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Chaotic Inflation in Spatially Homogeneous Anisotropic Bianchi Type I Space-Time

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Abstract: *Chaotic inflationary scenario in spatially homogeneous Bianchi Type I space-time following Linde [14] and using power law inflation i.e.* $R^3 = t^n$, $n > 1$ *Barrow [23], is discussed where R is scale factor. It is found that the inflationary parameters viz. slow roll parameters, non-Gaussianity parameter and anisotropic parameter are in excellent agreement with Planck results for canonical scalar field. It has also been observed that inflation is natural consequence of Chaotic initial conditions in the early universe and chaotic inflationary scenario is realized for anisotropic Bianchi Type I space-time.*

Keywords*: Bianchi I, chaotic inflation, anisotropic, spatially homogeneous. PACs Nos.: 98.80 .cq, 11.10 wx*

I. Introduction

There is a great interest in inflationary universe scenario since this scenario in its present form, solves many mysteries of modern cosmology like homogeneity, the isotropy, flatness of observed universe and primordial monopole problem. Historically, a model closely related to the inflationary universe scenario was first suggested by Starobinsky [1]. It was based on investigation of conformal anomalies in quantum gravity. Starobinsky's model did not suffer from the graceful exit problem and it was the first model predicting gravitational waves with a flat spectrum which are responsible for galaxy formation and these were found by the observations of the CMB anisotropy, was proposed by Mukhanov and Chibisov [2]. A much simpler inflationary model (old inflation) with a very clear physical motivation was proposed by Guth [3]. This model was based on the theory of super cooling during the cosmological phase transition (Kirzhnits and Linde [4]). According to this scenario, inflation is an exponential expansion state.

Guth[3] has also suggested that rapid expansion is due to false vacuum energy and after inflation, the universe is filled with bubbles. This inflationary scenario is also confirmed by CMB observations. Also as pointed out by Myrzakulov and Sebastiaini [5] that after the inflation, the fluid turns out to be a (perfect fluid) with equation of state parameter by putting $\gamma = 1/3$, we may recover the radiation/ultra relativistic matter universe of the standard model without invoking the reheating, as the energy density of fluid itself is converted into radiation. The inflationary period is divided into slow roll and re-heating regims. In slow roll epoch, the potential of inflation should remain large as compared to kinetic energy and universe inflated (Linde [6]). Also at the end of inflation, inflation starts oscillating about minimum of its potential while potential and kinetic energies are comparable during reheating regime (Albrecht and Steinhardt [7], Adams et al. [8], Sharif and Mohsaneen [9]). Inflation plays an important role in isotropization of universe so inflation does not start at the end of isotropization. Isotropization starts at the end of inflation as pointed out by after Hervik et al. [10] Bali [11]

False vacuum is a metastable state without any fields or particles but with large energy density. This simple and infinite picture of inflation in the false vacuum state is somewhat misleading because if the probability of the bubble formation is large, bubbles of the new phase are formed near each other, inflation is too short to solve any problem and the bubblewall collisions make the universe extremely inhomogeneous. If they are formed far away from each other which is the case if the probability of their formation is small and inflation long, each of these bubbles represents a separate open universe with a small density parameter. Both of these options are unacceptable which led to the conclusion that this scenario does not work and can not be improved.

A solution of this problem was found in 1981 with the invention of new inflationary theory [12]. In this theory, inflation may begin either in the false vacuum or in an unstable state at the top of the effective potential. Then the inflation field ϕ slowly rolls down to the minimum of its effective potential. The density perturbations produced during the slow-roll inflation are inversely proportional to $\dot{\phi}$ [13,14]. Thus the key difference between the new inflationary scenario and the old one is that the useful part of inflation in the new scenario which is responsible for the homogeneity of our universe, does not occur in false vacuum state where $\dot{\phi} = 0$. This new inflationary scenario becomes very popular. Unfortunately, this scenario was plagued by its own problems. In most versions of this scenario, the inflation field must have an extremely small coupling constant so it could not be thermal equilibrium with other matter fields as required in [15]. So new inflation theory does not work in this case. Moreover, thermal equilibrium requires many particles interacting with each other. This means new inflation theory could explain only why our universe was so large.

On the basis of available observations (CMB abundance of light elements) every body believed that universe was created in a hot big-bang. That is why it was so difficult to overcome a certain psychological barrier and abandon all of these assumptions. This was done in 1983 with the invention of the Chaotic inflation scenario (Linde [16]). This scenario resolved all of old and new inflation. According to this scenario, inflation may begin even if there was no thermal equilibrium in the early universe and it may occur in the theories with simplest potential as such $V(\phi) \sim \phi^2$. But it is not limited to the theories with polynomial potential. Chaotic inflation occurs in any theory where the potential has sufficiently flat region which allows the existence of slow-roll region [16].

To understand chaotic inflation, we consider a simplest model of a scalar field with a mass m and potential energy density $V(\phi) = \frac{m^2}{2}\phi^2$. Since this function has a minimum at $\phi = 0$, we may expect that scalar field should oscillate near this minimum. This is the case if the universe does not expand and equation of motion of scalar field coincides with equation for harmonic oscillator

$$\ddot{\phi} = -m^2\phi$$

However, because of the expansion of the universe with Hubble parameter $H = \dot{a}/a$, an additional term $3H\dot{\phi}$ appears in the harmonic oscillator equation

$$\ddot{\phi} + 3H\dot{\phi} = -m^2\phi \tag{1}$$

The term $3H\dot{\phi}$ can be interpreted as a friction term. The Einstein equation for a homogeneous universe containing scalar field ϕ takes the form as

$$H^2 + \frac{k}{a^2} = \frac{1}{6}(\dot{\phi}^2 + m^2\phi^2) \tag{2}$$

where k = - 1, 0,1 for open, flat or closed universe respectively. Here we have used $M_p^{-2} = 8\pi G = 1.$

If the scalar field ϕ initially was large, the Hubble parameter H was large too according to the second equation. This means that the friction $3H\dot{\phi}$ was very large and therefore the scalar field was moving very slowly as a ball in a viscous fluid. Therefore, at this stage, the energy density of scalar field remained almost constant and expansion of the universe continued with a much greater speed. Due to the rapid growth of the scale of the universe and slow motion of the field ϕ, we have

$$\ddot{\phi} << 3H\dot{\phi},\ H^2 >> \frac{k}{a^2}, \dot{\phi}^2 << m^2\phi^2$$

so that the system of equations leads to

$$H = \frac{\dot{a}}{a} = \frac{m\phi}{\sqrt{6}}, \dot{\phi} = -m\sqrt{2/3}$$

The first equation shows that if the field changes slowly, the size of the universe in this regime grows exponentially as e^{Ht} where $H = \frac{m\phi}{\sqrt{6}}$. This is the stage of inflation which ends when the field ϕ becomes much smaller than $M_p = 1$.

Linde [17] proposed chaotic model with an assumption that the present universe is originated from a chaotic distribution of scalar field when potential energy of field dominates over that of the kinetic energy. Later, it has been shown by Bunn et al. [18] that chaotic scenario can be realized even when scalar field is kinetic energy dominated. Paul et al. [19] have shown that Linde's chaotic inflationary scenario is fairly general and can be accommodated even if the universe is anisotropic. The first model of chaotic inflation was based on theories with polynomial potential $V(\phi)=\frac{1}{2}m^2\phi^2+\frac{\lambda}{4}\phi^4$ (Linde [20]). Bali [21] investigated chaotic inflationary universe in Bianchi Type I space-time assuming (i) $R^3 = e^{3H_0 t}$ (ii) $V(\phi)=\frac{1}{2}m^2\phi^2+\frac{\lambda}{n}\phi^n$ following Linde [20] assuming $\ddot{\phi} << \frac{dV}{d\phi}$.

In the present investigation, we consider spatially homogeneous anisotropic Bianchi Type I space-time and investigated chaotic inflationary scenario assuming $V(\phi)=\frac{1}{2}m^2\phi^2+\lambda\phi^n, \lambda$ being a constant. To get the deterministic model of the universe, we also assume that the scale factor $R^3 = t^n, n>1.$ (Power law inflation). It has been observed that inflation is the natural consequence of chaotic initial conditions in the early universe as pointed out by Linde [17].

II. The Metric and Field Equations

We consider spatially homogeneous Bianchi Type I space-time in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + C^2dz^2 \tag{3}$$

where metric potentials A, B, C are functions of t-alone. The Lagrangian is that of gravity minimally coupled to scalar field ϕ with effective potential $V(\phi)$ given by Stein-Schabes [22] as

$$S = \int \sqrt{-g}\left[R - \frac{1}{2}g^{\mu\nu}\partial_\mu\partial_\nu\phi - V(\phi)\right]d^4x \tag{4}$$

The variation of S with respect to the dynamical field, leads to the Einstein field equation

$$R_i^j - \frac{1}{2}R\,g_i^j = -T_i^j \tag{5}$$

(in geometrized unit $8\pi G = 1 = c$) where

$$T_{ij} = \partial_i\phi\partial_j\phi - \frac{1}{2}[\partial_\rho\phi\partial^\rho\phi + V(\phi)]g_{ij} \tag{6}$$

and

$$\frac{1}{\sqrt{-g}}\partial_i[\sqrt{-g}\,\partial_i\phi] = -\frac{dV}{d\phi} \tag{7}$$

The Einstein field equation (5) for the space-time (3) leads to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4C_4}{BC} = -\frac{1}{2}\phi_4^2 + V(\phi) \tag{8}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4C_4}{AC} = -\frac{1}{2}\phi_4^2 + V(\phi) \tag{9}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4B_4}{AB} = -\frac{1}{2}\phi_4^2 + V(\phi) \tag{10}$$

$$\frac{A_4B_4}{AB} + \frac{A_4C_4}{AC} + \frac{B_4C_4}{BC} = \frac{1}{2}\phi_4^2 + V(\phi) \tag{11}$$

Equation (7) for scalar field ϕ leads to

$$\phi_{44} + \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right)\phi_4 + \frac{dV}{d\phi} = 0 \tag{12}$$

III. Solution of the Field Equations

To get the deterministic scenario of chaotic universe, we assume that

(i) $R^3 = t^n$, $n > 1$, as considered by Barrow [23], R being scale factor (for power law of inflation)

(ii) $V(\phi)=\frac{1}{2}m^2\phi^2+\frac{\lambda}{n}\phi^n$

where $V(\phi)$ is effective potential, ϕ is scalar field, m and λ are constants.

Equations (8) and (9) lead to

$$\left(\frac{A_{44}}{A}-\frac{B_{44}}{B}\right)+\frac{C_4}{C}\left(\frac{A_4}{A}-\frac{B_4}{B}\right)=0 \tag{13}$$

Thus, we have

$$\frac{d}{dt}\left(\frac{A_4}{A}-\frac{B_4}{B}\right)+\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right)\left(\frac{A_4}{A}-\frac{B_4}{B}\right)=0 \tag{14}$$

Equation (14) leads to

$$\frac{A_4}{A}-\frac{B_4}{B}=\frac{L}{ABC}=\frac{L}{t^n} \tag{15}$$

where L is constant of integration.

Similarly equations (8) and (10) lead to

$$\frac{A_4}{A}-\frac{C_4}{C}=\frac{M}{ABC}=\frac{M}{t^n} \tag{16}$$

where we have used the conditions (i) $R^3 = ABC = t^n$ for metric (3). Also $R^3 = ABC = t^n$ leads to

$$\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}=\frac{n}{t} \tag{17}$$

Equations (15), (16) and (17) lead to

$$A=\alpha t^{n/3}e^{-\frac{(L+M)}{3(n-1)t^{n-1}}} \tag{18}$$

$$B=\alpha\beta t^{n/3}e^{\frac{(2L-M)}{3(n-1)t^{n-1}}} \tag{19}$$

$$C=\alpha\gamma t^{n/3}e^{\frac{2M-L}{3(n-1)t^{n-1}}} \tag{20}$$

where α,β,γ,M are constants of integration and $\alpha^3\beta\gamma=1.$

After suitable transformation of coordinates, the metric (3) leads to the form

$$ds^2=-dt^2+t^{2n/3}e^{-\frac{2(L+M)}{3(n-1)t^{n-1}}}dx^2+t^{2n/3}e^{\frac{2(2L-M)}{3(n-1)t^{n-1}}}dy^2+t^{2n/3}e^{\frac{2(2M-L)}{3(n-1)t^{n-1}}}dz^2 \tag{21}$$

IV. To find the Value of Scalar Field (ϕ)

Using (17) in equation (12), we have

$$\phi_{44}+\frac{n}{t}\phi_4=-\frac{dV}{d\phi} \tag{22}$$

which leads to

$$\phi_{44}+\frac{n}{t}\phi_4=-(m^2\phi+\lambda\phi^{n-1}) \tag{23}$$

where

$$V = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{n}\phi^n \tag{24}$$

Following Linde [20], we have

$$\phi_{44} << \frac{dV}{d\phi},$$

Now equation (23) leads to

$$\frac{d\phi}{m^2\phi + \lambda\phi^{n-1}} = -\frac{t}{n}dt \tag{25}$$

Equation (25) leads to

$$\frac{\phi^{n-3}d\phi}{\phi^{n-2}(a + \phi^{n-2})} = -\frac{\lambda(n-2)t}{n}dt \tag{26}$$

where $a = m^2/\lambda$. Equation (26) leads to

$$\frac{d\xi}{\xi(a+\xi)} = -\frac{(n-2)\lambda}{n}.tdt$$

where $\xi = \phi^{n-2}$. Thus we have

$$\left(\frac{1}{\xi} - \frac{1}{a+\xi}\right)d\xi = -\frac{(n-2)\lambda a}{n}.tdt$$

which leads to

$$\log\frac{\xi}{a+\xi} = -\frac{\lambda(n-2)a}{2n}t^2 + \log l$$

Now, we have

$\frac{\xi}{a+\xi} = le^{-bt^2}$, l is constant of integration and

b = $\frac{\lambda(n-2)a}{2n}$ and *l* is constant of integration.

Thus $\frac{a+\xi}{\xi} = \frac{1}{l}e^{bt^2}$

Therefore, $\frac{a}{\xi} = \frac{1}{l}e^{bt^2} - 1$

$$\frac{\xi}{a} = \frac{\ell e^{-bt^2}}{(1-\ell e^{-bt^2})}$$

Therefore, $\phi^{n-2} = a\left(\frac{\ell e^{-bt^2}}{1-\ell e^{-bt^2}}\right)$ (27)

For n = 4, equation (27) leads to

$$\phi^2 = a\left(\frac{\ell e^{-bt^2}}{1-\ell e^{-bt^2}}\right) \tag{28}$$

At t = 0, $\phi^2 = \frac{a\ell}{1-\ell} < 1$ where $\ell = \frac{1}{1+a}$

ϕ decreases fastly for large values of t.

V. Calculation of Inflationary Parameters

We calculate the inflationary parameters i.e. slow roll parameters, non-Gaussianity parameter f_{NL}^{equil} and anisotropic parameter for the model (21) to examine whether these parameters are in excellent agreement with Planck results for canonical scalar field.

To the first approximation, for the model (21), the scale factor (R) is given by

$$R = t^{\frac{n}{3}} \tag{29}$$

The Hubble parameter (H) is given by

$$H = \frac{\dot{R}}{R} = \frac{n}{3t} = \frac{\alpha}{\tau} \tag{30}$$

where $\alpha = \frac{n}{3}$ (31)

The slow roll parameter $\in$ and δ are defined as given by Unnikrishnan and Sahni [24]

$$\in = -\frac{\dot{H}}{H^2} = \frac{1}{\alpha} = \alpha^{-1} \tag{32}$$

and

$$\delta = \in - \frac{\dot{\in}}{2H\in} \tag{33}$$

$$= \in = \alpha^{-1} \tag{34}$$

Slow roll PLI (Power Law Inflation) corresponds to $\in << 1$ which occurs when $\alpha >> 1$.

In this paper, we have discussed a new Power Law Inflation model in which inflation is driven by a Canonical scalar field with the Lagrangian

$$L(\phi, X) = X - V(\phi) \tag{35}$$

where

$$X = \frac{\dot{\phi}^2}{2} \tag{36}$$

and ϕ is Scalar field. For a generic $L(\phi,X)$, it is convenient to introduce a third slow roll parameters S as given by Hu [25]:

$$S = \frac{\dot{C}_s}{HC_s} \tag{37}$$

where C_s is the speed of sound of the scalar field as given by Garriga and Mukhanov [26]:

$$C_s^2 = \frac{\frac{\partial L}{\partial X}}{\frac{\partial L}{\partial X} + 2X\left(\frac{\partial^2 L}{\partial X^2}\right)} = 1 \tag{38}$$

Thus S = 0 flow roll inflation requires not only $\in << 1$ and $|\delta| << 1$ but also $|S| << 1$. For a canonical scalar field, the value of S is identically zero and this is also the case for kinetically driven as well as the non-canonical model (Unnikrishnan et al. [27]).

Non-Gaussianity Parameter f_{NL}^{equil}

We first carry out a simple estimate of non-Gaussianity for non-Canonical model with the Lagrangian

$$L(X, \phi) = X\left(\frac{X}{M^4}\right)^{\alpha-1} - V(\phi) \tag{39}$$

where α is dimensionless while M is dimension of Mass.

For Canonical model α = 1, the non-Gaussianity parameter f_{NL}^{equil} is given by Chen et al. [28] as

$$f_{NL}^{equil} = \frac{5}{81}\left(\frac{1}{C_s^2} - 1 - \frac{2\lambda}{N}\right) - \frac{35}{108}\left(\frac{1}{C_s^2} - 1\right) \tag{40}$$

where

$$\lambda = X^2 L, XX + \frac{2}{3} X^3 L, XXX \tag{41}$$

and $$N = X L, X + 2X^2 L, XX \tag{42}$$

We find that

$$\frac{\lambda}{N} = \frac{\alpha - 1}{3} \tag{43}$$

Using (41) and $C_s = \frac{1}{\sqrt{2\alpha - 1}}$ in (40), we have

$$f_{NL}^{equil} \simeq -0.57(\alpha - 1) \simeq -0.28(C_s^{-2} - 1) \tag{44}$$

as given by Unnikrishnan and Sahni [24].

For canonical model $C_s = 1$ and α = 1 which leads to $f_{NL}^{equil} \simeq 0$.

For α = 6, equation (44) gives $f_{NL}^{equil} \simeq -2.8$ which is in excellent agreement with Planck result as given by (Ade et al. Planck 2013 results[29]).

Anisotropic Parameter

The Higgs field ϕ given by (27) for n ≥ 4 decreases slowly and tends to zero when t → ∞. The average anisotropy parameter ($\hat{A}_m$) is given by

$$\hat{A}_m = \frac{1}{3}\left[\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2\right] \tag{45}$$

where $\Delta H_1 = H_1 - H, \Delta H_2 = H_2 - H, \Delta H_3 = H_3 - H$ and $H_1 = \frac{A_4}{A}, H_2 = \frac{B_4}{B}, \quad H_3 = \frac{C_4}{C}$.

Equation (45) leads to

$$\hat{A}_m = \frac{2(L^2 + M^2 - LM)}{n^2 t^{2(n-1)}} \tag{46}$$

VI. Discussion and conclusion

The scale factor (R), the expansion (θ), the Hubble parameter (H) and the deceleration parameter (q) for the model (21) are given by

$$R^3 = t^n \tag{47}$$

$$\theta = \frac{n}{t} \tag{48}$$

$$H = \frac{n}{3t} \tag{49}$$

$$q = -\left(\frac{n-3}{n}\right) \tag{50}$$

The spatial volume increases with time. The average anisotropy parameter ($\hat{A}_m$) is not zero in general but tend to zero in special case i.e. if $L^2 + M^2 = LM$. The anisotropy is initially large but disappears for large values of t i.e. the model isotropizes at late time. This result agrees with the result as pointed out by Rothman and Ellis [30], Jensen and Stein-Schabes [31]. The deceleration parameter $q < 0$ if $n > 3$ and $q > 0$ if $n < 3$. Thus the model (21) represents decelerating and accelerating phases of universe. This result agrees with the results as obtained by Perlmutter et al. [32] and Riess et al. [33]. The inflationary parameters viz. slow roll parameters, non-Gaussianity parameter and anisotropic parameters are in excellent agreement with Planck results for canonical scalar field. The Higgs field (ϕ) evolves slowly but the universe expands. From equation (27), we find that $\phi^{n-2} < 1$ at $t = 0$ where $n \geq 4$ and $\ell < \frac{1}{1+a}$. Also $V(\phi) = \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{n} \phi^n < 1$ as $\phi^2 < 1$ and $n \geq 4$. Thus initial condition of inflation is satisfied. We also find that $\phi_4^2 < V(\phi)$. Therefore, chaotic inflationary scenario is realized for anisotropic homogeneous Bianchi Type I space-time.

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