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# Squeezing in the Sum and Difference of the Field Amplitude in Parametric Down Conversion Process

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***Abstract:*** We studied squeezing in the sum and difference of the field amplitude in parametric down conversion process under the short-time approximation based on a fully quantum mechanical approach. It is shown that for uncorrelated modes the normal squeezing in the sum and difference-frequency field depends on the sum and difference squeezing of input field modes respectively, which can generate normal squeezing in the sum and difference-frequency field mode. We shown that if the high-frequency mode is in a coherent state and the low-frequency mode is squeezed, the field state will be difference squeezed if the amplitude of the high-frequency mode is large enough; otherwise the state may or may not be difference squeezed. If both modes are squeezed, then the state may or may not be difference squeezed. Detection of sum and difference squeezing in this process is also studied. All the possibilities for obtaining sum and difference squeezing in two modes and its dependence on squeezing of individual field modes are investigated.

***Keywords:*** Sum and difference squeezing; Higher-order squeezing; Parametric down conversion process; Optical parametric oscillator; Photon number operator. ***PACS Code:*** 42.50.Dv; 42.50.Ar; 42.50.p

## I. Introduction

Over the past decades, the squeezing [1-6] in quantized electro-magnetic fields has received a great deal of attention because of its wide applications in many branches of science and technology especially for low noise property [7–9] with an applications in high quality telecommunication [10], quantum cryptography [11, 12], and so forth. The basic concept of squeezed light is concerned with reduction of quantum fluctuations in one of the quadrature, at the expense of increased fluctuations in the other quadrature. Squeezing has been focused on theoretical investigations and experimental observations in a variety of nonlinear optical processes, such as harmonic generation [13, 14], multiwave mixing processes [15–18], Raman [19–21], hyper-Raman [22] Hong and Mandel [23, 24], Hillery [25–27], and Zhan [28] for improving the performance of many optical devices and optical communication networks. Squeezing and photon statistical effect of the field amplitude in optical parametric and in Raman and hyper Raman scattering processes has also been reported by Perina [29]. Higher-order sub-poissonian photon statistics of light have also been studied by Kim and Yoon [30]. Recently, Prakash and Mishra [31, 32] have reported the higher-order sub-poissonian photon statistics and their use in detection higher-order squeezing. Furthermore, another type of higher-order squeezing, called sum and difference squeezing were proposed by Hillery [33] for the two modes which are in fact the simplest versions of multimode higher-order squeezing. These concepts have recently been generalized to include three modes for sum and difference squeezing [34-36] as well as an arbitrary number of modes for sum and difference squeezing [37-39]. More recently, Prakash [40] et al. has reported regarding detection of sum and difference squeezing and Zhan [41] and Truong et al. [42] has introduced the concept of entanglement using sum and difference squeezing.

The objective of this paper is to study for the first time the concept of sum and difference squeezing in Parametric down conversion process under the short-time approximation based on a fully quantum mechanical approach. The paper is organized as follows. Section II gives the definition of sum and difference squeezing in two-mode field. Sum squeezing of the field amplitude in the pump mode is investigated in section III. In this section it is shown that for uncorrelated modes the normal squeezing in the sum-frequency field depends on the sum squeezing of input field modes, which can generate normal squeezing in the sum-frequency field mode. Detection of sum squeezing of two-mode field in this process is also studied in section III. In Section IV, squeezing in the difference of the field amplitude in the signal and idler modes are investigated. All the possibilities for obtaining difference squeezing in two modes and its dependence on squeezing of individual field modes are investigated. The different conditions for obtaining difference squeezing state and their detection in this process are also studied in section III. Finally, we conclude the paper in Section V.

## II. Definition Of Two-Mode Sum And Difference Squeezing

### 1. Two-mode sum squeezing

In order to define two-mode sum and difference squeezing in Parametric down conversion process, let us consider first for two-mode of the electromagnetic field of frequency $\omega_2$ and $\omega_3$ with creation (annihilation) operators $b^\dagger(b)$ and $c^\dagger(c)$ and introduce two operators which correspond to real and imaginary parts respectively, of the sum of the field amplitude as

$$W_1 = \frac{1}{2}(BC + B^\dagger C^\dagger) \quad (1)$$

and $$W_2 = \frac{1}{2i}(BC - B^\dagger C^\dagger) \quad (2)$$

These operators satisfy the commutation relation

$$[W_1, W_2] = \frac{i}{2}(N_B + N_C + 1) \quad (3)$$

and the uncertainty relation

$$\Delta W_1 \, \Delta W_2 \geq \frac{1}{4}\langle N_B + N_C + 1 \rangle \quad (4)$$

where $N_B = B^\dagger B$ and $N_C = C^\dagger C$ are the photon number operator for the Stokes and idler mode respectively.
A state is said to be sum squeezed in the $W_1$ direction if

$$(\Delta W_1)^2 < \frac{1}{4}\langle N_B + N_C + 1 \rangle \quad (5)$$

### 2. Two-mode difference squeezing

For two modes of frequency $\omega_1$ and $\omega_3$ with creation (annihilation) operators $a^\dagger(a)$ and $c^\dagger(c)$ respectively, squeezing operators $U_1$ and $U_2$ may be defined as

$$U_1 = \frac{1}{2}(AC^\dagger + A^\dagger C) \quad (6)$$

and $$U_2 = \frac{1}{2i}(AC^\dagger - A^\dagger C) \quad (7)$$

The variables $U_1$ and $U_2$ obey the commutation relation

$$[U_1, U_2] = \frac{i}{2}(N_C - N_A) \quad (8)$$

and the uncertainty relation

$$\Delta U_1 \, \Delta U_2 \geq \frac{1}{4}\left|\langle N_C - N_A \rangle\right| \quad (9)$$

where $N_A = A^\dagger A$ is the photon number operator for the pump mode.
A state is said to be difference squeezed in the $U_1$ direction if

$$(\Delta U_1)^2 < \frac{1}{4}\left|\langle N_C - N_A \rangle\right| \quad (10)$$

Similarly, for two modes of frequency $\omega_1$ and $\omega_2$ with creation (annihilation) operators $a^\dagger(a)$ and $b^\dagger(b)$ respectively, squeezing operators $V_1$ and $V_2$ may be written as

$$V_1 = \frac{1}{2}(AB^\dagger + A^\dagger B) \quad (11)$$

and $$V_2 = \frac{1}{2i}(AB^\dagger - A^\dagger B) \quad (12)$$

These operators obey the commutation relation

$$[V_1, V_2] = \frac{i}{2}(N_B - N_A) \quad (13)$$

and the uncertainty relation

$$\Delta V_1 \, \Delta V_2 \geq \frac{1}{4}\left|\left\langle N_B - N_A \right\rangle\right| \tag{14}$$

A state is said to be difference squeezed in the $V_1$ direction if

$$(\Delta V_1)^2 < \frac{1}{4}\left|\left\langle N_B - N_A \right\rangle\right| \tag{15}$$

## III. Squeezing In The Sum Of The Field Amplitude In The Pump Mode

Parametric down conversion (PDC) process, shown in fig.1, is a three-wave interaction process where a pump of photon of frequency $\omega_p$ splits into two, signal and idler, photons with lower frequencies $\omega_s$ , $\omega_i$ respectively and the corresponding Hamiltonian can be written as

$$H = \omega_1 a^\dagger a + \omega_2 b^\dagger b + \omega_3 c^\dagger c + g\,(a b^\dagger c^\dagger + a^\dagger bc) \tag{16}$$

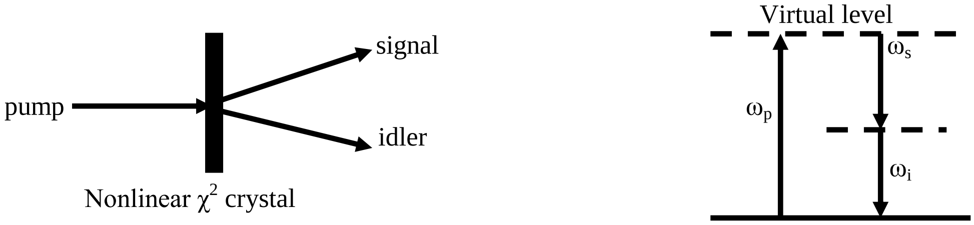


**Fig1: Schematic diagram showing the interaction of pump, signal and idler frequencies and energy level diagram for PDC process**

where $a^\dagger(a)$, $b^\dagger(b)$ and $c^\dagger(c)$ are the creation (annihilation) operators of the A, B and C modes respectively and g is the coupling constant in the interaction Hamiltonian, which is assumed to be real, describes the coupling between the two modes of the order of $10^2$–$10^4$ per second and is proportional to the nonlinear susceptibility of the medium as well as the complex amplitude of the pump field [43, 44]. However, to take care of complex g, we have used $|g|^2$ in the place of $g^2$ as we are not considering the phase terms. In the case of phase matching, g can also be treated as real [45].

Using the interaction Hamiltonian of the equation (16) in the coupled Heisenberg equation of motion

$$\dot{A}(t) = \frac{\partial A(t)}{\partial t} + i\,[\,H, A(t)\,] \qquad (\hbar = 1) \tag{17}$$

where the dot denotes time derivative.

Equation (17) leads to coupled Heisenberg equations of motion

$$\dot{A} = -igBC,\ \dot{B} = -igAC^\dagger \text{ and } \dot{C} = -igAB^\dagger \tag{18}$$

where A, B and C are slowly varying operators because the interaction between modes, the operators A(t) and $A^\dagger(t)$ induces a slower dependence on time as compared to fast variation, which are defined by $A = a\exp(i\omega_1 t)$, $B = b\exp(i\omega_2 t)$ and $C = c\exp(i\omega_3 t)$, with the relation $\omega_1 = \omega_2 + \omega_3$.

Note that the system evolution during a short period of time is practically relevant because the actual interaction is in fact very short. Hence the interaction time is taken to be short, of the order of $10^{-10}$ sec and a nanosecond or picosecond pulse laser can be used as the pump field. For real physical situation in the short-time scale $gt << 1$ ($gt \sim 10^{-6}$) and the number of photons are very large ($|\alpha|^2 >> 1$), it is possible to obtain much simpler approximate analytical formulas describing the variances. Expanding A(t) in Taylor's expansion and keep terms up to second order in gt, we get

$$A(t) = A - igtBC - \frac{1}{2}|g|^2 t^2 (N_B A + N_C A + A) \tag{19}$$

and

$$A^\dagger(t) = A^\dagger + igtB^\dagger C^\dagger - \frac{1}{2}|g|^2 t^2 (N_B A^\dagger + N_C A^\dagger + A^\dagger) \tag{20}$$

Similarly,

$$B(t) = B - igtAC^\dagger - \frac{1}{2}|g|^2 t^2 (N_C - N_A) B \tag{21}$$

and $B^{\dagger}(t) = B^{\dagger} + igtA^{\dagger}C - \frac{1}{2}|g|^2t^2 (N_C-N_A)B^{\dagger}$

(22)

also $C(t) = C - igtAB^{\dagger} - \frac{1}{2}|g|^2t^2 (N_B-N_A)C$ (23)

and $C^{\dagger}(t) = C^{\dagger} + igtA^{\dagger}B - \frac{1}{2}|g|^2t^2 (N_B-N_A)C^{\dagger}$ (24)

In order to examine the squeezing of the field amplitude of the fundamental A mode and its dependence on sum squeezing in the B and C modes as a function of time, we define two general quadrature components,

$$X_{1A}(t) = \frac{1}{2}[A(t) + A^{\dagger}(t)] \quad (25)$$

and $$X_{2A}(t) = \frac{1}{2i}[A(t) - A^{\dagger}(t)] \quad (26)$$

Using equations (19) and (20) in equations (25) and (26), we can obtain that to second order in gt

$$X_{1A}(t) = X_{1A} + |g|\, t\, W_2 - \frac{1}{2}|g|^2 t^2 (N_B + N_C + 1) X_{1A} \quad (27)$$

and $$X_{2A}(t) = X_{2A} - |g|\, t\, W_1 - \frac{1}{2}|g|^2 t^2 (N_B + N_C + 1) X_{2A} \quad (28)$$

For uncorrelated modes at t = 0, we get

$$[\Delta X_{1A}(t)]^2 = (\Delta X_{1A})^2 + |g|^2 t^2 [(\Delta W_2)^2 - \langle N_B + N_C + 1\rangle (\Delta X_{1A})^2] \quad (29)$$

and $$[\Delta X_{2A}(t)]^2 = (\Delta X_{2A})^2 + |g|^2 t^2 [(\Delta W_1)^2 - \langle N_B + N_C + 1\rangle (\Delta X_{2A})^2] \quad (30)$$

If the A mode is initially in a coherent state, then

$$(\Delta X_{1A})^2 = (\Delta X_{2A})^2 = \frac{1}{4} \quad (31)$$

and equations (29) and (30) reduce to

$$[\Delta X_{1A}(t)]^2 - \frac{1}{4} = |g|^2 t^2 [(\Delta W_2)^2 - \frac{1}{4}\langle N_B + N_C + 1\rangle] \quad (32)$$

and $$[\Delta X_{2A}(t)]^2 - \frac{1}{4} = |g|^2 t^2 [(\Delta W_1)^2 - \frac{1}{4}\langle N_B + N_C + 1\rangle] \quad (33)$$

These equations (32) and (33) give us the relation between sum squeezing and normal squeezing in sum-frequency generation. We see that if the input state is sum squeezed in the $W_2$ or $W_1$ direction, then sum-frequency generation will produce an output, which squeezed in the normal sense in the $X_{1A}$ or $X_{2A}$ direction respectively. In particular, sum squeezing in $W_1$ will lead to normal squeezing in $X_{2A}$, and sum squeezing in $W_2$ will produce normal squeezing in $X_{1A}$. This result suggests a method of detection for sum squeezing in PDC process.

It is also of interest to examine sum squeezing of the fundamental mode A as a function of time, we define the quadrature operators

$$W_{1A}(t) = \frac{1}{2}[B(t)C(t) + B^{\dagger}(t)C^{\dagger}(t)] \quad (34)$$

and $$W_{2A}(t) = \frac{1}{2i}[B(t)C(t) - B^{\dagger}(t)C^{\dagger}(t)] \quad (35)$$

Under short-time approximation we keep terms up to first-order in 'gt' in the Taylor's expansion to get

$$B(t) = B(0) + t\dot{B}(0) + \ldots\ldots\ldots\ldots \quad (36)$$

and $$C(t) = C(0) + t\dot{C}(0) + \ldots\ldots\ldots\ldots \quad (37)$$

Using equations (36) and (37) in equations (21-24), gives

$$B(t) = B - igtAC^{\dagger} \quad (38)$$

$$B^{\dagger}(t) = B^{\dagger} + igtA^{\dagger}C \quad (39)$$

and $$C(t) = C - igtAB^{\dagger}$$ (40)

$$C^{\dagger}(t) = C^{\dagger} + igtA^{\dagger}B \tag{41}$$

Using equations (38-41) in equation (34), we find only for $W_{1A}$ quadrature as,

$$W_{1A}(t) = \frac{1}{2}[(BC + B^{\dagger}C^{\dagger}) - igt\,(N_B + N_C + 1)\,(A - A^{\dagger})] \tag{42}$$

Now, we assume an initial quantum state as a product of coherent states $|\alpha>$ for the fundamental mode A, $|\beta>$ for the signal mode B and $|\gamma>$ for the idler mode C, i.e.

$$|\,\psi> = |\alpha>_A|\beta>_B|\gamma>_C \tag{43}$$

Using equation (43) in equation (42), we obtain the expectation values as

$$\left\langle\psi\middle|W_{1A}^2\middle|\psi\right\rangle = \frac{1}{4}[\beta^2\gamma^2 + \beta^{*2}\gamma^2 + 2|\beta|^2|\gamma|^2 + |\beta|^2 + |\gamma|^2 + 1 - 2igt(\alpha\beta\gamma|\beta|^2 + \alpha\beta\gamma|\gamma|^2 + 2\alpha\beta\gamma - \alpha^*\beta\gamma|\beta|^2$$
$$- 2\alpha^*\beta\gamma - \alpha^*\beta\gamma|\gamma|^2 + \alpha\beta^*\gamma^*|\beta|^2 + \alpha\beta^*\gamma^*|\gamma|^2 + 2\alpha\beta^*\gamma^* - \alpha^*\beta^*\gamma^*|\beta|^2 - \alpha^*\beta^*\gamma^*|\gamma|^2 - 2\alpha^*\beta^*\gamma^*)] \tag{44}$$

and

$$\left\langle\psi\middle|W_{1A}\middle|\psi\right\rangle^2 = \frac{1}{4}[\beta^2\gamma^2 + \beta^{*2}\gamma^2 + 2|\beta|^2|\gamma|^2 - 2igt(\alpha\beta\gamma|\beta|^2 + \alpha\beta\gamma|\gamma|^2 + \alpha\beta\gamma - \alpha^*\beta\gamma|\beta|^2 - \alpha^*\beta\gamma$$
$$- \alpha^*\beta\gamma|\gamma|^2 + \alpha\beta^*\gamma^*|\beta|^2 + \alpha\beta^*\gamma^*|\gamma|^2 + \alpha\beta^*\gamma^* - \alpha^*\beta^*\gamma^*|\beta|^2 - \alpha^*\beta^*\gamma^*|\gamma|^2 - \alpha^*\beta^*\gamma^*)] \tag{45}$$

Hence the field variance is

$$[\Delta W_{1A}(t)]^2 = \langle W_{1A}^2(t)\rangle - \langle W_{1A}(t)\rangle^2$$
$$= \frac{1}{4}[|\beta|^2 + |\gamma|^2 + 1 - 2igt\,(\alpha\beta\gamma - \alpha^*\beta\gamma + \alpha\beta^*\gamma^* - \alpha^*\beta^*\gamma^*)] \tag{46}$$

In order to determine sum squeezing, it is necessary to discuss the number of photons in the B and C modes. We find that to first order in 'gt'

$$N_B(t) = B^{\dagger}(t)B(t) = N_B - igt(AB^{\dagger}C^{\dagger} - A^{\dagger}BC) \tag{47}$$

and $$N_C(t) = C^{\dagger}(t)C(t) = N_C - igt(AB^{\dagger}C^{\dagger} - A^{\dagger}BC) \tag{48}$$

Using equation (43), we obtain

$$\frac{1}{4}\left\langle N_B(t) + N_C(t) + 1\right\rangle = = \frac{1}{4}[|\beta|^2 + |\gamma|^2 + 1 - 2igt\,(\alpha\beta^*\gamma^* - \alpha^*\beta\gamma)] \tag{49}$$

Subtraction of equation (46) from equation (49) yields

$$[\Delta W_{1A}(t)]^2 - \frac{1}{4}\left\langle N_B(t) + N_C(t) + 1\right\rangle = gt|\alpha\beta\gamma|\sin(\theta_1 + \theta_2 + \theta_3) \tag{50}$$

where $\alpha = |\alpha|\exp(i\theta_1), \beta = |\beta|\exp(i\theta_2)$ and $\gamma = |\gamma|\exp(i\theta_3)$. (51)

From this equation (50) we found that the squeezing of $W_{1A}$ will occur whenever $\sin(\theta_1 + \theta_2 + \theta_3) < 0$.

Let us now examine the dependence of sum squeezing for two-mode states on the squeezing of individual modes in which the modes are uncorrelated in Parametric down conversion process.

We define for two-mode sum squeezing as [33]

$$W_{A\varphi} = \frac{1}{2}(e^{i\varphi}B^{\dagger}C^{\dagger} + e^{-i\varphi}BC) \tag{52}$$

A field state is squeezed if $\Delta W_{A\varphi} < \frac{1}{2}$ for some $\varphi$.

Using equation (52), we obtain the following expectation values as

$$\left\langle W_{A\varphi}^2\right\rangle = \frac{1}{4}\left\langle e^{2i\varphi}(B^{\dagger}C^{\dagger})^2 + B^{\dagger}C^{\dagger}BC + BCB^{\dagger}C^{\dagger} + e^{-2i\varphi}(BC)^2\right\rangle$$

$$= \frac{1}{4}\left\langle e^{2i\varphi}(B^\dagger C^\dagger)^2 + 2B^\dagger BC^\dagger C + B^\dagger B + C^\dagger C + 1 + e^{-2i\varphi}(BC)^2 \right\rangle$$

(53)

$$\left\langle W_{A\varphi} \right\rangle^2 = \frac{1}{4}\left[\left\langle e^{2i\varphi} < B^\dagger C^\dagger >^2 + 2 < B^\dagger C^\dagger >< BC > + e^{-2i\varphi} < BC >^2 \right\rangle\right] \quad (54)$$

Hence, the field variance is

$$[\Delta W_{A\varphi}]^2 = < W_{A\varphi}^2 > - < W_{A\varphi} >^2$$

$$= \frac{1}{4}\{e^{2i\varphi}[< (B^\dagger C^\dagger)^2 > - < B^\dagger C^\dagger >^2] + 2 < B^\dagger BC^\dagger C > + < B^\dagger B > + < C^\dagger C > + 1$$

$$- 2 < B^\dagger C^\dagger >< BC > + e^{-2i\varphi}[< (BC)^2 > - < BC >^2]\} \quad (55)$$

Using equation (5) in equation (55), we find

$$[\Delta W_{A\varphi}]^2 - \frac{1}{4}\left\langle N_B + N_C + 1 \right\rangle = \frac{1}{4}\{e^{2i\varphi}[< (B^\dagger C^\dagger)^2 > - < B^\dagger C^\dagger >^2] + 2 < B^\dagger BC^\dagger C >$$

$$- 2 < B^\dagger C^\dagger >< BC > + e^{-2i\varphi}[< (BC)^2 > - < BC >^2]\} \quad (56)$$

A state is squeezed if the term in brackets becomes negative. This term is smallest when

$$\arg[< (BC)^2 > - < BC >^2] - 2\varphi = \pi \quad (57)$$

If $\varphi$ satisfies this condition, then

$$[\Delta W_{A\varphi}]^2 - \frac{1}{4}\left\langle N_B + N_C + 1 \right\rangle = \frac{1}{2}\left[< N_B N_C > - \left|< BC >\right|^2 - \left|< (BC)^2 > - < BC >^2\right|\right] \quad (58)$$

Therefore, a state is sum squeezed if and only if

$$|<(BC)^2> - <BC>^2| \;>\; <(N_B N_C)> - |<BC>|^2 \quad (59)$$

If the modes are uncorrelated, then the expectation values factorize into those for the B and C modes and the above equation (59) becomes

$$|<B^2><C^2> - <B>^2 <C>^2| \;>\; <N_B><N_C> - |<B><C>|^2 \quad (60)$$

Case-I: If the modes are uncorrelated and neither the B nor the C mode is squeezed, for which we have the condition [33-35]

$$|<B^2> - <B>^2 | \;\le\; <N_B> - |<B>|^2 \quad (61)$$

and $$|<C^2> - <C>^2| \;\le\; <N_C> - |<C>|^2 \quad (62)$$

Further, if none of the B and C modes are squeezed then none of the pairs can be sum squeezed [33-34], i.e.

$$|<B^2><C^2> - <B>^2 <C>^2| \;\le\; <N_B><N_C> - |<B><C>|^2 \quad (63)$$

Comparing this with equation (60) we see that the B and C modes are not sum squeezed in PDC process.

Case-II: If the B mode is squeezed and the C mode is in a coherent state of amplitude $\gamma$, we then have

$$|<B^2><C^2> - <B>^2 <C>^2| = |\gamma|^2 |(<B^2> - <B>^2)|$$

$$> |\gamma|^2 (<N_B> - |<B>|^2) \quad (64)$$

where $<N_C> = |\gamma|^2$ for coherent states and the inequality in equation (60) is fulfilled hence state is sum squeezed.

Case-III: If the C mode is squeezed and the B mode is in a coherent state of amplitude $\beta$, we then have

$$|<B^2><C^2> - <B>^2 <C>^2| = |\beta|^2 |(<C^2> - <C>^2)|$$

$$> |\beta|^2 (<N_C> - |<C>|^2) \quad (65)$$

where $<N_B> = |\beta|^2$ and the inequality in equation (60) is satisfied hence the B and C modes are sum squeezed.

Case-IV: If B and C modes are squeezed, then we have

$$|<B^2><C^2> - <B>^2 <C>^2| \;>\; <N_B><N_C> - |<B><C>|^2 \quad (66)$$

This satisfies the condition (60) and hence the state is sum squeezed.

## IV. Squeezing In The Difference Of The Field Amplitude In The Signal And Idler Modes

We now investigate the dependence of the occurrence of normal squeezing in the field amplitude of the signal mode B on difference squeezing in the A and C modes i.e. $\omega_2 = \omega_1 - \omega_3$ as a function of time, we define quadrature components

$$X_{1B}(t) = \frac{1}{2}[B(t) + B^\dagger(t)] \quad (67)$$

and $X_{2B}(t) = \frac{1}{2i}[B(t)-B^{\dagger}(t)]$

(68)

Using equations (21) and (22) in equations (67) and (68), we have

$$X_{1B}(t) = X_{1B} + |g|\, t\, U_2 - \frac{1}{2}|g|^2 t^2 (N_C - N_A)\, X_{1B} \tag{69}$$

and $$X_{2B}(t) = X_{2B} - |g|\, t\, U_1 - \frac{1}{2}|g|^2 t^2 (N_C - N_A)\, X_{2B} \tag{70}$$

For uncorrelated modes at t = 0, we get

$$[\Delta X_{1B}(t)]^2 = (\Delta X_{1B})^2 + |g|^2 t^2 [(\Delta U_2)^2 - \langle N_C - N_A \rangle (\Delta X_{1B})^2] \tag{71}$$

and $$[\Delta X_{2B}(t)]^2 = (\Delta X_{2B})^2 + |g|^2 t^2 [(\Delta U_1)^2 - \langle N_C - N_A \rangle (\Delta X_{2B})^2] \tag{72}$$

If the B mode is initially in a coherent state, then

$$(\Delta X_{1B})^2 = (\Delta X_{2B})^2 = \frac{1}{4} \tag{73}$$

and equations (71) and (72) reduce to

$$[\Delta X_{1B}(t)]^2 - \frac{1}{4} = |g|^2 t^2 [(\Delta U_2)^2 - \frac{1}{4}\langle N_C - N_A \rangle] \tag{74}$$

and $$[\Delta X_{2B}(t)]^2 - \frac{1}{4} = |g|^2 t^2 [(\Delta U_1)^2 - \frac{1}{4}\langle N_C - N_A \rangle] \tag{75}$$

Equations (74) and (75) show that $X_{1B}$ in the signal mode is squeezed if $U_2$ is squeezed and $X_{2B}$ is squeezed if $U_1$ is squeezed. In other words, the B mode is squeezed in the $X_{1B}$ direction if the A and C modes are difference squeezed in the $U_2$ direction and the B mode is squeezed in the $X_{2B}$ direction if the A and C modes are difference squeezed in the $U_1$ direction. Hence, difference-frequency generation changes difference squeezing into normal squeezing. In particular, difference squeezing in $U_1$ will be converted into squeezing in $X_{2B}$ and difference squeezing in $U_2$ will lead to squeezing in $X_{1B}$.

To examine difference squeezing of the fundamental mode A as a function of time, we define the quadrature operators

$$U_{1A}(t) = \frac{1}{2}[A(t)C^{\dagger}(t) + A^{\dagger}(t)C(t)] \tag{76}$$

and $$U_{2A}(t) = \frac{1}{2i}[A(t)C^{\dagger}(t) - A^{\dagger}(t)C(t)] \tag{77}$$

Using equations (19), (20), (23) and (24) to the first order coupling, we obtain

$$A(t)C^{\dagger}(t) = AC^{\dagger} - igt(N_C - N_A)B \tag{78}$$

and $$A^{\dagger}(t)C(t) = A^{\dagger}C + igt(N_C - N_A)B^{\dagger} \tag{79}$$

Using equations (78) and (79) in equation (76), we find only for $U_{1A}$ quadrature as,

$$U_{1A}(t) = \frac{1}{2}[(AC^{\dagger} + A^{\dagger}C) - igt\,(N_C - N_A)\,(B - B^{\dagger})] \tag{80}$$

Using equation (43) in equation (80), we obtain the expectation values as

$$\langle\psi|U_{1A}^2|\psi\rangle = \frac{1}{4}[\alpha^2\gamma^2 + \alpha^{*2}\gamma^2 + 2|\alpha|^2|\gamma|^2 + |\alpha|^2 + |\gamma|^2 - 2igt(\alpha\beta\gamma^*|\gamma|^2 - \alpha\beta\gamma^*|\alpha|^2$$
$$+ \alpha^*\beta\gamma|\alpha|^2 + \alpha^*\beta\gamma|\gamma|^2 + \alpha\beta^*\gamma^*|\alpha|^2 - \alpha\beta^*\gamma^*|\gamma|^2 + \alpha^*\beta^*\gamma|\alpha|^2 - \alpha^*\beta^*\gamma|\gamma|^2)] \tag{81}$$

and

$$\langle\psi|U_{1A}|\psi\rangle^2 = \frac{1}{4}[\alpha^2\gamma^2 + \alpha^{*2}\gamma^2 + 2|\alpha|^2|\gamma|^2 - 2igt(\alpha\beta\gamma^*|\gamma|^2 - \alpha\beta\gamma^*|\alpha|^2$$
$$+ \alpha^*\beta\gamma|\alpha|^2 + \alpha^*\beta\gamma|\gamma|^2 + \alpha\beta^*\gamma^*|\alpha|^2 - \alpha\beta^*\gamma^*|\gamma|^2 + \alpha^*\beta^*\gamma|\alpha|^2 - \alpha^*\beta^*\gamma|\gamma|^2)] \tag{82}$$

Hence the field variance is

$$\left[\Delta U_{1A}(t)\right]^2 = \langle U_{1A}^2(t)\rangle - \langle U_{1A}(t)\rangle^2 = \frac{1}{4}\left[|\alpha|^2 + |\gamma|^2\right] \tag{83}$$

Now, the number of photons of the A mode up to first order in 'gt'

$$N_A(t) = A^\dagger(t)A(t) = N_A - igt(AB^\dagger C^\dagger - A^\dagger BC) \tag{84}$$

Using equations (43), (48) and (84), we obtain

$$\frac{1}{4}\left|\langle N_C(t) - N_A(t)\rangle\right| = = \frac{1}{4}[|\alpha|^2 + |\gamma|^2 - 2igt\ (\alpha\beta^*\gamma^* + \alpha^*\beta\gamma)] \tag{85}$$

Subtraction of equation (83) from equation (85) yields

$$\left[\Delta U_{1A}(t)\right]^2 - \frac{1}{4}\left|\langle N_C(t) - N_A(t)\rangle\right| = igt|\alpha\beta\gamma|\cos\{\theta_1 - (\theta_2 + \theta_3)\} \tag{86}$$

From above equation (86) we found that the squeezing of $U_{1A}$ will occur whenever $\cos\{\theta_1 - (\theta_2 + \theta_3)\} < 0$.

Similarly, we now investigate the dependence of difference squeezing of the state on the squeezing of individual modes. For this case we define the two-mode operator as

$$U_{A\varphi} = \frac{1}{2}(e^{i\varphi}A^\dagger C + e^{-i\varphi}AC^\dagger) \tag{87}$$

Using equation (87), we obtain

$$\left\langle U_{A\varphi}^2\right\rangle = \frac{1}{4}\left\langle e^{2i\varphi}(A^\dagger C)^2 + A^\dagger CAC^\dagger + AC^\dagger A^\dagger C + e^{-2i\varphi}(AC^\dagger)^2\right\rangle$$

$$= \frac{1}{4}\left\langle e^{2i\varphi}(A^\dagger C)^2 + 2N_A N_C + N_A + N_C + e^{-2i\varphi}(AC^\dagger)^2\right\rangle \tag{88}$$

$$\left\langle U_{A\varphi}\right\rangle^2 = \frac{1}{4}\left[\left\langle e^{2i\varphi} < A^\dagger C >^2 + 2 < A^\dagger C >< AC^\dagger > + e^{-2i\varphi} < AC^\dagger >^2\right\rangle\right] \tag{89}$$

Hence,

$$[\Delta U_{A\varphi}]^2 = < U_{A\varphi}^2 > - < U_{A\varphi} >^2$$

$$= \frac{1}{4}\{e^{2i\varphi}[< (A^\dagger C)^2 > - < A^\dagger C >^2] + 2 < N_A N_C > + < N_A > + < N_C >$$

$$- 2 < A^\dagger C >< AC^\dagger > + e^{-2i\varphi}[< (AC^\dagger)^2 > - < AC^\dagger >^2]\} \tag{90}$$

Using equation (10) in equation (90), we find

$$[\Delta U_{A\varphi}]^2 - \frac{1}{4}\left|\langle N_C - N_A\rangle\right| = \frac{1}{4}\{e^{2i\varphi}[< (A^\dagger C)^2 > - < A^\dagger C >^2] + 2 < N_A N_C >$$

$$+ 2 < N_C > - 2 < A^\dagger C >< AC^\dagger > + e^{-2i\varphi}[< (AC^\dagger)^2 > - < AC^\dagger >^2]\} \tag{91}$$

A state is squeezed if the term in brackets becomes negative. This term is smallest when

$$\arg[< (AC^\dagger)^2 > - < AC^\dagger >^2] - 2\varphi = \pi \tag{92}$$

If $\varphi$ satisfies this condition, then

$$[\Delta U_{A\varphi}]^2 - \frac{1}{4}\left|\langle N_C - N_A\rangle\right| = \frac{1}{2}[< N_A N_C > - \left|< AC^\dagger >\right|^2 + < N_C > - \left|< (AC^\dagger)^2 > - < AC^\dagger >^2\right|] \tag{93}$$

Therefore, a state is difference squeezed if and only if

$$|<(AC^\dagger)^2> - <AC^\dagger>^2| \ > \ <N_A N_C> - |<AC^\dagger>|^2 + <N_C> \tag{94}$$

For uncorrelated modes the equation (94) becomes

$$|<A^2><(C^\dagger)^2> - <A>^2<C^\dagger>^2| \ > \ <N_A><N_C> - |<A><C^\dagger>|^2 + <N_C> \tag{95}$$

Case-I: If the modes are uncorrelated and both are unsqueezed, the condition for which is given by [33]

$$|<A^2> - <A>^2| \ \leq \ <N_A> - |<A>|^2 \tag{96}$$

Using equations (62) and (96), we then have

$$|\langle A^2\rangle \langle (C^\dagger)^2\rangle - \langle A\rangle^2\langle C^\dagger\rangle^2| \leq \langle N_A\rangle\langle N_C\rangle - |\langle A\rangle\langle C^\dagger\rangle|^2 \quad (97)$$

This implies that the condition given in equation (95) is not satisfied, so that the state is not difference squeezed.

Case-II: If the A mode squeezed and the C mode is in a coherent state of amplitude $\gamma$, we then have

$$|\langle A^2\rangle \langle (C^\dagger)^2\rangle - \langle A\rangle^2\langle C^\dagger\rangle^2| = |\gamma|^2 |(\langle A^2\rangle - \langle A\rangle^2)| \quad (98)$$

so that the state is difference squeezed if [33-35]

$$|\langle A^2\rangle - \langle A\rangle^2| > \langle N_A\rangle - |\langle A\rangle|^2 + 1 \quad (99)$$

Comparing with the condition as given in (96) we find that the above state may or may not be difference squeezed.

Case-III: If the A mode is in a coherent state of amplitude $\alpha$ and the C mode is squeezed, we then have

$$|\langle A^2\rangle \langle (C^\dagger)^2\rangle - \langle A\rangle^2\langle C^\dagger\rangle^2| = |\alpha|^2 |(\langle C^{\dagger 2}\rangle - \langle C^\dagger\rangle^2)| \quad (100)$$

where $\langle N_A\rangle = |\alpha|^2$ and the state is difference squeezed if

$$|\langle C^{\dagger 2}\rangle - \langle C^\dagger\rangle^2| > \langle N_C\rangle - |\langle C^\dagger\rangle|^2 + \frac{\langle N_C\rangle}{|\alpha|^2} \quad (101)$$

Comparing with the condition as given in (62) we find that the above state can be difference squeezed if

$$|\alpha|^2 \gg \langle N_C\rangle \quad (102)$$

i.e. the amplitude of mode A is large enough.

Case-IV: If both A and C modes are squeezed then

$$|\langle A^2\rangle \langle (C^\dagger)^2\rangle - \langle A\rangle^2\langle C^\dagger\rangle^2| > \langle N_A\rangle\langle N_C\rangle - |\langle A\rangle\langle C^\dagger\rangle|^2 \quad (103)$$

The above situation for A and C modes is similar to that for sum squeezing of B and C modes. Hence the state may or may not be difference squeezed.

In the same manner we define the amplitude quadrature components in the idler mode for the difference squeezing condition $\omega_3 = \omega_1 - \omega_2$ as

$$X_{1C}(t) = \frac{1}{2}[C(t) + C^\dagger(t)] \quad (104)$$

and $$X_{2C}(t) = \frac{1}{2i}[C(t) - C^\dagger(t)] \quad (105)$$

Using equations (23) and (24) in equations (104) and (105) for uncorrelated modes at t = 0, we have

$$[\Delta X_{1C}(t)]^2 = (\Delta X_{1C})^2 + |g|^2 t^2 [(\Delta V_2)^2 - \langle N_B - N_A\rangle (\Delta X_{1C})^2] \quad (106)$$

and $$[\Delta X_{2C}(t)]^2 = (\Delta X_{2C})^2 + |g|^2 t^2 [(\Delta V_1)^2 - \langle N_B - N_A\rangle (\Delta X_{2C})^2] \quad (107)$$

If the C mode is initially in a coherent state i.e.,

$$(\Delta X_{1C})^2 = (\Delta X_{2C})^2 = \frac{1}{4} \quad (108)$$

Then equations (106) and (107) reduce to

$$[\Delta X_{1C}(t)]^2 - \frac{1}{4} = |g|^2 t^2 [(\Delta V_2)^2 - \frac{1}{4}\langle N_B - N_A\rangle] \quad (109)$$

and $$[\Delta X_{2C}(t)]^2 - \frac{1}{4} = |g|^2 t^2 [(\Delta V_1)^2 - \frac{1}{4}\langle N_B - N_A\rangle] \quad (110)$$

Equations (109) and (110) show that $X_{1C}$ in the idler mode is squeezed if $V_2$ is squeezed and $X_{2C}$ is squeezed if $V_1$ is squeezed. In other words, the C mode is squeezed in the $X_{1C}$ direction if the A and B modes are difference squeezed in the $V_2$ direction and the C mode is squeezed in the $X_{2C}$ direction if the A and B modes are difference squeezed in the $V_1$ direction. Hence, difference-frequency generation changes difference squeezing into normal squeezing. In particular, difference squeezing in $V_1$ will be converted into squeezing in $X_{2C}$ and difference squeezing in $V_2$ will lead to squeezing in $X_{1C}$.

Now, the difference squeezing of the fundamental mode A as a function of time, we define the quadrature operators for A and B, as

$$V_{1A}(t) = \frac{1}{2}[A(t)B^\dagger(t) + A^\dagger(t)B(t)] \quad (111)$$

and $$V_{2A}(t) = \frac{1}{2i}[A(t)B^\dagger(t) - A^\dagger(t)B(t)] \quad (112)$$

Using equations (19-22) to the first order coupling in equation (111), we find for $V_{1A}$ quadrature as,

$$V_{1A}(t) = \frac{1}{2}\,[(AB^{\dagger} + A^{\dagger}B) - igt\,(N_B - N_A)\,(C - C^{\dagger})] \tag{113}$$

Using equation (43) in equation (113), we obtain the field variance as

$$[\Delta V_{1A}(t)]^2 = \langle V_{1A}^2(t)\rangle - \langle V_{1A}(t)\rangle^2 = \frac{1}{4}\left[|\alpha|^2 + |\beta|^2\right] \tag{114}$$

Hence using equations (15) and (114), we obtain

$$[\Delta V_{1A}(t)]^2 - \frac{1}{4}\left|\langle N_B(t) - N_A(t)\rangle\right| = igt|\alpha\beta\gamma|\cos\{\theta_1 - (\theta_2 + \theta_3)\} \tag{115}$$

From equation (115) we found the same result as given in equation (86) that the squeezing of $V_{1A}$ will occur whenever $\cos\{\theta_1 - (\theta_2 + \theta_3)\} < 0$.

Now, similarly the condition for the state to be difference squeezed in the present case is given by

$$|\langle (AB^{\dagger})^2\rangle - \langle AB^{\dagger}\rangle^2| \;>\; \langle N_A N_B\rangle - |\langle AB^{\dagger}\rangle|^2 + \langle N_B\rangle \tag{116}$$

For uncorrelated modes the equation (116) becomes

$$|\langle A^2\rangle\langle (B^{\dagger})^2\rangle - \langle A\rangle^2\langle B^{\dagger}\rangle^2| \;>\; \langle N_A\rangle\langle N_B\rangle - |\langle A\rangle\langle B^{\dagger}\rangle|^2 + \langle N_B\rangle \tag{117}$$

Case-I: If the modes are uncorrelated and both are unsqueezed.

Using equations (61) and (96), we then have

$$|\langle A^2\rangle\langle (B^{\dagger})^2\rangle - \langle A\rangle^2\langle B^{\dagger}\rangle^2| \;\le\; \langle N_A\rangle\langle N_B\rangle - |\langle A\rangle\langle B^{\dagger}\rangle|^2 \tag{118}$$

This implies that the condition given in equation (117) is not satisfied, so that the state is not difference squeezed.

Case-II: If the A mode is squeezed and the B mode is in a coherent state of amplitude β, we then have

$$|\langle A^2\rangle\langle (B^{\dagger})^2\rangle - \langle A\rangle^2\langle B^{\dagger}\rangle^2| \;=\; |\beta|^2\,|(\langle A^2\rangle - \langle A\rangle^2)| \tag{119}$$

so that the state is difference squeezed if

$$|\langle A^2\rangle - \langle A\rangle^2| > \langle N_A\rangle - |\langle A\rangle|^2 + 1 \tag{120}$$

Comparing with the condition as given in (96) we find that the state may or may not be difference squeezed.

Case-III: If the A mode is in a coherent state of amplitude α and the B mode is squeezed, we then have

$$|\langle B^{\dagger 2}\rangle - \langle B^{\dagger}\rangle^2| \;>\; \langle N_B\rangle - |\langle B^{\dagger}\rangle|^2 + \frac{\langle N_B\rangle}{|\alpha|^2} \tag{121}$$

Comparing this with equation (61) we see that the state can be difference squeezed if

$$|\alpha|^2 \gg \langle N_B\rangle \tag{122}$$

i.e. the amplitude of mode A is large enough.

Case-IV: If both A and B modes are squeezed, then the situation is similar to that for difference squeezing of the modes A and C i.e. the state may or may not be difference squeezed.

## V. Conclusions

In this paper we have found that the squeezing of the sum frequency field depends on the sum squeezing of the signal and idler modes in pump mode of PDC process.

In pump mode, we have established the relation between sum and normal squeezing in sum-frequency generation. That is sum squeezing can be turned into normal squeezing via sum-frequency generation. This result suggests a method of generation and also detection for sum squeezing in PDC process. We have also observed that the sum squeezing will occur whenever the condition will follow as $\sin(\theta_1 + \theta_2 + \theta_3) < 0$ to case of first order coupling. To generate sum squeezing, we have shown all kind of possibilities for an uncorrelated two modes state. If both modes are not squeezed, then the state is not sum squeezed. If one mode is squeezed and the second one is in a coherent state, then the state is sum squeezed. Finally, if both modes are squeezed, then the state may or may not be sum squeezed.

In signal and idler modes, difference squeezing is turned into normal squeezing by difference-frequency generation. The effect of first order coupling on squeezing in signal and idler modes have also studied and found that the difference squeezing will occur whenever the condition will follow as $\cos\{\theta_1 - (\theta_2 + \theta_3)\} < 0$. Various possibilities for obtaining difference squeezing in two modes and its dependence on squeezing of individual field modes are investigated. If both modes are not squeezed, the resulting state is not difference squeezed. If the high-frequency mode is in a coherent state and the low-frequency mode is squeezed, the field state will be difference squeezed if the amplitude of the high-frequency mode is large enough; otherwise the state may or may not be difference squeezed. If both modes are squeezed, then the state may or may not be difference squeezed.

These results suggest ways to generate a squeezed sum and difference-frequency fields in a nonlinear optical process. As a result, this family of higher-order squeezing effects can be used as a resource to improve high quality optical telecommunication [46].

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