# Using Spreadsheets and Computer Simulation to Approximate π

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***Abstract:*** *The role of information technology has been quite instrumental in the field of pedagogy. The use of Spreadsheets and computer simulation to understand the basic concepts of physics and mathematics has created powerful impact. From the pedagogical constructivist point of view it is important to select appropriate physics experiments. Combinations of motivational teaching techniques result in an upgrading of students' motivation for physics education. In this article, the use of spreadsheets and visual basic for applications in class room environment is presented to understand the approximation of universal mathematical constant π. In the class room teaching, the value of π is simply thrown at the students irrespective of where it comes from. Moreover, the graphical illustration of the problem shows a way to increase students' appreciation in the class. A simple practical device is fabricated to simulate this activity in the lab session.*
***Keywords:*** *ICT, Physics Education, Pi, Simulation, Spreadsheets, VBA*

## I. Introduction

An experiment is the most important educational and motivational tool in science education. Every educational activity based on experiments can be a strong motivational stimulus. Science teachers are often interested in creation of experiments. But they usually do not think about the concrete implementation of the experiments in teaching and learning science. A use of the wrong teaching method with an experiment results in a loss of educational objectives including motivation. It is observed that demonstration based teaching has yielded good results in understanding of students [1, 2]. In this article, an investigative approach is used to understand π constant-both with an experiment and computer simulation. The number π is a mathematical constant, conventionally defined as the ratio of a circle's circumference to its diameter and approximately equal to 3.14159. It is represented by the Greek letter "π" since the mid-18th century. Being an irrational number [3], π cannot be expressed exactly as a common fraction- the ratio of two integers, although fraction such as $\frac{22}{7}$ is commonly used to approximate π. Consequently its decimal representation never ends and there is no permanent repeating pattern in the decimal digits of this constant. The digits are randomly distributed. Other feature of π is that it is a transcendental number [4], a number that is not the root of any non-zero polynomial having rational coefficients.

For many years, mathematicians and other scientists have attempted to extend their understanding of π; most of times computing its value to a high degree of accuracy. In the beginning, mathematicians such as Archimedes and Liu Hui used geometrical techniques, based on polygons, to estimate the value of π. Around the 15th century, algorithms based on infinite series [5] revolutionized the computation of π. In the 20th and 21st centuries mathematicians and computer scientists discovered new approaches that, when combined with increasing computational power, extended the decimal representation of π to over 10 trillion (1011) digits [6]. In general, the scientific applications require no more than a few digits of π, so the primary motivation for these computations is the human desire to break records. Because its definition relates to the circle, π is found in many formulae in trigonometry and geometry, especially those concerning circles, ellipses or spheres. It is also found in formulae used in other branches of science such as cosmology, number theory, statistics, fractals, thermodynamics, mechanics and electromagnetism. The ubiquity of π makes it one of the most widely known mathematical constants both inside and outside the scientific community. Such a wide presence of π in many fields of sciences and its use in the school physics and mathematics puts pressure on the field of pedagogy i.e. teacher and taught. Although teacher can explain that π is just another constant which is equal to the ratio of circumference of a circle to its diameter. But this approach definitely blocks the logical thinking of the students. Why this ratio i.e. π is constant still remains a puzzle for the students. Here computer simulation is used to demonstrate the approximation of π, just as another constant occurring in the area computation of circle.

## II. Mathematical Model and Practical Design

Let us start from very common figure - Square. In geometry, a square is a regular quadrilateral, which means that it has four equal sides and four equal angles (90° each). It can also be defined as a rectangle in which two adjacent sides have equal length. A square with vertices A, B, C, and D is represented by ABCD as in Fig. 1. The circle - a locus of all points which are at equal distance from a fixed point, is another common figure as

shown in Fig. 2. The two figures are drawn in such a way that Circle is inscribed with in Square (see Fig. 3). From the Figure 3, it is obvious that the radius of the Circle and side of the Square are related as given below:

$$r = \frac{a}{2}$$
$$= f \times a \tag{1}$$

where $f = \frac{1}{2}$, is the fraction – ratio of radius of circle and size of square. This fraction can have any value between 0.5 and 0 for a circle lying within Square.
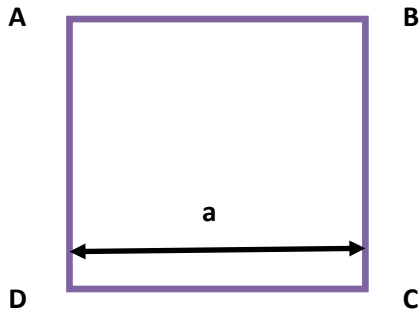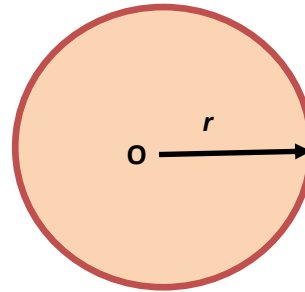
A ⬜ B

a

D ⬜ C

Figure 1: Square 'ABCD' with side 'a'.

O → r

Figure 2: Circle of radius 'r'.
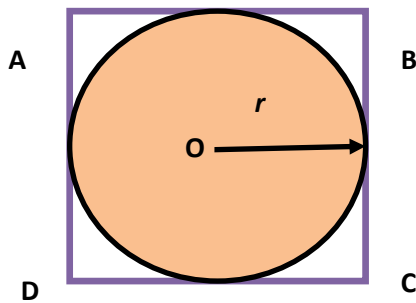
A

r

O →

B

D

C

Figure 3: Circle inscribed with in square 'ABCD'

Now let us try to find out the relationship between areas of Circle and Square for situation as depicted in Figure 3 for $0 \leq f \leq 0.5$. Suppose we do not know the exact expression for the area of Circle. It is also known that the dimensions of area are same whether it is area of Circle or area of Square. Moreover, the area of a Square contains product of two lengths ($a \times a$). This implies that the area of a Circle should be directly proportional to product of any two lengths. For simplicity both the lengths in case of a Circle are taken equal to radius, $r$ of the circle. Hence the area of the Circle can be expressed as:

$$A_c \propto r^2 \tag{2}$$

That implies

$$A_c = Constant \times r^2 \tag{3}$$

where $Constant$ is not known (Later it turns out to be well known constant $\pi$). After substituting the value of $r = f \times a$ , we get

$$A_c = Constant \times f^2 \times a^2 \tag{4}$$

The area of a Square with side $'a'$ is:

$$A_s = a^2 \tag{5}$$

Practically a square board is fabricated and circles (for $0 \leq f \leq 0:5$) are engraved within it. Let us now throw small coins one by one so that they stay within the board. All this is done while standing blindfolded before board. After certain number of successful trials, find the number of coins lying within any of the circle. Let it is $N_c$. Let the total number of successful trials is $N_s$ i.e. it is the number of coins on the board.

Theoretically, I assume that area is made of many small points of same size. It is well known that the probability of coin landing on the board at any point is same. There is no favorable point or part of area within the board. If the area is large, it will accommodate more points within it and hence coin will have large probability of landing within it. Then according to common wisdom, one can easily say that the number of coins in the Circle would be directly proportional to its area. And the number of coins lying on the board i.e. Square would also be directly proportional to the area of the square. This implies

$$\frac{area\ of\ Square}{are\ of\ Circle}$$

$$= \frac{N_s}{N_c} \tag{6}$$

$$=> \frac{N_s}{N_c}$$

$$= \frac{a^2}{Contant \times f^2 \times a^2} \tag{7}$$

$$=> \frac{N_s}{N_c} = \frac{4}{Constant}\ (for\ f$$

$$= \frac{1}{2}) \tag{8}$$

$$or\quad Constant$$

$$= \frac{N_c}{N_s \times f^2}\ (in\ general) \tag{9}$$

The experiment is attempted in the lab session. The value of $Constant$ computed was found close to $\frac{22}{7}$ for different values of fraction, $f$. However, using computer simulation, we can also determine the value of this $Constant$ to greater accuracy. While at the same time visualizations may help the students to appreciate and understand the fact that value of $\pi$ is not puzzle but in fact it is a constant associated with spherical shapes and could be evaluated inside the class room.

### III. Computer Simulation

Information and Communication Technology (ICT) has been instrumental in improvement of class room teaching. Computer simulation of real life problems and their presentation before the students in class room environment has enhanced the logical thinking and subject understanding among the students [1, 2]. To achieve the desired results, the above problem is modeled with the use of MS Excel and Visual Basic for Applications. The concept of Monte Carlo simulation is used. Here, a square of size '1 arbitrary unit' is drawn with its center at (0.5, 0.5). A circle of radius, $r = f$ arbitrary unit is plotted with its center at (0.5, 0.5). Now the random numbers $(x, y)$ lying between 0 and 1 are generated. For any point to lie within Square the following conditions must be satisfied:

$-0{:}5 \leq x \leq 0{:}5$ and $-0{:}5 \leq y \leq 0{:}5$.

The random number generation process is purely random and it resembles with the throwing of a coin on the board. Hence one trial corresponds to generation of one pair of random numbers i.e. $(x, y)$. All those points which lie on and within the Circle satisfy the relation $\sqrt{x^2 + y^2} = f$. A spreadsheet is prepared to simulate the problem using Visual Basic for Applications (VBA).

**3.1 Spreadsheet Environment**

Excel is a highly programmable product, and it is easily the best choice for developing spreadsheet based applications. Excel uses the VBA language, which is now in widespread use. This language allows the user to create structured programs directly in Excel. On the other hand, with Excel it's easy to access the controls such as scrollbars, spin buttons etc. because their implementation requires little or no macro programming.

In this case our problem is to draw a square and circle inside it. The size of the square is converted to 1 in arbitrary units and so the radius of circle depending on the value of $f$. The position (0.5, 0.5) is chosen the center of circles and Square. The input parameters in the spreadsheet are entered as shown in Table 1.

| Row | Column A | Column B |
|---|---|---|
| 3 | Size of Square= | 15 |
| 4 | Number of Trials= | 1000 |

Table 1: Input data

All other values appearing in spreadsheet are generated by the program. The output is generated with a small program hidden under spreadsheet control buttons - START, Reset and Double Arrow Button. The circle is plotted using data from Columns C and D. The random co-ordinates (XX, YY) generated in Columns E and F represents the hits within square and circle. The number of hits within circle is calculated using boundary condition $\sqrt{XX^2 + YY^2} \le f$ . With all these parameters defined it is possible to build a graphic to illustrate simulation of throwing of coins into the board. The screen shot of the spreadsheet is shown is Fig. 4.
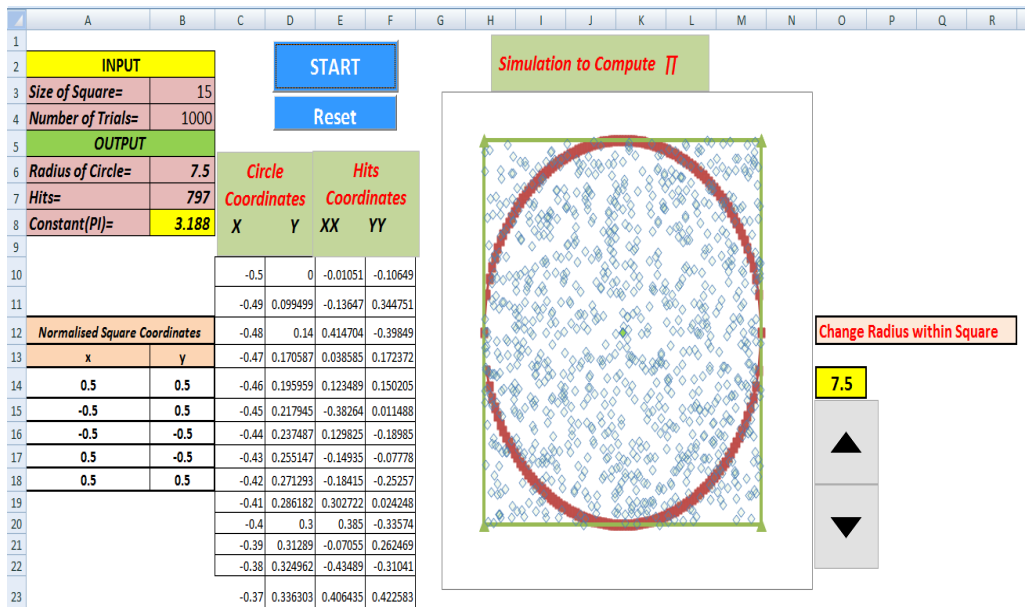


Figure 4: Screen Shot of Spreadsheet.

The small program written in VBA to simulate the throwing of coins is:

```
Private Sub CommandButton1_Click()
Dim r As Double, x As Double, y As Double, st As Double, xx As Double
Dim yy As Double, Pi As Double, Sq As Double
Dim N As Integer
Sq = Cells(3, 2)    'Size of square
N = Cells(4, 2)     'Number of trials/throws
pp = 0
kk = 0
Hits = 0            'Number of Hits inside circle
r = Sq / 2
Cells(6, 2) = Sq / 2
Sq = Sq / (Sq * 2)  'Size of unit square with centre (0.5,0.5)
'Coordinates of square vertices A,B,C,D and A
Cells(14, 1) = Sq   ' x of B
Cells(17, 1) = Sq   ' x of C
Cells(15, 1) = -Sq  ' x of D
Cells(16, 1) = -Sq  ' x of A
Cells(18, 1) = Sq   ' x of B
Cells(14, 2) = Sq   ' y of B
Cells(16, 2) = -Sq  ' y of C
Cells(17, 2) = -Sq  ' y of D
Cells(15, 2) = Sq   ' y of A
Cells(18, 2) = Sq   ' y of B
```

```
r = r / (2 * r)      'Circle of 0.5 radius-Normalised & centre (0.5,0.5)

'Circle data...................

r1 = Cells(16, 15) / 10 'new radius assigment-different 'f'

If r1 = "" Then
r = r
Else
r = r1
End If
'Generation of circle coordinates
st = r / 50
For x = -r To r Step st

y = Sqr(r * r - x * x)

Cells(10 + kk, 3) = x

Cells(10 + kk, 4) = y

Cells(110 + kk, 3) = -x

Cells(110 + kk, 4) = -y

 kk = kk + 1

'DoEvents
Next x

'End of Circle Data...........

'Number of hits inside the circle
For nn = 1 To N

xx = Rnd - 0.5
yy = Rnd - 0.5

If (xx <= 0.5 And yy <= 0.5) Then
pp = pp + 1
Cells(9 + pp, 5) = xx
Cells(9 + pp, 6) = yy

If (Sqr(xx * xx + yy * yy) <= r) Then

Hits = Hits + 1
Else
End If
Else
End If
'DoEvents
Next nn
'Calculate Constant
f = r
Cells(7, 2) = Hits
Pi = 1 / (f ^ 2) * (Hits / N)
Cells(8, 2) = Pi
```

End Sub

```
'Resetting the data
Private Sub CommandButton2_Click()
For jj = 1 To 30000
 Cells(9 + jj, 5) = 0
Cells(9 + jj, 6) = 0

Cells(9 + jj, 3) = 0

Cells(9 + jj, 4) = 0
Next jj

Cells(14, 1) = 0
Cells(15, 1) = 0
Cells(16, 1) = 0
Cells(17, 1) = 0
Cells(18, 1) = 0
Cells(14, 2) = 0
Cells(15, 2) = 0
Cells(16, 2) = 0
Cells(17, 2) = 0
Cells(18, 2) = 0

End Sub
```

The $(XX, YY)$ coordinate pair (Figure 4) represents the position of coin in the board. In this simulation one has the option to change number of trials (throws), size of square and radius of the circle inside the square. The value of *Constant* is evaluated by varying many parameters.

### 3.2 Results

Out of total number of trials, $N_t = N_s$ $(or\ N)$ for generation of $(XX, YY)$ $pairs$ of random numbers, the ones which satisfy the $\sqrt{XX^2 + YY^2} \leq f$ relation are said to be present within the circle (called *Hits* and represented by $N_c$). The ratio $\frac{N_c}{N_s \times f^2}$, gives the value of *Constant*. Every time program is run while keeping $N$ constant, different values of *Constant* are observed.


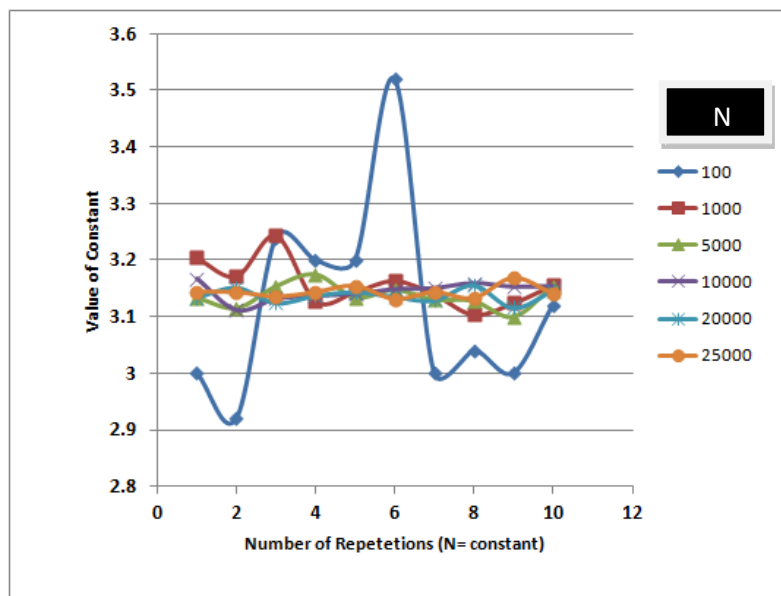
Figure 5: Number of Repetitions versus value of *Constant*.

In this simulation experiment, following three graphs are plotted:

i). Graph between number of repetition (for constant number of trials) and value of *Constant* (see Fig. 5)

ii). Graph between 'Number of Trials, *N* and value of *Constant* (see Fig. 6) and

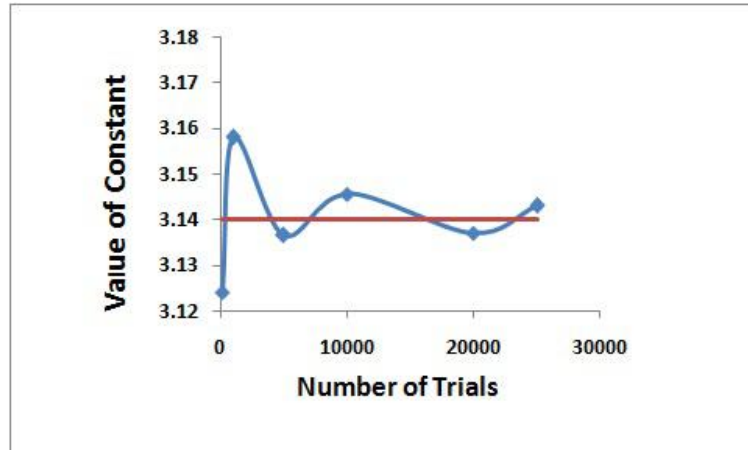iii). Graph between radius of Circle, $r$ and value of *Constant* (Fig. 7).



Figure 6: Number of Trials (N) versus value of *Constant*.

From the Fig. 5, it is observed that there are large variations in the value of *Constant* when the number of trials is less and variations decrease when the number of trials is large enough. However, as the number of trials increases, the average value of the *Constant* converges to $\frac{22}{7}$ (as in Figure 6). The observed estimate of the *Constant* is very close to the frequently encountered mathematical constant π. The Figure 7 reflects the change in the behavior of *Constant* with respect to the radius of circle drawn inside the square. This plot (Figure 7) shows that if the radius of the circle is increased, the statistical variations also decrease and the *Constant* tends to converge towards $\frac{22}{7}$ .
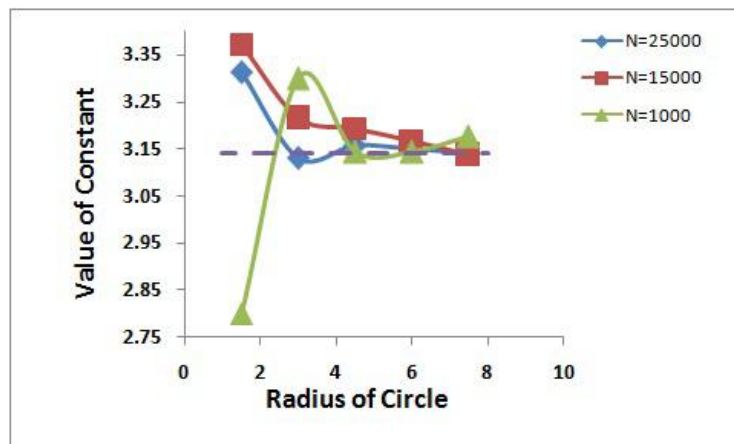


Figure 7: Radius of Circle ($r$) versus value of *Constant*.

## IV. Conclusions

In this article, a simple experiment to estimate the value of π is presented. The use of MS Excel along with Visual Basic for Applications (VBA) is demonstrated to estimate the value of π by giving visual dimension to the problem. This technique is used to teach the students in the classroom environment along with designed lab session. A little operational knowledge of computer is required from the teacher to use this application. Secondary physics experiment is significant instrument for effective and motivational physics education. Simple physics experiments have strong motivational effectiveness and can be used in several cognitive motivational teaching techniques. In this case the students and teachers have shown interest and appreciated the simple visual concept to estimate the constant, π.

## Acknowledgments

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