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Transverse Resolution Enhancement beyondthe Diffraction Limit by Three-Zone Complex Pupil Filters

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Abstract:*We propose the use three-zone complex pupil filters to shape the intensity profile in the focal region of an optical imaging system to enhance its resolution.The enactment of these filters has been examined in terms of transverse resolution by means of PSF, spot size and position of first minima and intensity of first optical side-lobe. It has been observed that the presence of these filters enhancing the resolution of an optical system beyond the diffraction limit.*

Keywords:*Apodization, complex pupil filters, optical side-lobe, point spread function, transverse resolution, spot size*

I. Introduction

The response of any physically realizable optical system to a point object is never a point, but is a smear of light energy over a finite region, known as the point spread function (PSF) [1]. In diffraction theory, this is more commonly known as Airy pattern. The problem was probably first studied by Airy [2] and then developed by Rayleigh [3] by investigating light distribution in the images of discs. Lommel [4] studied both theoretically and experimentally the light distribution in three dimensions at and near the focus of an optical system. The control of the light intensity profile at and near the geometrical focus of an optical imaging system is highly desirable in many applications in diverse potential fields such as image processing, confocal microscopy, spectroscopy, microscopy, laser printing, optical data storage, lithography, medical imaging and microelectronics.It is well known that through proper apodization in the exit pupil of an optical system, the PSF of an optical system at focal plane may be shaped [5] into a desired form giving rise to superresolution over a confined region of the field of the instrument. This is the basis for our investigation.

Many methods have been proposed for the design of apodizer structure to achieve superresolution based on variable transmittance [6-13], phase-only profile [14-17] and both amplitude-phase profiles (complex pupil functions) [18, 19].In this paper we present the three-zone complex pupil filters to further enhancement in theresolution. This can be achieved by introducing amplitude filter in the central region of the pupil structure proposed byMaojin Yun et al [19]. At the same time we Paid particular attention to how best each filter enhances the quality and resolution of an imaging system for a given zone boundaries for a constant and variabletransmittance of the first and second zones, respectively, of the pupil filter in the presence of phase change.

We have organized the paper in the following manner: In Section II, we derive the mathematical expression for the amplitude and intensity distribution in the focal region of an optical system. Then in Section III, some interesting numerical results and our observations on the same are put forward. Finally we finish with a brief conclusion in section IV.

II. Theory

The far-field effects due to a circular aperture in an optical system can be derived from its amplitude response or the amplitude PSF. The diffracted light amplitude $G(v,u)$ associated with a rotationally symmetric pupil is given by

$$G(v,u) = 2\int_0^1 P(\rho)exp\left(-\frac{1}{2}iu\rho^2\right)J_0(v\rho)\rho d\rho \qquad (1)$$

where$P(\rho)$ is the generalized pupil function, ρ is the distance of the general point on the exit pupil expressed as a fraction of the radius of the pupil; J_0 is the Bessel function of the first kind andzero order. Here v and u are the simplified radial and axial optical coordinates, respectively.

$$v = kr\sin\alpha \qquad (2)$$

$$u = 4kz\sin^2\alpha \qquad (3)$$

Where $k=2\pi/\lambda$, $sin\alpha$ is the numerical aperture of the system, while r and z denote the radial and axial distances from the focal point.

In the focal plane we have

$$G(v,0) = 2\int_0^1 P(\rho) J_0(v\rho)\rho d\rho \quad (4)$$

so that in the focal plane the amplitude is the Hankel transform of the pupil function, whereas along the axis we have

$$G(0,u) = 2\int_0^1 P(\rho) exp\left(-\frac{1}{2}iu\rho^2\right)\rho d\rho \quad (5)$$

When we introduce the variable $t=\rho^2$, the pupil function can be written as $Q(t)$, which can be defined as the equivalent pupil function. Equation (5) becomesas

$$G(0,u) = \int_0^1 Q(t) exp\left(-\frac{1}{2}iut\right) dt \quad (6)$$

Thus the axial amplitude is the Fourier transform of the equivalent pupil function $Q(t)$. It is clear that the axial and transverse behaviors are not independent. Hence, the modification of the three-dimensional intensity distribution can be done by introducing proper pupil filter. So that superresolution can be realized.

In our investigations, we propose the complex pupil filters as superresolvers to modify the focusing properties of the rotationally symmetric optical imaging systems and can be written as

$$P(\rho) = T(\rho) exp[i\varphi(\rho)] \quad (7)$$

Here, $T(\rho)$ and $\varphi(\rho)$ are the two functions responsible for the amplitude and phase changes in the focusing light, respectively. The structure of the three zone complex pupil filter of unit radius is as shown in Fig.1 and it can be expressed as

$$P(\rho) = T(\rho) exp[i\varphi(\rho)] = \begin{cases} texp(0.6i\pi) & 0 \le \rho \le a \\ \beta\rho^2 a < \rho \le b \\ exp(0.6i\pi) b < \rho \le 1 \end{cases} \quad (8)$$

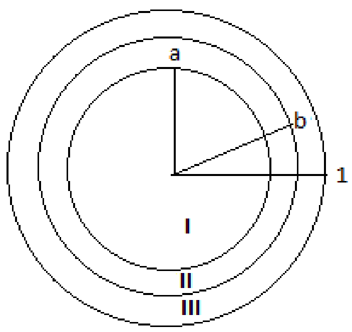

Fig. 1 Structure of the three-zone complex pupil filter.

wherea, b and 1are radii of three zones.The transmittances of three zones are t, βr^2 and 1; the corresponding phases are 0.6π, 0, and 0.6π. Here β is the apodization parameter which controls the degree of the non-uniform transmission over the specified region of an exit pupil. The range values it takes are $0 \le \beta \ge 1$. It is clear that$T(\rho)$= 0 for $\beta=0$, which implies no transmittance over this specified zone. On the other hand, $T(\rho)$= 1 for t = 1, that indicates uniform and complete transmittance over the central/first region of the pupil.

III. Results And Discussion

The results of investigations on the effects of complex pupil filters on intensity profile in the image plane of an optical system have been obtained from Eq. (4) as a function of radial co-ordinate v varying from –10 to +10 by employing a twelve-point Gauss quadrature numerical method of integration. An iterative method has been developed and applied to find the positions and peak intensities of various minima and maxima in the image plane.We mainly focused on positions of first minima (FMP) and intensity of first maxima (FMI) since they are the important parameter in judging the superresolution of an optical system. It may be mentioned here that we obtained these values for different cases and neglected higher order side-lobes and minima. In terms of these parameters, we assessed the quality of an image produced by an optical system in which the complex pupil filters are introduced at the exit pupil.All these results are obtained by choosing zone boundaries a=0.6, b=0.8 and 1of complex three level pupil filters.

The performance of the designed pupil filters is characterized by the transverse point spread functions and depicted in Fig. 2. In this figure the Airy PSF is added for easy comparison. The PSFs are graphically represented in this figure for different t values when the second-zone of the pupil filter is completely non-transmittance, β = 0. This figure reveals that the central lobe is narrowing as the transmittance of the first zone is decreasing. In addition to that the intensity of first side-lobe decreases for all the values of t, except t = 0, related to that of Airy case. This manifests the reduction in the spot size subsequently enhancing the resolution. However the magnitude of reduction in the spot size and suppression of side-lobe depends on t value. As the transmittance $t$of first zone increases from 0 to 1 intensity of side-lobes decreases from 0.0198 to 0.008, it means effectively suppressing the side-lobes thereby increasing the resolution of an imaging system. This can be seen

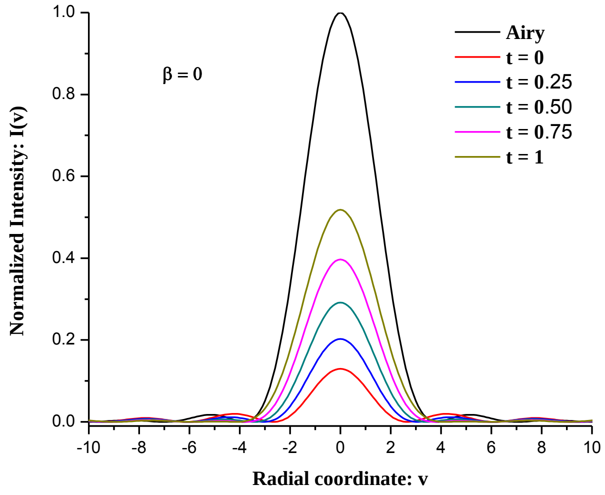

Fig. 2 Intensity profile for different t values when a second region is non-transmittance, β = 0.

in detail in Fig. 3 which is the magnified version of the region $2 \leq v \leq 7$of Fig. 2. This is an evidence for amenable result of controlling of first zone transmittance and second zone non-transmittance by narrowing central-lobe consequently occurring small spot as well as suppressing the side-lobes to a great extent.

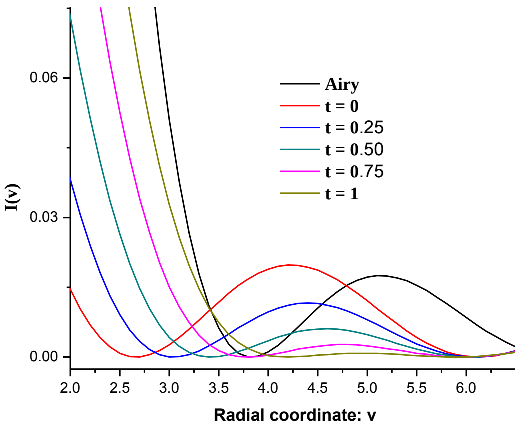

Fig. 3 Magnified version of the feet of figure 2.

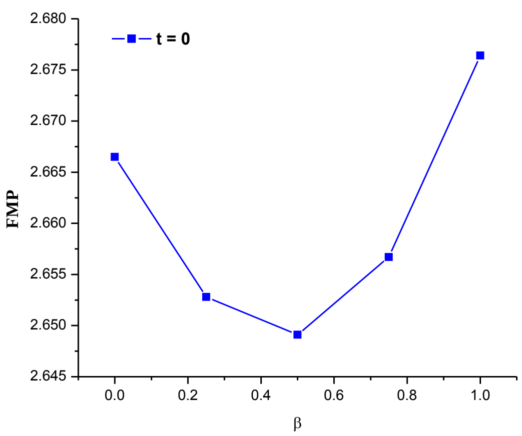

Fig.4: For a given value of t, the effect of an apodization parameter*β*on the position of first minimum (FMP).

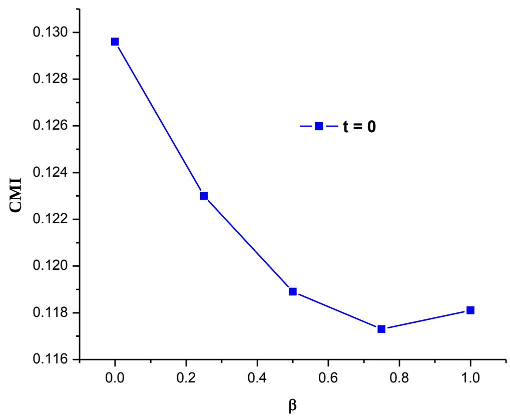

Fig.5: For a given value of t, the effect of an apodization parameter β on the intensity of central maximum (CMI).

Fig. 4 shows the effect of an apodization parameter β on the position of first minimum when the first zone is non-transmitted one (t = 0). Position of first minimum decreases as β increases and it becomes lowest value 2.6491 at β = 0.5 and there onwards it increases with β. In the case of t = 0 and β = 0, the corresponding value is 2.6665 and 3.822 in the Airy case. It emphasizes that how effectively the variable transmittance in the second zone reducing the spot size for lower transmittance of second zone. Under the same conditions the relation between the intensity of central maximum and β is observed and depicted in Fig 5. This shows that it increases first and then decreases with β. However, it is not lowest value at β = 0.5. Hence, for these combination of a, b and t, β = 0.5 may treat as an optimum value. This is facilitating to observe extremely faint object present very close to bright object as well as to resolve the two spectral lines which are widely varying in their intensities in the spectrum. This is the breakthrough in the transverse resolution that may callas superresolutionof an optical imaging system.

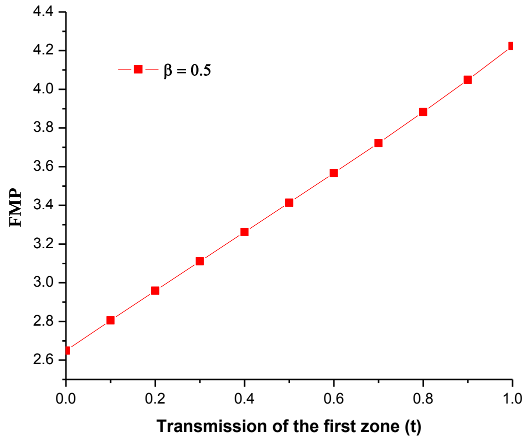

Fig.6: Relation between the first minimum position (FMP) and the transmittance of the first-zone of a complex pupil filter when β =0.5.

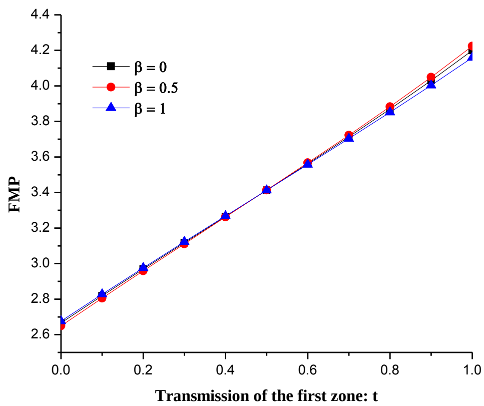

Fig.7: For different values of an apodization parameter β, the variation of first minimum position (FMP) as a function of transmittance of the first-zone.

The position of first minimum is particularly significant because it defines the width of the central maximum, where most of the intensity is located. In our investigation it is noticed that the position of first minimum decreases with t value whenβ = 0.5. The same is true for all βvalues. These are depicted in figure 6 and 7. They specify the existence of linear relationship between the position of first minimum and the transmittance of first zone for any given βvalue. Nevertheless, for lower values of $t$the position of first minimum increases with $\beta$and higher values of t it decreases with β.Fig. 4 and 7 are strengthening this fact. In the presence of variable transmittance of second zone the optical side-lobes suppression is much more and fairly crossing the diffraction barrier in reducing the spot size comparatively when it is completely transmitted one.

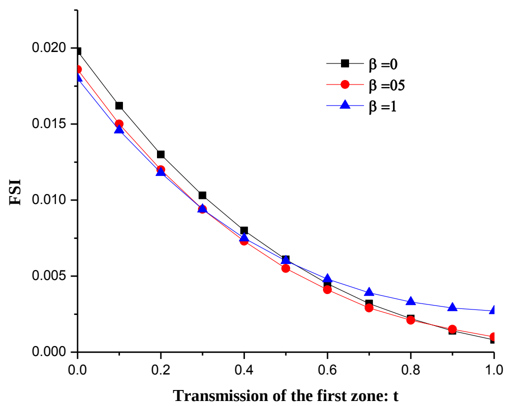

Fig.8: For different values of an apodization parameter β, the variation of first side-lobe intensity (FSI) with transmittance of the first-zone of a pupil filter.

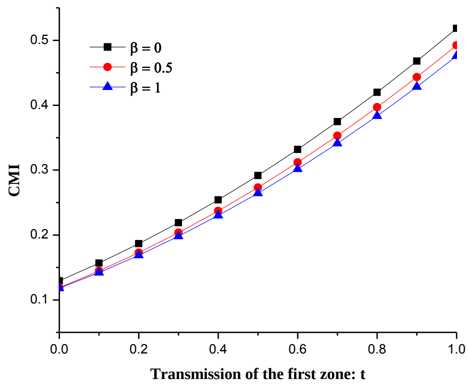

Fig9: For different values of an apodization parameter β, the variation of central-lobe intensity (CMI) with transmittance of the first-zone of a pupil filter.

Fig 8 shows the intensity of first side-lobe of the transverse PSFs for different values of first zone transmittance *t*. These curves are obtained for various amount of the apodization parameter β. As the figure reveals, an intensity of side-lobe decreases for all values of *b* as *t* increases from 0 to 1. However, it is clear from the observation that the magnitude of suppression increases with the apodization parameter β for lower values and decreases for higher values of *t*. As an example, when $t = 0$, side-lobe intensity reduces from 0.0198 to 0.0180 as *b* increases from 0 to 1 and when $t = 1$, side-lobe intensity increases from 0.008 to 0.027 for the same variation of β.

The peak intensity of central maximum (CMI) has been evaluated for different amount of first zone transmittance *t*. It has been observed that the intensity increases with *t*, for any given value of β. However, it decreases with the apodization parameter β. It means intensity of principal maximum is very much depending on both *t* as well as β.Fig. 9 shows this phenomenon in more detail.

IV. Conclusions

We may conclude that our work aimed at enhancing the resolution beyond diffraction barrier called superresolution has been carried out by considering three-zone complex pupil filters. Transverse PSFs are obtained with highly reduced side-lobes and a narrow central maximum. This is achieved at the cost of central spot brightness. The gliding of first minimum towards the centre of the main-lobe is accompanied by decrement of the side-lobes relative to that of Airy case is typically leads to enhance the resolution and image quality of an imaging systems as well as it provides high contrast. Unfortunately, for β =0 and t = 1 values the intensity of first maximum is slightly higher than the Airy case which may be drawback. But these values yield a better result by obtaining small spot size acceptable intensity of central maximum and extremely suppressed side lobes for the whole range of interest.

On the whole it can be emphasized that an additional improvement for resolution could be obtained by the use of complex pupil filters instead of phase only filters or amplitude only filters. This would make the system fabrication little bit complicate but it could be a solution for cases in which a very high contrast or resolution is required. Further investigations are being carried out on this subject to obtain 3D resolution.

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