose analysis, the respective vector is introduced. Example: The effect vector$\vec{E}_1$ indicates that it was introduced under the analysis of one dimensional forces and hence, must be the effect vector of a one-dimensional force.

1a.1. For A Constant One-Dimensional Force

When the force is constant and varies with respect to just distance along a single dimension, then we can represent the force as,

$\vec{F}_1$ = F (x), φ_1= φ(x) where x represents the dimension along which the force varies.

Note: how the number of dimensions the test particle is capable of undergoing displacement in, due to the force, is equal to the number of dimensions of the force that is acting on it.

Let this force act on the test particle such that it undergoes displacement according to a function φ_1 = φ(x). Therefore, effect $\vec{E}_1$ = acceleration of the test particle = $\frac{\mathrm{d}}{\mathrm{dt}}(\frac{\mathrm{d}(\varphi 1)}{\mathrm{dt}})$

Here, φ_1 = φ(x) because, the force is constant with respect to time. Thus the displacement of the test particle, at a given distance from the reference frame's origin, will also be constant. It is eminent in the graph below.

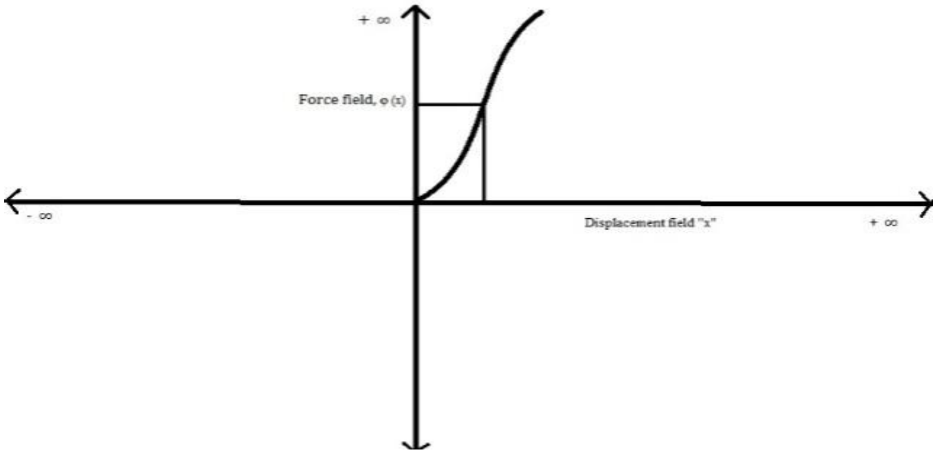

Figure 1.a.1 1-*dimensional steady force field vs displacement curve.*

Now, from Chain rule for multivariable functions, [17][18]

$\mathrm{d}\varphi_1 = \frac{\partial \varphi 1}{\partial x}(\mathrm{dx})$

or, $\frac{d\varphi 1}{dt} = \frac{\partial \varphi 1}{\partial x}(\frac{dx}{dt})$

Let us define a velocity function $\Psi_1 = \Psi\,(x, t) = \frac{d\varphi 1}{dt}$

Therefore $\vec{E}_1 = \frac{d\Psi 1}{dt}$

Now, $\mathrm{d}\Psi_1 = \frac{\partial\Psi 1}{\partial x}(\mathrm{dx}) + \frac{\partial\Psi 1}{\partial t}(\mathrm{dt})$ as Ψ is a function of both x and t.

$\frac{d\Psi 1}{dt} = \frac{\partial\Psi 1}{\partial x}(\frac{dx}{dt}) + \frac{\partial\Psi 1}{\partial t}$

Where, $\frac{\partial\Psi 1}{\partial x}(\frac{dx}{dt}) = \{\frac{\partial}{\partial x}(\frac{\partial\varphi 1}{\partial x})\}(\frac{dx}{dt})^2 = \partial_x^{\,2}(\Delta_x^{\,1})^2$

And $\frac{\partial\Psi 1}{\partial t} = \frac{\partial\varphi 1}{\partial x}\{\frac{d}{dt}(\frac{dx}{dt})\} = \partial_x^{\,1}(\Delta_x^{\,2})$ as here, φ_1 is constant with respect to time t.

or,$\vec{E}_1 = \frac{\mathrm{d}}{\mathrm{dt}}\{\frac{\mathrm{d}(\Psi 1)}{\mathrm{dt}}\} = \{\frac{\partial}{\partial x}(\frac{\partial\varphi 1}{\partial x})\}(\frac{dx}{dt})^2 + \frac{\partial\varphi 1}{\partial x}\{\frac{d}{dt}(\frac{dx}{dt})\} = \partial_x^{\,2}(\Delta_x^{\,1})^2 + \partial_x^{\,1}(\Delta_x^{\,2})$.

Therefore, for a constant one-dimensional force, the effect as observed is

$\vec{E}_1 = \partial_x^{\,2}(\Delta_x^{\,1})^2 + \partial_x^{\,1}(\Delta_x^{\,2})$……….Eq(1)

1a.2. For A Varying One-Dimensional Force.

When the force varies with respect to time along with displacement along a single dimension, then we can represent the force as,

$\vec{F}_1$ = F (x, t), $\varphi_1 = \varphi(x, t)$. where t is the time dimension.
Note: how again the number of dimensions the test particle is capable of undergoing displacement in due to the force, is equal to the number of dimensions of the force that is acting on it.
Displacement curve of the test particle is governed by the force acting on it. Since the force in this case changes with time, thus the displacement function of the test particle also changes with time. It can be seen in the graph below.

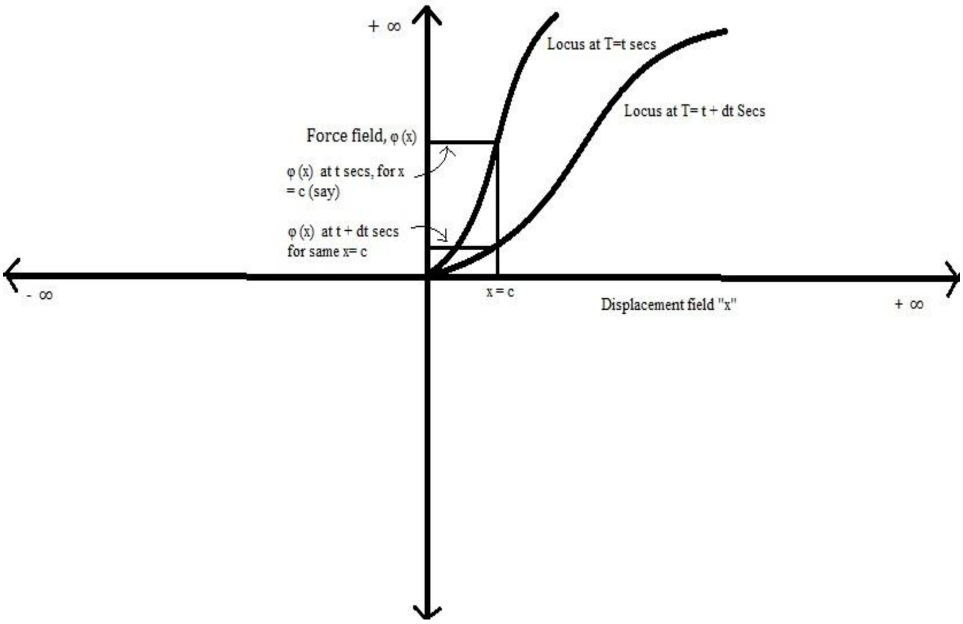

Figure 1.a.2 1*-dimensional varying force field vs displacement curve*

Again, from Chain rule for multivariable functions,
$d\varphi_1 = \frac{\partial \varphi 1}{\partial x}(dx) + \frac{\partial \varphi 1}{\partial t}(dt)$
or, $\frac{d\varphi 1}{dt} = \frac{\partial \varphi 1}{\partial x}(\frac{dx}{dt}) + \frac{\partial \varphi 1}{\partial t}$
Therefore, the velocity function in this case will be,
$\Psi_1 = \Psi(x, t) = \frac{d\varphi 1}{dt} = \frac{\partial \varphi 1}{\partial x}(\frac{dx}{dt}) + \frac{\partial \varphi 1}{\partial t}$
Now,
$d\Psi_1 = \frac{\partial \Psi 1}{\partial x}(dx) + \frac{\partial \Psi 1}{\partial t}(dt)$
or, $\frac{d\Psi 1}{dt} = \frac{\partial \Psi 1}{\partial x}(\frac{dx}{dt}) + \frac{\partial \Psi 1}{\partial t}$
Here, $\frac{\partial \Psi 1}{\partial x}(\frac{dx}{dt}) = \{\frac{\partial}{\partial x}(\frac{\partial \varphi}{\partial x})\}(\frac{dx}{dt})^2 + \{\frac{\partial}{\partial x}(\frac{\partial \varphi}{\partial t})\}(\frac{dx}{dt}) = \partial_x^{\,2}(\Delta_x^{\,1})^2 + \partial_{x,\,t}^{\,2}(\Delta_x^{\,1})$ as here, φ_1 not is constant with respect to time t.
and $\frac{\partial \Psi 1}{\partial t} = \{\frac{\partial}{\partial t}(\frac{\partial \varphi}{\partial x})\}(\frac{dx}{dt}) + \frac{\partial \varphi 1}{\partial x}\{\frac{d}{dt}(\frac{dx}{dt})\} + \frac{\partial}{\partial t}(\frac{\partial \varphi}{\partial t}) = \partial_{x,\,t}^{\,2}(\Delta_x^{\,1}) + \partial_x^{\,1}(\Delta_x^{\,2}) + \partial_t^{\,2}$
Since $\vec{E}_1 = \frac{d\Psi 1}{dt}$
Therefore, $\vec{E}_1 = \{ \frac{\partial}{\partial x}(\frac{\partial \varphi}{\partial x})\}(\frac{dx}{dt}) + \frac{\partial}{\partial t}(\frac{\partial \varphi}{\partial t}) \} + 2\{ \frac{\partial}{\partial x}(\frac{\partial \varphi}{\partial t})\}(\frac{dx}{dt}) \} + \frac{\partial \varphi 1}{\partial x}\{\frac{d}{dt}(\frac{dx}{dt})\}$
$= \{ \partial_x^{\,2}(\Delta_x^{\,1})^2 + \partial_t^{\,2} \} + 2\{ \partial_{x,\,t}^{\,2}(\Delta_x^{\,1}) \} + \partial_x^{\,1}(\Delta_x^{\,2})$
Thus, for a varying one-dimensional force, the effect $\vec{E}_1$ is,
$\vec{E}_1 = \{ \partial_x^{\,2}(\Delta_x^{\,1})^2 + \partial_t^{\,2} \} + 2\{ \partial_{x,\,t}^{\,2}(\Delta_x^{\,1}) \} + \partial_x^{\,1}(\Delta_x^{\,2})$……….Eq(2)
This is the generalized equation for the effect of a one-dimensional force.
We can see that, for a constant one dimensional force, any value of ∂ with subscript t will become 0 as φ does not vary with time and thus, $\partial_t^{\,2}$ and $\partial_{x,\,t}^{\,2}$ will become 0 and we will get back Eq(1).

1b: Analysis Of Two-Dimensional Force And Its Effect.

Again, let there be a two-dimensional force, represented by the vector $\vec{F}_2$ that imparts a displacement curve (the curve traced out by the body during its motion under the influence of force $\vec{F}_2$) φ_2 to the test particle.

Let the observed effect be $\vec{E}_{2.}$. Again, like in previous case, the subscripts indicate the number of dimensions of the force, under whose analysis, the respective vector is introduced.

1b.1. For A Constant Two-Dimensional Force

When the force is constant and varies with respect to just distance along two dimensions, then we can represent the force as,

$\vec{F}_2$ = F (x, y), $\varphi_2 = \varphi(x, y)$ where x and y represents the dimensions along which the force varies.

Note: how the number of dimensions the test particle is capable of undergoing displacement in, due to the force, is equal to the number of dimensions of the force that is acting on it.

Let this force act on the test particle such that it undergoes displacement according to a function $\varphi_2 = \varphi(x, y)$. Therefore, effect $\vec{E}_2$ = acceleration of the test particle = $\frac{d}{dt}(\frac{d(\varphi 2)}{dt})$

Here, $\varphi_2 = \varphi(x, y)$ because, the force is constant with respect to time. Thus the displacement of the test particle (which happens in xy plane), at a given distance from the reference frame's origin, will also be constant. It is eminent in the graph below.

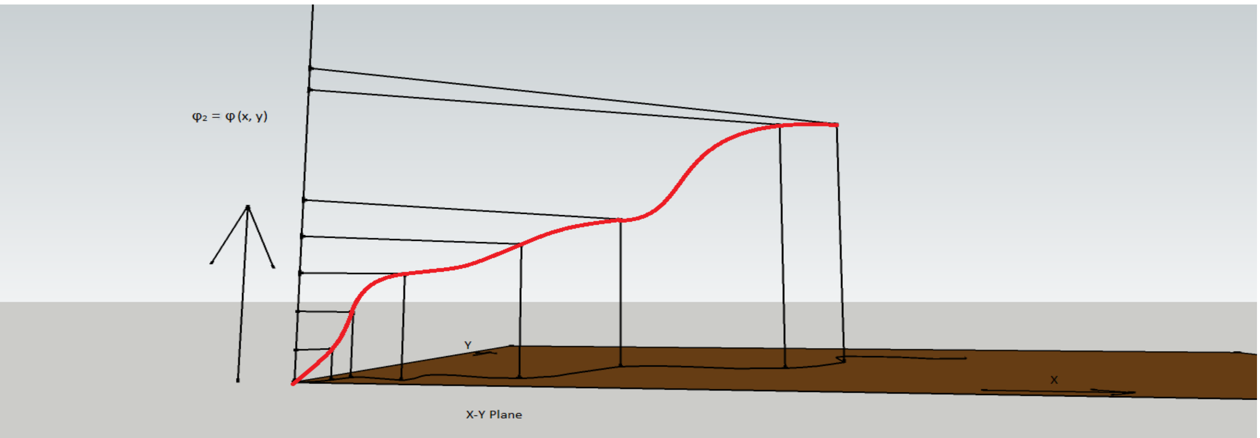

Figure 1.b.1 ***2****-dimensional steady force field vs displacement curve, view 1.*

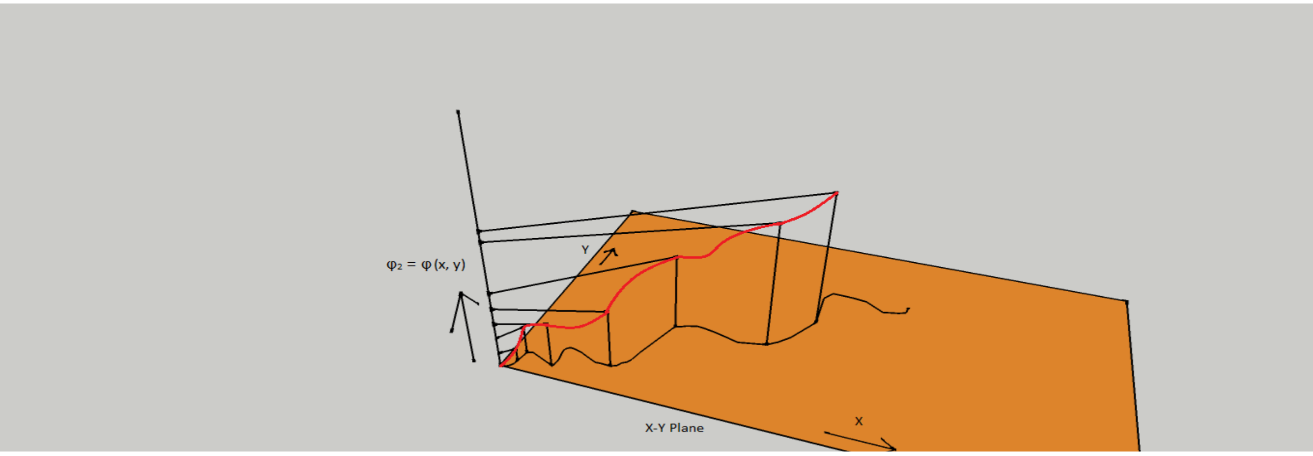

Figure 1.b.1 ***2****-dimensional steady force field vs displacement curve, view 2.*

Again, from Chain rule for multivariable functions,

$d\varphi_2 = \frac{\partial \varphi 2}{\partial x}(dx) + \frac{\partial \varphi 2}{\partial y}(dy)$

or, $\frac{d\varphi 2}{dt} = \frac{\partial \varphi 2}{\partial x}(\frac{dx}{dt}) + \frac{\partial \varphi 2}{\partial y}(\frac{dy}{dt})$

Therefore, the velocity function in this case will be,

$\Psi_2 = \Psi (x, y, t) = \frac{d\varphi 2}{dt} = \frac{\partial \varphi 2}{\partial x}(\frac{dx}{dt}) + \frac{\partial \varphi 2}{\partial y}(\frac{dy}{dt})$ as Ψ is a function of x, y and t.

Therefore, from solving we get,

$$\vec{E}_2 = \frac{d}{dt}\{\frac{d(\Psi 2)}{dt}\} = \{\frac{\partial}{\partial x}(\frac{\partial \varphi 2}{\partial x})\}(\frac{dx}{dt})^2 + \{\frac{\partial}{\partial x}(\frac{\partial \varphi 2}{\partial y})\}(\frac{dx}{dt})(\frac{dy}{dt}) + \{\frac{\partial}{\partial y}(\frac{\partial \varphi 2}{\partial y})\}(\frac{dy}{dt})^2 + \{\frac{\partial}{\partial y}(\frac{\partial \varphi 2}{\partial x})\}(\frac{dx}{dt})(\frac{dy}{dt}) + \frac{\partial \varphi 2}{\partial x}\{\frac{d}{dt}(\frac{dx}{dt})\} + \frac{\partial \varphi 2}{\partial y}\{\frac{d}{dt}(\frac{dy}{dt})\}$$

Or, $\vec{E}_2 = \partial_x^{\,2}(\Delta_x^{\,1})^2 + \partial_{x,\,y}^{\,2}(\Delta_x^{\,1})(\Delta_y^{\,1}) + \partial_y^{\,2}(\Delta_y^{\,1})^2 + \partial_{x,\,y}^{\,2}(\Delta_x^{\,1})(\Delta_y^{\,1}) + \partial_x^{\,1}(\Delta_x^{\,2}) + \partial_y^{\,1}(\Delta_y^{\,2})$

Or, $\vec{E}_2 = \{\partial_x^{\,2}(\Delta_x^{\,1})^2 + \partial_y^{\,2}(\Delta_y^{\,1})^2\} + 2\{\partial_{x,\,y}^{\,2}(\Delta_x^{\,1})(\Delta_y^{\,1})\} + \partial_x^{\,1}(\Delta_x^{\,2}) + \partial_y^{\,1}(\Delta_y^{\,2})$……….Eq(3)

1b.2. For A Varying Two-Dimensional Force.

When the force varies with respect to time along with displacement along two dimensions, then we can represent the force as,

$\vec{F}_2 = F(x, y, t)$, $\varphi_2 = \varphi(x, y, t)$. where t is the time dimension.

Note: how again the number of dimensions the test particle is capable of undergoing displacement in, due to the force, is equal to the number of dimensions of the force that is acting on it.

Let this force act on the test particle such that it undergoes displacement according to a function $\varphi_2 = \varphi(x, y, t)$.

Therefore, effect $\vec{E}_2$ = acceleration of the test particle = $\frac{d}{dt}(\frac{d(\varphi 2)}{dt})$

Displacement curve of the test particle is governed by the force acting on it. Since the force in this case changes with time, thus the displacement function of the test particle also changes with time. It can be seen in the graph below.

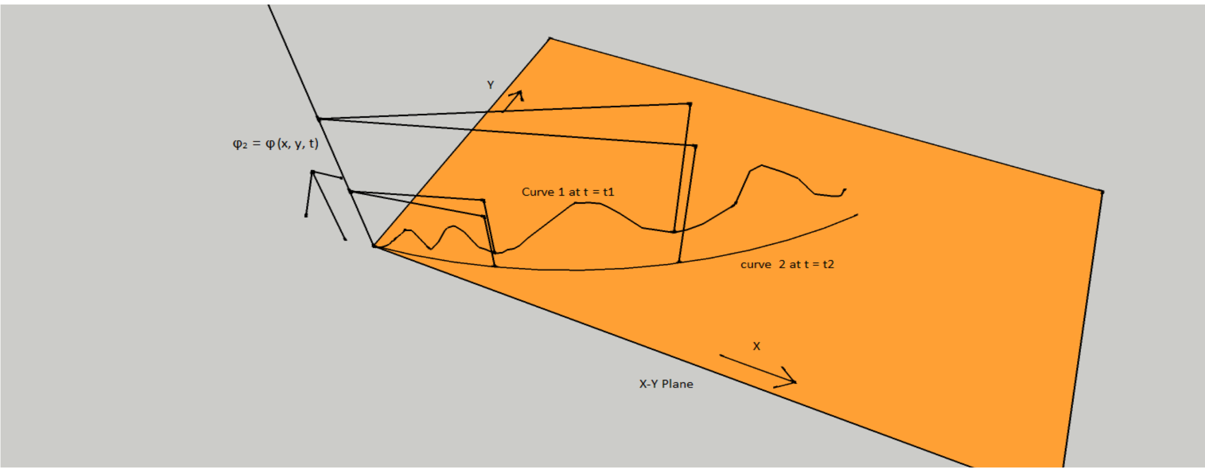

***Figure 1.b.2** 2-dimensional non-steady field vs displacement curve*

Again, from Chain rule for multivariable functions,

$d\varphi_2 = \frac{\partial \varphi 2}{\partial x}(dx) + \frac{\partial \varphi 2}{\partial y}(dy) + \frac{\partial \varphi 2}{\partial t}(dt)$ as φ_2 varies with time too.

$\frac{d\varphi 2}{dt} = \frac{\partial \varphi 2}{\partial x}(\frac{dx}{dt}) + \frac{\partial \varphi 2}{\partial y}(\frac{dy}{dt}) + \frac{\partial \varphi 2}{\partial t}$

Therefore, here, the velocity function will be

$\Psi_2 = \Psi(x, y, t) = \frac{d\varphi 2}{dt} = \frac{\partial \varphi 2}{\partial x}(\frac{dx}{dt}) + \frac{\partial \varphi 2}{\partial y}(\frac{dy}{dt}) + \frac{\partial \varphi 2}{\partial t}$

Therefore, again solving, we get,

$\vec{E}_2 = \frac{d}{dt}\{\frac{d(\Psi 2)}{dt}\} = \{\frac{\partial}{\partial x}(\frac{\partial \varphi 2}{\partial x})\}(\frac{dx}{dt})^2 + \{\frac{\partial}{\partial x}(\frac{\partial \varphi 2}{\partial y})\}(\frac{dx}{dt})(\frac{dy}{dt}) + \{\frac{\partial}{\partial x}(\frac{\partial \varphi 2}{\partial t})\}(\frac{dx}{dt}) + \{\frac{\partial}{\partial y}(\frac{\partial \varphi 2}{\partial y})\}(\frac{dy}{dt})^2 + \{\frac{\partial}{\partial y}(\frac{\partial \varphi 2}{\partial x})\}(\frac{dx}{dt})(\frac{dy}{dt}) + \{\frac{\partial}{\partial y}(\frac{\partial \varphi 2}{\partial t})\}(\frac{dy}{dt}) + \{\frac{\partial}{\partial t}(\frac{\partial \varphi 2}{\partial x})\}(\frac{dx}{dt}) + \frac{\partial \varphi 2}{\partial x}\{\frac{d}{dt}(\frac{dx}{dt})\} + \{\frac{\partial}{\partial t}(\frac{\partial \varphi 2}{\partial y})\}(\frac{dy}{dt}) + \frac{\partial \varphi 2}{\partial y}\{\frac{d}{dt}(\frac{dy}{dt})\} + \frac{\partial}{\partial t}(\frac{\partial \varphi 2}{\partial t})$

$= \partial_x^{\,2}(\Delta_x^{\,1})^2 + \partial_{x,\,y}^{\,2}(\Delta_x^{\,1})(\Delta_y^{\,1}) + \partial_{x,\,t}^{\,2}(\Delta_x^{\,1}) + \partial_y^{\,2}(\Delta_y^{\,1})^2 + \partial_{x,\,y}^{\,2}(\Delta_x^{\,1})(\Delta_y^{\,1}) + \partial_{y,\,t}^{\,2}(\Delta_y^{\,1}) + \partial_{x,\,t}^{\,2}(\Delta_x^{\,1}) + \partial_x^{\,1}(\Delta_x^{\,2}) + \partial_{y,\,t}^{\,2}(\Delta_y^{\,1}) + \partial_y^{\,1}(\Delta_y^{\,2}) + \partial_t^{\,2}$

$= \partial_x^{\,2}(\Delta_x^{\,1})^2 + \partial_y^{\,2}(\Delta_y^{\,1})^2 + \partial_t^{\,2}\ 2\{\partial_{x,\,y}^{\,2}(\Delta_x^{\,1})(\Delta_y^{\,1}) + \partial_{x,\,t}^{\,2}(\Delta_x^{\,1}) + \partial_{y,\,t}^{\,2}(\Delta_y^{\,1})\} + \partial_x^{\,1}(\Delta_x^{\,2}) + \partial_y^{\,1}(\Delta_y^{\,2})$

Thus,

$\vec{E}_2 = \{\partial_x^{\,2}(\Delta_x^{\,1})^2 + \partial_y^{\,2}(\Delta_y^{\,1})^2 + \partial_t^{\,2}\} + 2\{\partial_{x,\,y}^{\,2}(\Delta_x^{\,1})(\Delta_y^{\,1}) + \partial_{x,\,t}^{\,2}(\Delta_x^{\,1}) + \partial_{y,\,t}^{\,2}(\Delta_y^{\,1})\} + \partial_x^{\,1}(\Delta_x^{\,2}) + \partial_y^{\,1}(\Delta_y^{\,2})$ …..Eq(4)

This is the generalized equation for the effect of a two-dimensional force.

We can see that, for a constant two-dimensional force, any value of ∂ with subscript t will become 0 as φ does not vary with time and thus, $\partial_t^{\,2}$, $\partial_{x,\,t}^{\,2}$ and $\partial_{y,\,t}^{\,2}$ will become 0 and we will get back Eq (3)

1.c.1. For A Constant Three-Dimensional Force

When the force is constant and varies with respect to just distance along three dimensions, then we can represent the force as,

$\vec{F}_3 = F(x, y, z)$, $\varphi_3 = \varphi(x, y, z)$ where x and y represents the dimensions along which the force varies.

Let this force act on the test particle such that it undergoes displacement according to a function $\varphi_3 = \varphi(x, y, z)$.
Therefore, effect $\vec{E}_3$ = acceleration of the test particle = $\frac{d}{dt}(\frac{d(\varphi 3)}{dt})$
Here, $\varphi_3 = \varphi(x, y, z)$ because, the force is constant with respect to time. Thus the displacement of the test particle, at a given distance from the reference frame's origin, will also be constant.
This cannot be shown in graph since the displacement function needs a fourth dimension to be plotted.
Again, from Chain rule for multivariable functions,
$d\varphi_3 = \frac{\partial \varphi 3}{\partial x}(dx) + \frac{\partial \varphi 3}{\partial y}(dy) + \frac{\partial \varphi 3}{\partial z}(dz)$
or, $\frac{d\varphi 3}{dt} = \frac{\partial \varphi 3}{\partial x}(\frac{dx}{dt}) + \frac{\partial \varphi 3}{\partial y}(\frac{dy}{dt}) + \frac{\partial \varphi 3}{\partial z}(\frac{dz}{dt})$
Therefore, the velocity function in this case will be,
$\Psi_3 = \Psi(x, y, z, t) = \frac{d\varphi 3}{dt} = \frac{\partial \varphi 3}{\partial x}(\frac{dx}{dt}) + \frac{\partial \varphi 3}{\partial y}(\frac{dy}{dt}) + \frac{\partial \varphi 3}{\partial z}(\frac{dz}{dt})$ as Ψ is a function of x, y, z and t.
Therefore, we get,
$\vec{E}_3 = \frac{d}{dt}\{\frac{d(\Psi 3)}{dt}\} = \{\frac{\partial}{\partial x}(\frac{\partial \varphi 3}{\partial x})\}(\frac{dx}{dt})^2 + \{\frac{\partial}{\partial x}(\frac{\partial \varphi 3}{\partial y})\}(\frac{dx}{dt})(\frac{dy}{dt}) + \{\frac{\partial}{\partial x}(\frac{\partial \varphi 3}{\partial z})\}(\frac{dx}{dt})(\frac{dz}{dt}) + \{\frac{\partial}{\partial y}(\frac{\partial \varphi 3}{\partial y})\}(\frac{dy}{dt})^2 + \{\frac{\partial}{\partial y}(\frac{\partial \varphi 3}{\partial x})\}(\frac{dx}{dt})(\frac{dy}{dt}) + \{\frac{\partial}{\partial y}(\frac{\partial \varphi 3}{\partial z})\}(\frac{dz}{dt})(\frac{dy}{dt}) + \{\frac{\partial}{\partial z}(\frac{\partial \varphi 3}{\partial z})\}(\frac{dz}{dt})^2 + \{\frac{\partial}{\partial z}(\frac{\partial \varphi 3}{\partial x})\}(\frac{dx}{dt})(\frac{dz}{dt}) + \{\frac{\partial}{\partial z}(\frac{\partial \varphi 3}{\partial y})\}(\frac{dz}{dt})(\frac{dy}{dt}) + \{\frac{\partial}{\partial z}(\frac{\partial \varphi 3}{\partial z})\}(\frac{dz}{dt})^2 + \{\frac{\partial}{\partial z}(\frac{\partial \varphi 3}{\partial x})\}(\frac{dx}{dt})(\frac{dz}{dt}) + \{\frac{\partial}{\partial z}(\frac{\partial \varphi 3}{\partial y})\}(\frac{dz}{dt})(\frac{dy}{dt})$
$= \partial_x^2(\Delta_x^1)^2 + \partial_{x,y}^2(\Delta_x^1)(\Delta_y^1) + \partial_{x,z}^2(\Delta_x^1)(\Delta_z^1) + \partial_y^2(\Delta_y^1)^2 + \partial_{x,y}^2(\Delta_x^1)(\Delta_y^1) + \partial_{y,z}^2(\Delta_y^1)(\Delta_z^1) + \partial_z^2(\Delta_z^1)^2 + \partial_{x,z}^2(\Delta_x^1)(\Delta_z^1) + \partial_{y,z}^2(\Delta_y^1)(\Delta_z^1) + \partial_x^1(\Delta_x^2) + \partial_y^1(\Delta_y^2) + \partial_z^1(\Delta_z^2)$
$= \partial_x^2(\Delta_x^1)^2 + \partial_y^2(\Delta_y^1)^2 + \partial_z^2(\Delta_z^1)^2 + 2\{\partial_{x,y}^2(\Delta_x^1)(\Delta_y^1) + \partial_{y,z}^2(\Delta_y^1)(\Delta_z^1) + \partial_{x,z}^2(\Delta_x^1)(\Delta_z^1)\} + \partial_x^1(\Delta_x^2) + \partial_y^1(\Delta_y^2) + \partial_z^1(\Delta_z^2)$
Or,
$\vec{E}_3 = \{\partial_x^2(\Delta_x^1)^2 + \partial_y^2(\Delta_y^1)^2 + \partial_z^2(\Delta_z^1)^2\} + 2\{\partial_{x,y}^2(\Delta_x^1)(\Delta_y^1) + \partial_{y,z}^2(\Delta_y^1)(\Delta_z^1) + \partial_{x,z}^2(\Delta_x^1)(\Delta_z^1)\} + \partial_x^1(\Delta_x^2) + \partial_y^1(\Delta_y^2) + \partial_z^1(\Delta_z^2)$ ………..Eq(5)

1.c.2 For A Varying Three-Dimensional Force
When the force varies with respect to time along with displacement along three dimensions, then we can represent the force as,
$\vec{F}_3 = F(x, y, z, t)$, $\varphi_3 = \varphi(x, y, z, t)$. where t is the time dimension.
Note: how again the number of dimensions the test particle is capable of undergoing displacement in, due to the force, is equal to the number of dimensions of the force that is acting on it.
Let this force act on the test particle such that it undergoes displacement according to a function $\varphi_3 = \varphi(x, y, z, t)$. Therefore, effect $\vec{E}_3$ = acceleration of the test particle = $\frac{d}{dt}(\frac{d(\varphi 3)}{dt})$
Again, from Chain rule for multivariable functions,
$d\varphi_3 = \frac{\partial \varphi 3}{\partial x}(dx) + \frac{\partial \varphi 3}{\partial y}(dy) + \frac{\partial \varphi 3}{\partial z}(dz) + \frac{\partial \varphi 3}{\partial t}(dt)$
or, $\frac{d\varphi 3}{dt} = \frac{\partial \varphi 3}{\partial x}(\frac{dx}{dt}) + \frac{\partial \varphi 3}{\partial y}(\frac{dy}{dt}) + \frac{\partial \varphi 3}{\partial z}(\frac{dz}{dt}) + \frac{\partial \varphi 3}{\partial t}$
Therefore, the velocity function in this case will be,
$\Psi_3 = \Psi(x, y, z, t) = \frac{d\varphi 3}{dt} = \frac{\partial \varphi 3}{\partial x}(\frac{dx}{dt}) + \frac{\partial \varphi 3}{\partial y}(\frac{dy}{dt}) + \frac{\partial \varphi 3}{\partial z}(\frac{dz}{dt}) + \frac{\partial \varphi 3}{\partial t}$
as Ψ is a function of x, y, z and t
Therefore,
$\vec{E}_3 = \partial_x^2(\Delta_x^1)^2 + \partial_{x,y}^2(\Delta_x^1)(\Delta_y^1) + \partial_{x,z}^2(\Delta_x^1)(\Delta_z^1) + \partial_{x,t}^2(\Delta_x^1) + \partial_y^2(\Delta_y^1)^2 + \partial_{x,y}^2(\Delta_x^1)(\Delta_y^1) + \partial_{y,z}^2(\Delta_y^1)(\Delta_z^1) + \partial_{y,t}^2(\Delta_y^1) + \partial_z^2(\Delta_z^1)^2 + \partial_{x,z}^2(\Delta_x^1)(\Delta_z^1) + \partial_{y,z}^2(\Delta_y^1)(\Delta_z^1) + \partial_{z,t}^2(\Delta_z^1) + \partial_{x,t}^2(\Delta_x^1) + \partial_x^1(\Delta_x^2) + \partial_{y,t}^2(\Delta_y^1) + \partial_y^1(\Delta_y^2) + \partial_{z,t}^2(\Delta_z^1) + \partial_z^1(\Delta_z^2) + \partial_t^2$
Or, $\vec{E}_3 = \{\partial_x^2(\Delta_x^1)^2 + \partial_y^2(\Delta_y^1)^2 + \partial_z^2(\Delta_z^1)^2 + \partial_t^2\} + 2\{\partial_{x,y}^2(\Delta_x^1)(\Delta_y^1) + \partial_{y,z}^2(\Delta_y^1)(\Delta_z^1) + \partial_{x,z}^2(\Delta_x^1)(\Delta_z^1) + \partial_{x,t}^2(\Delta_x^1) + \partial_{y,t}^2(\Delta_y^1) + \partial_{z,t}^2(\Delta_z^1)\} + \partial_x^1(\Delta_x^2) + \partial_y^1(\Delta_y^2) + \partial_z^1(\Delta_z^2)$ ………..Eq(6)
This is the generalized equation for the effect of a three-dimensional force.
We can see that, for a constant three dimensional force, any value of ∂ with subscript t will become 0 as φ does not vary with time and thus, ∂_t^2, $\partial_{x,t}^2$, $\partial_{y,t}^2$ and $\partial_{z,t}^2$ will become 0 and we will get back Eq(5).
Summarizing the results obtained in 1.a, 1.b and 1.c, the generalized equations for effect of different dimensional forces are:

Table 1.1

1D	$\partial_x^2(\Delta_x^1)^2 + \partial_t^2$	$+ 2\{ \partial_{x,t}^2(\Delta_x^1)\}$	$+\partial_x^1(\Delta_x^2)$
2D	$\partial_x^2(\Delta_x^1)^2+\partial_y^2(\Delta_y^1)^2 + \partial_t^2$	$+ 2\{ \partial_{x,y}^2(\Delta_x^1)(\Delta_y^1) + \partial_{x,t}^2(\Delta_x^1) + \partial_{y,t}^2(\Delta_y^1)\}$	$+\partial_x^1(\Delta_x^2) + \partial_y^1(\Delta_y^2)$
3D	$\partial_x^2(\Delta_x^1)^2 +\partial_y^2(\Delta_y^1)^2 + \partial_z^2(\Delta_z^1)^2 + \partial_t^2$	$+ 2\{ \partial_{x,y}^2(\Delta_x^1)(\Delta_y^1) + \partial_{y,z}^2(\Delta_y^1)(\Delta_z^1) + \partial_{x,z}^2(\Delta_x^1)(\Delta_z^1) + \partial_{x,t}^2(\Delta_x^1) + \partial_{y,t}^2(\Delta_y^1) + \partial_{z,t}^2(\Delta_z^1)\}$	$+\partial_x^1(\Delta_x^2) +\partial_y^1(\Delta_y^2) + \partial_z^1(\Delta_z^2)$

For which, the respective values of forces and the displacement functions are:

Table1.2

1D	$\vec{F}_1$ = F (x, t)	$\varphi_1 = \varphi(x, t)$
2D	$\vec{F}_2$ = F (x, y, t)	$\Phi_2 = \varphi(x, y, t)$
3D	$\vec{F}_3$ = F (x, y, z, t)	$\Phi_3 = \varphi(x, y, z, t)$

Note: Here the dimension of time is taken more as a case of varying force rather than a separate dimension. Nonetheless, the conclusions that will be drawn from the above summary will remain the same even if one considers time as an individual dimension.

III. The Theorem

2.a:The Impairing

What happens when we try to measure a two-dimensional force from a one-dimensional frame of reference?We know from Table 1.2 that for a two dimensional force varying with time $\vec{F}_2$ = F (x, y, t), the displacement curve is given by $\Phi_2 = \varphi(x, y, t)$ and the effect (from Table 1.1) by $\vec{E}_2 = \{\partial_x^2(\Delta_x^1)^2+\partial_y^2(\Delta_y^1)^2 + \partial_t^2\}+ 2\{ \partial_{x,y}^2(\Delta_x^1)(\Delta_y^1) + \partial_{x,t}^2(\Delta_x^1) + \partial_{y,t}^2(\Delta_y^1)\} + \partial_x^1(\Delta_x^2) + \partial_y^1(\Delta_y^2)$.However, since we are measuring from a one-dimensional frame of reference, we can measure displacement along just one dimension (say x). Thus the displacement curve perceived by us, for the test particle will be $\Phi_2' = \varphi(x, t)$ which will be the subset of the original displacement curve. When we will try to see the effect of the force, having only one dimension to work with, we won't be able to perceive any displacement or excitation due to the force in the second dimension. We can observe and perceive only the effects that takes place in the dimension we are measuring in. So, if we are measuring in x dimension then we can only measure displacements along x. Thus the differential equation for the effect obtained by us will contain only derivatives that are with respect to x or t (Again note how the dimension of time is implicitly taken into consideration so as to represent practicality). Thus the equation of the effect perceived by us will be:$\vec{E}_2' = \{ \partial_x^2(\Delta_x^1)^2 + \partial_t^2 \} + 2\{ \partial_{x,t}^2(\Delta_x^1) \} + \partial_x^1(\Delta_x^2)$.As we can see, the equation is same as equation(2) which is the equation for the effect a one-dimensional varying force, whose imparted displacement curve on the test particle is given by $\varphi_1 = \varphi(x, t)$ which, is again same as our perceived displacement curve $\Phi_2' = \varphi(x, t)$.

Therefore, the results so obtained will actually indicate a one-dimensional varying force. Also, since Φ_2' is a subset of Φ_2 that is, all values of Φ_2' lies within the range of Φ_2, therefore, the results obtained will also be consistent with the original phenomenon, $\vec{F}_2$.Similarly, for a three-dimensional varying force, if we try to measure from a two-dimensional frame of reference then, the results we will get will be consistent and will indicate towards a two-dimensional varying force. Building upon the notion, we can see that to measure an 'n' dimensional force, we require a frame of reference that supports at least 'n' dimensions. Else, while measuring from a smaller dimensional frame of reference, we will perceive the force as an 'm' dimensional force where m is equal to the number of dimensions supported by the frame of reference. Also, we cannot do any experiment,that can indicate towards the fact that it is actually, a higher dimensional phenomenon. Also note, at this point, we are capable of understanding the fact that here, time is just used as a dimension against which, the rates of change are measured. We can measure the rates of change against a different parameter. In that case, time will be treated just like any other dimensional parameter (x, y, z etc.) and their respective differential coefficients will switch places.

2.b The Unifying Prediction

Consider a general case of two-dimensional force:

$\vec{F}_2$ = F (x, y, t), $\Phi_2 = \varphi(x, y, t)$ and $\vec{E}_2 = \{\partial_x^2(\Delta_x^1)^2+\partial_y^2(\Delta_y^1)^2 + \partial_t^2\}+ 2\{ \partial_{x,y}^2(\Delta_x^1)(\Delta_y^1) + \partial_{x,t}^2(\Delta_x^1) + \partial_{y,t}^2(\Delta_y^1)\} + \partial_x^1(\Delta_x^2) + \partial_y^1(\Delta_y^2)$. Say our frame of reference supports only one dimension (excluding time). That is, either we can measure along x, or along y, but not both.From the conclusions of (2.a), When measuring along x, the results obtained will be:

$\vec{F}_2^{'x}$ = F (x, t), $\Phi_2^{'x}$ = φ(x, t) and $\vec{E}_2^{'x}$= { $\partial_x^2(\Delta_x^1)^2 + \partial_t^2$ } + 2{ $\partial_{x,t}^{\ 2}(\Delta_x^1)$ } + $\partial_x^1(\Delta_x^2)$ and when measuring along y, the results obtained will be:$\vec{F}_2^{'y}$ = F (y, t), $\Phi_2^{'y}$ = φ(y, t) and $\vec{E}_2^{'y}$= { $\partial_y^2(\Delta_y^1)^2 + \partial_t^2$} + 2{ $\partial_{y,t}^{\ 2}(\Delta_y^1)$} + $\partial_y^1(\Delta_y^2)$.As we can see, both the results are consistent with the original phenomenon i.e., $\Phi_2^{'x}$ and $\Phi_2^{'y}$ both are subsets of Φ_2 and $\vec{E}_2^{'x}$ &$\vec{E}_2^{'y}$ both are constituents of $\vec{E}_2$ thus we will perceive both of them as two dimensional forces that is varying with time however, they won't be similar. i.e. generally, $\vec{F}_2^{'x} \neq \vec{F}_2^{'y}$, $\Phi_2^{'x} \neq \Phi_2^{'y}$ and $\vec{E}_2^{'x} \neq \vec{E}_2^{'y}$. Therefore, they might be perceived as two different individual 2-dimensional forces that varies with time and we would not be able to perform any experiment that can hint to the fact that they are actually a manifestation of a single higher dimensional force, that they unify, in higher dimension.One might argue that for this to be a valid argument, the measurement axes x and y must be different from each other to which the answer will be no! the important concept here is how we can divide higher dimensional phenomenon into different lower dimensional phenomenon. Think the above example like this, there is a 2-dimensional phenomenon, with one component along x axis and one along y axis. We just have a 1-dimensional frame to measure with. So we try to measure the x component with our 1-d (one dimensional) frame and then, we try to measure the y component. We perceive them as different phenomenon. We can also take two 1-d frames and measure the x component with one and the y component with the other. We will still perceive them as different. The frame from which we are measuring is not changing, it remains the same. Measuring with two 1-d frames at a time is not same as measuring with one 2-d frame. The phenomena we are measuring might just be a component of a higher dimensional phenomena.

2.c: Analogy: With Special & General relativity:

Special [19][20][21]&General [22] relativity tells us that one cannot perform any experiment to tell about the absolute state of motion [23] of any particle. One can only tell its state of motion, relative to a frame of reference. Similarly, Banerjee-Dzubur Dimensional Impairing Theorem puts a restriction on the absoluteness of a force field. One cannot say with absolution, the properties of a force field acting on the test particle, and likewise differentiate a force field from other, all one can do is define and differentiate the force fields, with respect to the frame of reference of the measurement.

2.d: Does It Contradicts The Unified Field Theory?

Not necessarily. From Higgs mechanism [24][25][26] to spontaneous symmetry breaking[27][28][29], allproperties and mechanism are tied to the 4 dimensional spacetime. It is how we perceive,how the respective fields and particles interacts however, it may or may not interact in the same way, when studied from a higher dimensional frame of reference. According to Banerjee-Dzubur Dimensional Impairing Theorem, we will never know whether how we perceive these interactions, are the actual phenomenon or just a truncated part of the real one, until and unless, we re-perform the experiments from a frame of reference, which supports higher number of dimensions than the 3 spacial and 1 temporal dimensions of the spacetime we seem to know and love.

2.e: Conclusion – Banerjee-Dzubur Dimensional Impairing Theorem

Banerjee-Dzubur Dimensional Impairing Theorem thus concludes the following three statements.

1. No force field can ever be defined completely with absoluteness. It can only be defined, with respect to some frame of reference
2. An 'n' dimensional phenomenon may not be perceived fully, depending upon the number of dimensions supported by the frame of reference used for the analysis of the phenomenon. It is perceived as an 'm' dimensional phenomenon instead where, m is equals to n if, the frame of reference used for analysis of the phenomenon is capable of supporting number of dimensions greater than or equals n else, m is equals to the number of dimensions the frame of reference used for the analysis of the phenomenon, supports.
3. The results so obtained in the afore mentioned case will be always consistent with the original phenomenon and we cannot perform any test or experiment, that will indicate to the fact that it is indeed a higher dimensional phenomenon.
4. Two different kinds of, but similar dimensional phenomenon, may unify at a higher dimension.

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