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The Role of Conductivity and Susceptibility on Having Powerful Magnetic Field

M. H. Saad [a,b] , M. D. Abd-Alla[b]
[a] *Physics department, College of Science, Taibah University, Yanbu Branch*
[b] *Physics department, College of Science, Sudan University of Science & Technology, Khartoum, 11113, Sudan*

Abstract: *Maxwell equations are utilized to obtain a useful expression for the equation of motion of the magnetic moment. By imposing certain physical constrains on the conductivity, magnetic and electric susceptibility it is possible to have a powerful magnetic field.*
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I. Introduction

Super conductivity was discovered in 1911 in the Leiden Laboratory. [1] Of a so called "Blue Boy" noticed that the resistivity of Hg metal vanished abruptly at about 4k.

Super conductors have been studied intensively and for its technological applications. Te powerful magnetic field produced by superconductors is widely used in industry. Superconducting magnets are used in prototype levitated trains, electric generators and Magnetic Resonance Imaging (MRI).

Despite these successes applications of superconductivity in industry suffer from same longstanding problems. For instance superconducting state are need cooling to a very low temperature.

Magnetic Moment Dynamical Equation:

According to Maxwell equations the magnetic field and the electric field obeys the relations:

$$\nabla . E = 0 \qquad (1)$$

$$\nabla . H = 0 \qquad (2)$$

$$\nabla \times E = -\frac{dB}{dt} \qquad (3)$$

$$\nabla \times H = \varepsilon \frac{dE}{dt} + \sigma E \qquad (4)$$

Where:
E: the electric field.
H: the magnetic field strength.
B: the magnetic flux density.
ε: epsilon permittivity (dielectric constant of the medium).
σ: conductivity.

The relations between the electric displacements, the magnetic flux density, and the magnetic and electric field strength are:

$$B = \mu_0 \left(H + M \right) \qquad (5)$$

$$B = \mu H \qquad (6)$$

$$M = X_m H \qquad (7)$$

Where:

μ_0 : The magnetic moment in vacuum (permeability).

μ : The magnetic moment in the medium.

X_m : Susceptibility.

M : The magnetization.

3- Magnetization equation of motion:

By taking the curl of the fourth Maxwell equation (4) one gets:

$$\nabla \times \left(\nabla \times H \right) = \varepsilon \frac{d}{dt} \left(\nabla \times E \right) + \sigma \nabla \times E \qquad (8)$$

The Left hand side can be deduced to be:

$$\nabla\times(\nabla\times H)=\nabla(\nabla.H)-\nabla^2 H \qquad (9)$$

Using equations (8) and (9), equation (7) can be rewritten as:

$$-\nabla^2 H=\varepsilon\frac{d}{dt}(\nabla\times E)+\sigma\nabla\times E \qquad (10)$$

Where from the first Maxwell (1):

$$\nabla.H\Longrightarrow\nabla(\nabla.H)=0 \qquad (11)$$

From the third Maxwell equation (3) by substituting equation (10) one gets:

$$-\nabla^2 H=-\varepsilon\frac{d}{dt}\frac{dB}{dt}-\sigma\frac{dB}{dt} \qquad (12)$$

From the relation of the magnetic flux density:

$$B=\mu H \qquad (13)$$

By substituting the expression above in equation (12) one find:

$$-\nabla^2 H+\mu\varepsilon\frac{d^2H}{dt^2}+\mu\sigma\frac{dH}{dt}=0 \qquad (14)$$

Multiplying equation (14) by X_m to obtains:

$$-\nabla^2 X_m H+\mu\varepsilon\frac{d^2X_mH}{dt^2}+\mu\sigma\frac{dX_mH}{dt}=0 \qquad (15)$$

The Magnetization M can be written as:

$$M=X_mH \qquad (16)$$

Substituting this form in equation (15) one obtains:

$$-\nabla^2 M+\mu\varepsilon\frac{d^2M}{dt^2}+\mu\sigma\frac{dM}{dt}=0 \qquad (17)$$

Spatial varying Amplitude Solution of the magnetization equation:

The magnetization equation (17) can be solved by assuming the magnetization vector to be in the form of a traveling wave with spatial varying amplitude.

For simplicity one can assume the amplitude to vary in the z direction in the form:

$$M(z)=M_m(z)e^{i(kz-\omega t)} \qquad (18)$$

Therefore:

$$\frac{dM(z)}{dt}=-i\omega M_m(z)e^{i(kz-\omega t)} \qquad (19)$$

$$\frac{d^2M(z)}{dt^2}=-\omega^2M_m(z)e^{i(kz-\omega t)} \qquad (20)$$

$$\nabla M=\left(\frac{\partial M_m}{\partial Z}+ikM_m\right)e^{i(kz-\omega t)} \qquad (21)$$

$$\nabla^2 M=\left(\frac{\partial^2M_m}{\partial Z^2}+2ik\frac{\partial M_m}{\partial Z}-k^2M_m\right)e^{i(kz-\omega t)} \qquad (22)$$

Utilizing the above relations (19), (20), (21), (22) in equation (17) and eliminating the exponential term on both sides yields

$$\frac{\partial^2M_m}{\partial Z^2}+2ik\frac{\partial M_m}{\partial Z}=\left(k^2+\mu\varepsilon\omega^2\right)M_m-i\mu\sigma\omega M_m \qquad (23)$$

Equating real and imaginary part on both sides one gets:

$$\frac{\partial^2 M_m}{\partial Z^2} = \left(k^2 + \mu\varepsilon\omega^2\right)M_m \quad (24)$$

$$\frac{\partial M_m}{\partial Z} = -\frac{\mu\sigma\omega}{2k} M_m \quad (25)$$

In corpora ting (25) in (24) it follows that:

$$\frac{\partial}{\partial Z}\left(\frac{\partial M_m}{\partial Z}\right) = \frac{\partial}{\partial Z}\left(-\frac{\mu\sigma\omega M_m}{2k}\right) = \left(k^2 + \mu\varepsilon\omega^2\right)M_m$$

$$\frac{\partial M_m}{\partial Z} = -\frac{2k\left(k^2 + \mu\varepsilon\omega^2\right)}{\mu\sigma\omega} M_m$$

$$\int \frac{\partial M_m}{M_m} = -\frac{2k\left(k^2 + \mu\varepsilon\omega^2\right)}{\mu\sigma\omega}\int dZ + c$$

Hence:

$$M_m(Z) = M_{mo} e^{\alpha z} \quad (26)$$

Where:

$$\alpha = -\frac{2k\left(k^2 + \mu\varepsilon\omega^2\right)}{\mu\sigma\omega}$$

$$e^c = M_{mo} \quad (27)$$

Hence the solution of (17) takes the form:

$$M = M_m(z)e^{i(kz-\omega t)} = M_{m0} e^{\alpha z} e^{i(kz-\omega t)} \quad (28)$$

To have a powerful magnetic field the parameter α should be positive. This can be obtained if

$$\mu = -\mu = -|\mu| \quad (29)$$

Where:

$$\mu = \mu_0\left(1 + X_m\right) \quad (30)$$

Where: X_m represent the magnetic susceptibility. Therefore μ is negative if:

$$X_m = -|X_m| \quad (31)$$

And

$$\mu = -\mu_0\left(|X_m| - 1\right) \quad (32)$$

This requires also that:

$$|X_m| > 1 \quad (33)$$

In this case (24): $\alpha = +\frac{2k\left(k^2 + \mu\varepsilon\omega^2\right)}{|\mu|\sigma\omega} = +$ (34)

Hence amplification of a magnetic field inside any medium can be obtained if the magnetic susceptibility is negative and its absolute value exceeds one. In other word amplification exists in a diamagnetic materials having absolute magnetic susceptibility greater than one.

Magnetic amplification can also be achieved if the conductivity is negative, i.e.

$$\sigma = -|\sigma| \quad (35)$$

In this case:

$$\alpha = \frac{\left(2k\left(k + \mu\varepsilon\omega^2\right)\right)}{\mu\omega|\sigma|} = + \quad (36)$$

The conductivity relates the electric field strength to the current density according to the formula.

$$J = \sigma E \quad (37)$$

It is well known that an electromagnetic conducting material obeys the relation:

$$E = -L\frac{di}{dt} = -LA\frac{dJ}{dt} \quad (38)$$

For instance if the current density takes the form:

$$J = J_o e^{i\omega_0 t} \quad (39)$$

Then:

$$E = -iLA\omega_0 J \quad (40)$$

Hence

$$\sigma = -iLA\omega_0 \quad (41)$$

If $\omega_o = i\omega_2$ then $\sigma = -\omega_2 AL$, hence the conductivity is negative and amplification can be achieved.

4- Slowly varying Magnetic Moment:

Another solution can be suggested by considering slowly varying amplitude a long the z axis .i.e.

$$M = M_m(z)e^{i(kz-\omega t)} \quad (43)$$

$$\frac{\partial^2 M_m}{\partial z^2} << \frac{\partial M_m}{\partial z} \quad (44)$$

In this case any term containing terms like ($\frac{\partial^2 M_m}{\partial dz^2}$) or higher orders can be neglected. As a result equation (23) reads

$$2ik\frac{dM_m}{dz} = \left(k^2 + \mu\varepsilon\omega^2 - i\mu\sigma\omega\right)M_m \quad (45)$$

This equation can be solved by suggesting solution in the form:

$$M_m = M_{m0}e^{\gamma z} \quad (46)$$

$$\gamma = \frac{1}{2ik}\left(k^2 + \mu\varepsilon\omega^2 - i\varpi\sigma\omega\right) \quad (47)$$

Where: $M_{m0} = e^{C_0}; C_0$ is the integration constant.

Hence:

$$M_m = M_{m0}\exp\left(-\frac{\mu\sigma\omega}{2k}z\right)\exp\left[-\frac{i}{2k}\left(k^2 + \mu\varepsilon\omega^2\right)z\right] \quad (48)$$

From equation (18):

$$M = M_m \exp[i(kz - wt)]$$

$$= M_{m0}\exp\left(-\frac{\mu\sigma\omega}{2k}z\right)\exp\left[-\frac{i}{2k}\left(k^2 + \mu\varepsilon\omega^2\right)z\right]\exp[i(kz - wt)] (49)$$

For very small ω and very large σ the third and fourth terms can be neglected to get:

$$M = M_{m0}\exp\left(-\frac{\mu\sigma\omega}{2k}z\right)\exp\left(\frac{ik}{2}z\right) \quad (50)$$

From equation (49) and (50) it is clear that amplification can be achieved again if:

$$\mu = -|\mu| \text{ Or } \sigma = -|\sigma| \quad (51)$$

But not both simultaneously.

There for amplification to take place the medium should be either a diamagnetic material having large magnetic susceptibility of absolute value exceeding one .i.e.

$$|x_m| > 1 \quad (52)$$

Or in a material having an electromagnetic inductance interaction where the induced current apposes the applied field.

Exponential Amplitude Solution:

In the previous section (3) and (4) general solutions are considered by suggesting the solution (18) and by equating real and imaginary parts or by considering slowly varying amplitude.
In this section the amplitude is assumed to be an exponential function in the form:

$$M_m = M_0 e^{\theta z} \qquad (53)$$

The solution is obtained without imposing any restrictions on the equation or solutions substituting:

$$\frac{\partial M_m}{\partial z} = \theta M_0 e^{\theta z} = \theta M_m$$

$$\frac{\partial^2 M_m}{\partial z^2} = \theta^2 M_0 e^{\theta z} = \theta^2 M_m \qquad (54)$$

Equation (54) can be substituted in equation (23) to get:

$$\theta^2 + 2ik\theta + \left(i\mu\sigma\omega - \mu\varepsilon\omega^2 - k^2\right) \qquad (55)$$

Using the identity: $x = \frac{-b \pm \sqrt{b2 - 4ac}}{2a}$

With: $x = \theta,\quad a = 1,\quad b = 2ik,\qquad c = i\mu\sigma\omega - \mu\varepsilon\omega 2 - k \qquad (56)$

One obtains:

$$\theta = \frac{-2ik \pm \sqrt{-4k^2 + 4\mu\varepsilon\omega^2 + 4k^2 - 4i\mu\sigma\omega}}{2}$$

$$\theta = -ik \pm \left(\mu\varepsilon\omega^2 - i\mu\sigma\omega\right)^{\frac{1}{2}} \qquad (57)$$

For poor conductors or very high frequency or even very large magnetic or electric susceptibility equation (57) reads:

$$\theta = ik \pm \left(\mu\varepsilon\omega^2\right)^{\frac{1}{2}} = ik \pm \omega\sqrt{\mu\varepsilon} \qquad (58)$$

There fore:

$$M_m = M_0 e^{\left(ik \pm \omega\sqrt{\mu\varepsilon}\right)z} \qquad (59)$$

Taking the plus sign:

$$M_m = M_0 e^{ikz + \omega\sqrt{\mu\varepsilon}z} \qquad (60)$$

As a result equation (18) becomes:

$$M = M_0 e^{\omega z\sqrt{\mu\varepsilon}} . e^{i(2kz - \omega_t)} \qquad (61)$$

With the restriction imposed on equation (57) amplification can take place in a very poor conductor when the relation frequency is extremely large.

II. Conclusion

When a solution (18) is considered with slowly spatially varying amplitude or when equating real and imaginary part, magnetic amplification can be achieved for diamagnetic materials, having extremely large magnetic susceptibility, having absolute value exceeding one, or for a medium having negative conductivity like ores having self inductance.

If the amplitude is assumed to very exponentially amplification can be observed in materials having poor conductivity or very large magnetic or electric susceptibility.

Amplification can also depend on the properties of the incident radiation as well where magnetic amplification can also be switched on by high frequency radiation.

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