IOSR Journal of Applied Physics (IOSR-JAP)
e-ISSN: 2278-4861.Volume 8, Issue 1 Ver. II (Jan. - Feb. 2016), PP 35-43
www.iosrjournals

Form Factors in VVA Triangle Diagram Using Modified Pre-regularization Method

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Abstract: *We have evaluated the form factors in VVA triangle diagram for on-shell electrons of finite mass m using Modified Pre-regularization method. From the result the behavior of the form factors for different values of axial vector momentum has been discussed and found the magnetic moment and anomalous magnetic moment.*

Keywords: *Modified Pre-regularization method, Dimensional regularization method, Loop integrals, Renormalization, QED, Ward identity, Anomalies, Form factors.*

I. Introduction

We know renormalization of a theory is a necessary condition for an acceptable gauge theory. Similarly, to find the most accurate result of the inherent physical quantities such as electric charge, magnetic moment, etc are also equally important which can be found from the finite parts of the result, by writing it in the form of form factors. It is interesting to note that the contribution of quantum effects is the key point to get most accurate result that can be found from loop diagrams.

As we know the VVA triangle diagram give anomalous result for the two currents. That means if the vector currents are conserved then the axial vector current is anomalous and vice-versa. Hence, it is interesting to evaluate the form factors of these diagrams.

Long time ago Ward, Takahashi, Taylor, Slavnor [1]-[4] gave an identity to find the renormalization of a theory. That means using this identity one can check the conservation of currents in case of loop diagrams. As we know in most cases the loop diagrams are divergent, so one has to use some kind of regularization method to evaluate these. *Pre-regularization* method [5] is one of the best methods to evaluate the loop diagrams. Using Pre-regularization method Chowdhury [6] has shown that in the VVA triangle diagram there is an anomaly in axial current if the vector currents are conserved. Using this prescription Chowdhury et al [7]-[9] have also shown anomalies or ambiguities in the conservation of currents in many other loop diagrams. However, he did not worked out anything from the finite parts of the result. Recently, Chowdhury [10] modified their old prescription so as to incorporate and to find some other important features of the underling theory, which cannot be found through the existing prescription and it is called Modified Pre-regularization. Using this prescription we have evaluated the *form factors* [11] in QED which arises from the finite parts of the result. Since the VVA triangle diagram gives anomalous result for the conservation of currents so it is interesting to check whether the *form factors* for VVA triangle diagram can be evaluated using this new prescription. In this paper using the *Pre-regularization* and *Modified Pre-regularization* methods and following the procedure adopted in [11] we have first evaluated the Ward identities for the two currents viz; *Vector Ward identity* and *Axial Vector Ward Identity*, then fixing the vector identity we have found the result for axial vector identity which is very important. Using this result we have evaluated the form factors of these diagrams. A thorough discussion of the form factors for different values of axial vector momentum has been given in this paper. From these result the magnetic moment and anomalous magnetic moment have been found. Also an explanation is given why the charge form factor is absent in this case.

II. Ward Identities and VVA Triangle Diagram

The three point Feynman diagrams for two vectors and one axial vector current can be represented by the two Feynman diagrams (see, Figure 1). Now using Pre-regularization method [5] and following ref. [6] the total three point function is given by:

$$\Gamma^{T}_{\mu\nu\rho}(p_1, p_2) = \Gamma^{1}_{\mu\nu\rho}(p_1, p_2) + \Gamma^{2}_{\mu\rho\nu}(p_2, p_1) \tag{1}$$

where $\Gamma^{2}_{\mu\nu\rho}(p_1, p_2)$ and $\Gamma^{2}_{\mu\rho\nu}(p_2, p_1)$ are three point functions of diagrams 1(a) and 1(b) respectively.

For the conserved vector currents and axial-vector current the Ward identities to be satisfied are as follows

Vector current ward identities:

$$p_1^{\nu}\Gamma^{T}_{\mu\nu\rho}(p_1, p_2) = 0 \tag{2}$$

$$p_2^{\rho}\Gamma^{T}_{\mu\nu\rho}(p_1, p_2) = 0 \tag{3}$$

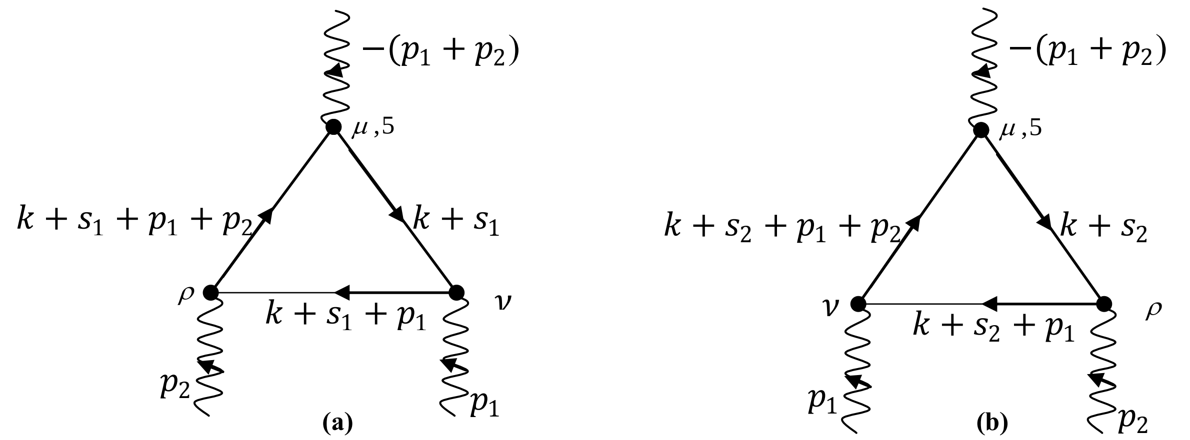

Figure 1: Three Point Diagrams

Axial-Vector current ward identity:

$$-(p_1+p_2)^{\mu}\Gamma^{T}_{\mu\nu\rho}(p_1,p_2)=0 \tag{4}$$

It is interesting to note that after calculation one can easily find that it would be almost impossible to satisfy the conditions (2) - (4) simultaneously. However, if we fixed the vector currents conserved, then the axial vector current is anomalous.

According to Pre-regularization prescription [5], we add arbitrary parameters s_1 and s_2 to diagrams in 1(a) and 1(b) respectively. These momentum ambiguities are used to ensure that the Ward identities of equations (2) - (4) are respected. With these parameters equation (1) yields

$$\Gamma^{T}_{\mu\nu\rho}(p_1,p_2)=\Gamma^{1}_{\mu\nu\rho}(p_1,p_2,s_1)+\Gamma^{2}_{\mu\rho\nu}(p_2,p_1,s_2) \tag{5}$$

where,

$$\Gamma^{1}_{\mu\nu\rho}(p_1,p_2,s_1)=-\int\frac{d^4k}{(2\pi)^4}tr\left[\gamma_\mu\gamma_5\frac{i}{\not{k}+\not{s}_1}(-ie\gamma_\nu)\frac{i}{\not{k}+\not{p}_1+\not{s}_1}(-ie\gamma_\rho)\frac{i}{\not{k}+\not{p}_1+\not{p}_2+\not{s}_1}\right] \tag{6}$$

$$\Gamma^{2}_{\mu\rho\nu}(p_2,p_1,s_2)=-\int\frac{d^4k}{(2\pi)^4}tr\left[\gamma_\mu\gamma_5\frac{i}{\not{k}+\not{s}_2}(-ie\gamma_\rho)\frac{i}{\not{k}+\not{p}_2+\not{s}_2}(-ie\gamma_\nu)\frac{i}{\not{k}+\not{p}_1+\not{p}_2+\not{s}_2}\right] \tag{7}$$

Here, these integrals lead to be linearly divergent, but computation shows that the sum of the two integrals is finite [6]. If the external momentum p_1 dotted with $\Gamma^{1}_{\mu\nu\rho}$ and $\Gamma^{2}_{\mu\rho\nu}$ at γ_ν vertex of equations (6) - (7) then (5) can be written as

$$p_1^{\nu}\Gamma^{T}_{\mu\nu\rho}(p_1,p_2)=-ie^2\int\frac{d^4k}{(2\pi)^4}tr\left[\gamma_\mu\gamma_5\frac{1}{\not{k}+\not{s}_1}\gamma_\rho\frac{1}{\not{k}+\not{s}_1+\not{p}_1+\not{p}_2}-\gamma_\mu\gamma_5\frac{1}{\not{k}+\not{s}_2}\gamma_\rho\frac{1}{\not{k}+\not{s}_1+\not{p}_1+\not{p}_2}\right]$$

$$-ie^2\int\frac{d^4k}{(2\pi)^4}tr\left[\gamma_\mu\gamma_5\frac{1}{\not{k}+\not{s}_2}\gamma_\rho\frac{1}{\not{k}+\not{s}_2+\not{p}_2}-\gamma_\mu\gamma_5\frac{1}{\not{k}+\not{s}_1+\not{p}_1}\gamma_\rho\frac{1}{\not{k}+\not{s}_1+\not{p}_1+\not{p}_2}\right] \tag{8}$$

We see that by power counting each term of (8) is quadratic divergent. Hence in this type of divergent integrals if we are allowed to shift the variable of integration naively then all integrals cancel out exactly. However, for divergent integrals the naive shifting is not allowed because of surface terms. The advantage of Pre-regularization method is that in this kind of situation when we shift the variable of integration then there is a procedure for keeping track of the appropriate surface terms. In this method it is seen that after calculation the divergent integrals cancels each other leaving with the finite surface terms. This finite part is now easy to evaluate with our prescription, where as in other methods it is not that much easy.

After performing all manipulations we get,

$$p_1^{\nu}\Gamma^{T}_{\mu\nu\rho}(p_1,p_2)=-\frac{ie^2}{8\pi^2}i\varepsilon_{\mu\rho\alpha\beta}\,p_1^{\alpha}p_2^{\beta}(B_1-B_2+1) \tag{9}$$

where we have used $s_i=A_ip_1+B_ip_2$.

Similarly, if we dotted p_2 with $\Gamma^1_{\mu\nu\rho}$ and $\Gamma^2_{\mu\rho\nu}$ at γ_ρ vertex and go through all the techniques, then we end up with

$$p_2^\rho \Gamma^T_{\mu\nu\rho}(p_1, p_2) = -\frac{ie^2}{8\pi^2} i\varepsilon_{\mu\nu\alpha\beta}\ p_1^\alpha p_2^\beta (A_1 - A_2 + 1) \tag{10}$$

For Axial vector current divergence, we have to dot $-(p_1 + p_2)$ with $\Gamma^T_{\mu\nu\rho}$ at γ_μ vertex and we get

$$-(p_1 + p_2)^\mu \Gamma^T_{\mu\nu\rho}(p_1, p_2) = -\frac{ie^2}{8\pi^2} i\varepsilon_{\nu\rho\alpha\beta}\ p_1^\alpha p_2^\beta (A_1 - A_2 - B_1 + B_2 + 2) \tag{11}$$

Now for conserved Vector and Axial vector Ward identities (9) - (11) must be zero. That is,

$$B_1 - B_2 + 1 = 0 \tag{12}$$

$$A_1 - A_2 + 1 = 0 \tag{13}$$

$$A_1 - A_2 - B_1 + B_2 + 2 = 0 \tag{14}$$

Here we can see it is not possible to find any values for A 's and B 's which satisfy both the equations simultaneously. That is, we cannot find any specific values of s_1 and s_2 to satisfy the Ward identities [6]. If we find values for A 's and B 's to satisfy vector Ward identities then axial vector Ward identity is anomalous and vice-versa. We will use this result in the calculation of form factors.

III. Evaluation of One-loop VVA Diagram with Modified Pre-regularization Method

The amplitude of the two diagrams has been given in equations (6) - (7). Let us take equation (6) we get

$$\Gamma^1_{\mu\nu\rho} = -ie^2 \int \frac{d^4k}{(2\pi)^4} tr[\gamma_\mu \gamma_5 \gamma_\alpha \gamma_\nu \gamma_\beta \gamma_\rho \gamma_\sigma] \frac{(k + s_1)^\alpha (k + p_1 + s_1)^\beta (k + p_1 + p_2 + s_1)^\sigma}{(k + s_1)^2 (k + p_1 + s_1)^2 (k + p_1 + p_2 + s_1)^2} \tag{15}$$

Combining the denominators using the Feynman identity and applying Pre-regularization prescription we obtain

$$\Gamma^1_{\mu\nu\rho} = 2ie^2 \int \frac{d^4k}{(2\pi)^4} tr[\gamma_5 \gamma_\mu \gamma_\alpha \gamma_\nu \gamma_\beta \gamma_\rho \gamma_\sigma] \int_0^1 dx \int_0^{1-x} dy \times$$

$$\frac{\{k - (p_1(x+y) + p_2 y\}^\alpha \{k + (p_1(1-x-y) - p_2 y\}^\beta \{k + (p_1(1-x-y) + p_2(1-y))\}^\sigma}{\left[k^2 - \{(p_1(x+y) + p_2 y\}^2 + p_1^2 x + (p_1 + p_2)^2 y\right]^3}$$

$$+ 2ie^2 tr[\gamma_5 \gamma_\mu \gamma_\alpha \gamma_\nu \gamma_\beta \gamma_\rho \gamma_\sigma] \frac{1}{(2\pi)^4} \int_0^1 dx \int_0^{1-x} dy \left(\frac{-i\pi^2}{12}\right) [\{s_1 + p_1(x+y) + p_2 y\}^\alpha g^{\beta\sigma}$$

$$+ \{s_1 + p_1(x+y) + p_2 y\}^\beta g^{\alpha\sigma} + \{s_1 + p_1(x+y) + p_2 y\}^\sigma g^{\alpha\beta}] \tag{16}$$

We have, $tr[\gamma_5 \gamma_\mu \gamma_\alpha \gamma_\nu \gamma_\beta \gamma_\rho \gamma_\sigma] = 4i(g_{\alpha\nu}\varepsilon_{\beta\rho\sigma\mu} - g_{\mu\nu}\varepsilon_{\beta\rho\sigma\alpha} + g_{\mu\alpha}\varepsilon_{\beta\rho\sigma\nu}$

$$- g_{\rho\sigma}\varepsilon_{\mu\alpha\nu\beta} + g_{\beta\sigma}\varepsilon_{\mu\alpha\nu\rho} - g_{\beta\rho}\varepsilon_{\mu\alpha\nu\sigma}) \tag{17}$$

Using (17) in (16) and for performing k integration taking the advantage of symmetric integration, we can write (16) as

$$\Gamma^1_{\mu\nu\rho} = -2e^2 C' \int \frac{d^4k}{(2\pi)^4} \int_0^1 dx \int_0^{1-x} dy \times$$

$$\frac{k^2 g^{\alpha\beta}(p_1(1-x-y) + p_2(1-y))^\sigma - k^2 g^{\beta\sigma}(p_1(x+y) + p_2 y)^\alpha + k^2 g^{\alpha\sigma}(p_1(1-x-y) - p_2 y)^\beta}{[k^2 - \{p_1(x+y) + p_2 y\}^2 + p_1^2 x + (p_1 + p_2)^2 y]^3}$$

$$+ 8e^2 C' \int \frac{d^4k}{(2\pi)^4} \int_0^1 dx \int_0^{1-x} dy \frac{(p_1(x+y) + p_2 y)^\alpha (p_1(1-x-y) - p_2 y)^\beta (p_1(1-x-y) - p_2(1-y))^\sigma}{[k^2 - \{p_1(x+y) + p_2 y\}^2 + p_1^2 x + (p_1 + p_2)^2 y]^3}$$

$$+ \frac{ie^2}{24\pi^2} C' \int_0^1 dx \int_0^{1-x} dy \left(\frac{-i\pi^2}{12}\right) [\{s_1 + p_1(x+y) + p_2 y\}^\alpha g^{\beta\sigma}$$

$$+\{s_1 + p_1(x+y) + p_2 y\}^{\beta} g^{\alpha\sigma} + \{s_1 + p_1(x+y) + p_2 y\}^{\sigma} g^{\alpha\beta}] \qquad (18)$$

where, $C' = g_{\alpha\nu}\varepsilon_{\beta\rho\sigma\mu} - g_{\mu\nu}\varepsilon_{\beta\rho\sigma\alpha} + g_{\mu\alpha}\varepsilon_{\beta\rho\sigma\nu} - g_{\rho\sigma}\varepsilon_{\mu\alpha\nu\beta} + g_{\beta\sigma}\varepsilon_{\mu\alpha\nu\rho} - g_{\beta\rho}\varepsilon_{\mu\alpha\nu\sigma}$ (19)

Performing the integration over k and integrating over x and y, it yields,

$$\Gamma^1_{\mu\nu\rho} = -\frac{ie^2}{48\pi^2} C'\Gamma(0)[g^{\alpha\beta}(p_1^{\sigma} + 2p_2^{\sigma}) - g^{\beta\sigma}(2p_1^{\alpha} + p_2^{\alpha}) + g^{\alpha\sigma}(p_1^{\beta} - p_2^{\beta})]$$
$$+\frac{e^2}{4\pi^2} C'\int_0^1 dx\int_0^{1-x} dy \frac{(p_1(x+y)+p_2 y)^{\alpha}(p_1(1-x-y)-p_2 y)^{\beta}(p_1(1-x-y)+p_2(1-y))^{\sigma}}{\{p_1(x+y)+p_2 y\}^2 - p_1^2 x - (p_1+p_2)^2 y}$$
$$+\frac{ie^2}{144\pi^2} C'[(3s_1^{\alpha} + 2p_1^{\alpha} + p_2^{\alpha})g^{\beta\sigma} + (3s_1^{\beta} + 2p_1^{\beta} + p_2^{\beta})g^{\alpha\sigma} + (3s_1^{\sigma} + 2p_1^{\sigma} + p_2^{\sigma})g^{\alpha\beta}] \qquad (20)$$

Using (19) only in the divergent part of (20) and simplifying we get,

$$\Gamma^1_{\mu\nu\rho} = -\frac{ie^2}{24\pi^2}\Gamma(0)[\varepsilon_{\mu\sigma\nu\rho}(p_1^{\sigma} + 2p_2^{\sigma}) - \varepsilon_{\mu\alpha\nu\rho}(2p_1^{\alpha} + p_2^{\alpha}) + \varepsilon_{\mu\beta\nu\rho}(p_1^{\beta} - p_2^{\beta})]$$
$$+\frac{e^2}{4\pi^2} C'\int_0^1 dx\int_0^{1-x} dy \frac{(p_1(x+y)+p_2 y)^{\alpha}(p_1(1-x-y)-p_2 y)^{\beta}(p_1(1-x-y)+p_2(1-y))^{\sigma}}{\{p_1(x+y)+p_2 y\}^2 - p_1^2 x - (p_1+p_2)^2 y}$$
$$+\frac{ie^2}{72\pi^2}[(3s_1^{\alpha} + 2p_1^{\alpha} + p_2^{\alpha})\varepsilon_{\mu\alpha\nu\rho} + (3s_1^{\beta} + 2p_1^{\beta} + p_2^{\beta})\varepsilon_{\mu\beta\nu\rho} + (3s_1^{\sigma} + 2p_1^{\sigma} + p_2^{\sigma})\varepsilon_{\mu\sigma\nu\rho}] \qquad (21)$$

Similarly, for diagram 1(b), we get

$$\Gamma^2_{\mu\rho\nu} = \frac{ie^2}{24\pi^2}\Gamma(0)[\varepsilon_{\mu\sigma\nu\rho}(p_2^{\sigma} + 2p_1^{\sigma}) - \varepsilon_{\mu\alpha\nu\rho}(2p_2^{\alpha} + p_1^{\alpha}) + \varepsilon_{\mu\beta\nu\rho}(p_2^{\beta} - p_1^{\beta})]$$
$$+\frac{e^2}{4\pi^2} C''\int_0^1 dx\int_0^{1-x} dy \frac{(p_2(x+y)+p_1 y)^{\alpha}(p_2(1-x-y)-p_1 y)^{\beta}(p_2(1-x-y)+p_1(1-y))^{\sigma}}{\{p_2(x+y)+p_1 y\}^2 - p_2^2 x - (p_1+p_2)^2 y}$$
$$-\frac{ie^2}{72\pi^2}[(3s_2^{\alpha} + 2p_2^{\alpha} + p_1^{\alpha})\varepsilon_{\mu\alpha\nu\rho} + (3s_2^{\beta} + 2p_2^{\beta} + p_1^{\beta})\varepsilon_{\mu\beta\nu\rho} + (3s_2^{\sigma} + 2p_2^{\sigma} + p_1^{\sigma})\varepsilon_{\mu\sigma\nu\rho}] \qquad (22)$$

where, $C'' = g_{\alpha\rho}\varepsilon_{\beta\nu\sigma\mu} - g_{\mu\rho}\varepsilon_{\beta\nu\sigma\alpha} + g_{\mu\alpha}\varepsilon_{\beta\nu\sigma\rho} - g_{\nu\sigma}\varepsilon_{\mu\alpha\rho\beta} + g_{\beta\sigma}\varepsilon_{\mu\alpha\rho\nu} - g_{\beta\nu}\varepsilon_{\mu\alpha\rho\sigma}$ (23)

Adding equations (21) and (22) and using Modified Pre-regularization method [10] we get

$$\Gamma^1_{\mu\nu\rho} + \Gamma^2_{\mu\rho\nu} = \frac{ie^2}{24\pi^2}\Gamma(\frac{\varepsilon}{2})[(p_1^{\sigma} - p_2^{\sigma})\varepsilon_{\mu\sigma\nu\rho} + (p_1^{\alpha} - p_2^{\alpha})\varepsilon_{\mu\alpha\nu\rho} - 2(p_1^{\beta} - p_2^{\beta})\varepsilon_{\mu\beta\nu\rho}]$$
$$+\frac{e^2}{4\pi^2} C'\int_0^1 dx\int_0^{1-x} dy \frac{(p_1(x+y)+p_2 y)^{\alpha}(p_1(1-x-y)-p_2 y)^{\beta}(p_1(1-x-y)+p_2(1-y))^{\sigma}}{\{p_1(x+y)+p_2 y\}^2 - p_1^2 x - (p_1+p_2)^2 y}$$
$$+\frac{e^2}{4\pi^2} C''\int_0^1 dx\int_0^{1-x} dy \frac{(p_2(x+y)+p_1 y)^{\alpha}(p_2(1-x-y)-p_1 y)^{\beta}(p_2(1-x-y)+p_1(1-y))^{\sigma}}{\{p_2(x+y)+p_1 y\}^2 - p_2^2 x - (p_1+p_2)^2 y}$$
$$+\frac{ie^2}{72\pi^2}[(3s_1^{\alpha} - 3s_2^{\alpha} + p_1^{\alpha} - p_2^{\alpha})\varepsilon_{\mu\alpha\nu\rho} + (3s_1^{\beta}$$
$$- 3s_2^{\beta} + p_1^{\beta} - p_2^{\beta})\varepsilon_{\mu\beta\nu\rho} + (3s_1^{\sigma} - 3s_2^{\sigma} + p_1^{\sigma} - p_2^{\sigma})\varepsilon_{\mu\sigma\nu\rho}] \qquad (24)$$

Now to compute the Feynman parameter integrals in equation (24) let us define the following two vectors [12]:

$Q^{\mu} = -(p_1 + p_2)^{\mu}$ and $\Delta^{\mu} = (p_1 - p_2)^{\mu}$ (25)

On mass-Shell: $p_1^2 = p_2^2 = -m^2$ (26)

Then the following expressions can be written as

$$\{p_1(x+y)+p_2 y\}^2 - p_1^2 x - (p_1+p_2)^2 y = m^2 x(1-x) - Q^2 y(1-x-y) \tag{27}$$

$$\begin{aligned}
&(p_1(x+y)+p_2 y)^\alpha (p_1(1-x-y)-p_2 y)^\beta (p_1(1-x-y)+p_2(1-y))^\sigma \\
&= \frac{1}{4}[Q^2(y^2-y^3) - m^2(xy - x^2 y)] g^{\beta\sigma} Q^\alpha + \frac{1}{4} m^2 x^2 y g^{\sigma\alpha} Q^\beta \\
&\quad + \frac{1}{4} m^2 (x - x^2 - xy + x^2 y) g^{\alpha\beta} Q^\sigma - \frac{1}{4}[Q^2(xy - xy^2) \\
&- m^2(x^2 - x^3)] g^{\beta\sigma} p_1^\alpha + \frac{1}{4} Q^2 (y - xy - y^2 + xy^2)] g^{\sigma\alpha} p_1^\beta + \frac{1}{4} Q^2 xy^2 g^{\alpha\beta} p_1^\sigma
\end{aligned} \tag{28}$$

$$\begin{aligned}
&(p_2(x+y)+p_1 y)^\alpha (p_2(1-x-y)-p_1 y)^\beta (p_2(1-x-y)+p_1(1-y))^\sigma \\
&\quad = \frac{1}{4}[Q^2(y^2-y^3) - m^2(xy - x^2 y)] g^{\beta\sigma} Q^\alpha + \frac{1}{4} m^2 x^2 y g^{\sigma\alpha} Q^\beta + \frac{1}{4} m^2 (x - x^2 - xy \\
&\qquad + x^2 y) g^{\alpha\beta} Q^\sigma - \frac{1}{4}[Q^2(xy - xy^2) - m^2(x^2 - x^3)] g^{\beta\sigma} p_2^\alpha \\
&\qquad + \frac{1}{4} Q^2 (y - xy - y^2 + xy^2)] g^{\sigma\alpha} p_2^\beta + \frac{1}{4} Q^2 xy^2 g^{\alpha\beta} p_2^\sigma
\end{aligned} \tag{29}$$

Using the equations (27) – (29) and some related algebra, we can write the Feynman parameter integral part in (24) as follows

$$\begin{aligned}
&\frac{e^2}{4\pi^2} C' \int_0^1 dx \int_0^{1-x} dy \, \frac{(p_1(x+y)+p_2 y)^\alpha (p_1(1-x-y)-p_2 y)^\beta (p_1(1-x-y)+p_2(1-y))^\sigma}{\{p_1(x+y)+p_2 y\}^2 - p_1^2 x - (p_1+p_2)^2 y} \\
&+ \frac{e^2}{4\pi^2} C'' \int_0^1 dx \int_0^{1-x} dy \, \frac{(p_2(x+y)+p_1 y)^\alpha (p_2(1-x-y)-p_1 y)^\beta (p_2(1-x-y)+p_1(1-y))^\sigma}{\{p_2(x+y)+p_1 y\}^2 - p_2^2 x - (p_1+p_2)^2 y} \\
&\quad = \frac{e^2}{16\pi^2} \int_0^1 dx \int_0^{1-x} dy \left[\frac{Q^2(y^2-y^3) - m^2(xy - x^2 y)}{m^2 x(1-x) - Q^2 y(1-x-y)} (C' + C'') g^{\beta\sigma} Q^\alpha \right. \\
&\qquad + \frac{m^2 x^2 y}{m^2 x(1-x) - Q^2 y(1-x-y)} (C' + C'') g^{\sigma\alpha} Q^\beta + \frac{m^2 (x - x^2 - xy + x^2 y)}{m^2 x(1-x) - Q^2 y(1-x-y)} \\
&\qquad \left. (C' + C'') g^{\alpha\beta} Q^\sigma \right] + \frac{e^2}{8\pi^2} \int_0^1 dx \int_0^{1-x} dy \left[\frac{-\{Q^2(xy - xy^2) - m^2(x^2 - x^3)\}}{m^2 x(1-x) - Q^2 y(1-x-y)} \varepsilon_{\mu\alpha\nu\rho} \Delta^\alpha \right. \\
&\quad \left. + \frac{Q^2(y - y^2 - xy + xy^2)}{m^2 x(1-x) - Q^2 y(1-x-y)} \varepsilon_{\mu\beta\nu\rho} \Delta^\beta + \frac{Q^2 xy^2}{m^2 x(1-x) - Q^2 y(1-x-y)} \varepsilon_{\mu\sigma\nu\rho} \Delta^\sigma \right]
\end{aligned} \tag{30}$$

The relations for Ward identities that found in [6] have been written in equations (12) - (14). In [6] it is clearly shown that if the vector Ward identity is satisfied then the axial vector Ward identity is anomalous. Let us consider here that the vector Ward identity is satisfied then this implies that we can take

$$s_1 - s_2 = -(p_1 + p_2) = Q \tag{31}$$

where we have used the result in (12) – (13).

Substituting relations (25) - (30) in (24), we get

$$\begin{aligned}
\Gamma^1_{\mu\nu\rho} + \Gamma^2_{\mu\rho\nu} &= F_1^{(1)}(Q^2) \varepsilon_{\mu\alpha\nu\rho} \Delta^\alpha + F_1^{(2)}(Q^2) \varepsilon_{\mu\beta\nu\rho} \Delta^\beta + F_1^{(3)}(Q^2) \varepsilon_{\mu\sigma\nu\rho} \Delta^\sigma \\
&+ \left\{ F_2^{(1)}(Q^2)(C' + C'') g^{\beta\sigma} + \frac{ie^2}{24\pi^2} \varepsilon_{\mu\alpha\nu\rho} \right\} Q^\alpha + \left\{ F_2^{(2)}(Q^2)(C' + C'') g^{\sigma\alpha} \right. \\
&\left. + \frac{ie^2}{24\pi^2} \varepsilon_{\mu\beta\nu\rho} \right\} Q^\beta + \left\{ F_2^{(3)}(Q^2)(C' + C'') g^{\alpha\beta} + \frac{ie^2}{24\pi^2} \varepsilon_{\mu\sigma\nu\rho} \right\} Q^\sigma
\end{aligned} \tag{32}$$

where,

$$F_1^{(1)}(Q^2) = \frac{ie^2}{24\pi^2}\left\{\Gamma(\frac{\varepsilon}{2}) + \frac{1}{3}\right\} + \frac{e^2}{8\pi^2}\int_0^1 dx \int_0^{1-x} dy \; \frac{-\{Q^2(xy - xy^2) - m^2(x^2 - x^3)\}}{m^2 x(1-x) - Q^2 y(1-x-y)} \tag{33}$$

$$F_1^{(2)}(Q^2) = -\frac{ie^2}{24\pi^2}\left\{2\Gamma(\frac{\varepsilon}{2}) - \frac{1}{3}\right\} + \frac{e^2}{8\pi^2}\int_0^1 dx \int_0^{1-x} dy \; \frac{Q^2(y - y^2 - xy + xy^2)}{m^2 x(1-x) - Q^2 y(1-x-y)} \tag{34}$$

$$F_1^{(3)}(Q^2) = \frac{ie^2}{24\pi^2}\left\{\Gamma(\frac{\varepsilon}{2}) + \frac{1}{3}\right\} + \frac{e^2}{8\pi^2}\int_0^1 dx \int_0^{1-x} dy \; \frac{Q^2 xy^2}{m^2 x(1-x) - Q^2 y(1-x-y)} \tag{35}$$

$$F_2^{(1)}(Q^2) = \frac{e^2}{16\pi^2}\int_0^1 dx \int_0^{1-x} dy \; \frac{Q^2(y^2 - y^3) - m^2(xy - x^2 y)}{m^2 x(1-x) - Q^2 y(1-x-y)} \tag{36}$$

$$F_2^{(2)}(Q^2) = \frac{e^2}{16\pi^2}\int_0^1 dx \int_0^{1-x} dy \; \frac{m^2 x^2 y}{m^2 x(1-x) - Q^2 y(1-x-y)} \tag{37}$$

$$F_2^{(3)}(Q^2) = \frac{e^2}{16\pi^2}\int_0^1 dx \int_0^{1-x} dy \; \frac{m^2(x - x^2 - xy + x^2 y)}{m^2 x(1-x) - Q^2 y(1-x-y)} \tag{38}$$

Now we perform the Feynman parameter integrals y in equations (33) - (38) to yield

$$F_1^{(1)}(Q^2) = \frac{ie^2}{24\pi^2}\{\Gamma(\frac{\varepsilon}{2}) + \frac{1}{3}\} + \frac{e^2}{8\pi^2}\left[\frac{1}{6} - 2\int_0^1 dx \; x^2 \frac{1-x}{4t}\log\frac{1 - \frac{2t}{1-x}}{1 + \frac{2t}{1-x}}\right] \tag{39}$$

$$F_1^{(2)}(Q^2) = -\frac{ie^2}{24\pi^2}\left\{2\Gamma(\frac{\varepsilon}{2}) - \frac{1}{3}\right\} + \frac{e^2}{8\pi^2}\left[-\frac{1}{3} + 2\left(1 + \frac{2m^2}{Q^2}\right)\int_0^1 dx\,(x - x^2)\frac{1-x}{4t}\log\frac{1 - \frac{2t}{1-x}}{1 + \frac{2t}{1-x}}\right] \tag{40}$$

$$F_1^{(3)}(Q^2) = \frac{ie^2}{24\pi^2}\left\{\Gamma(\frac{\varepsilon}{2}) + \frac{1}{3}\right\} + \frac{e^2}{8\pi^2}\left[\frac{1}{6} + 2\int_0^1 dx \left\{x - x^2\left(1 + \frac{2m^2}{Q^2}\right)\right\}\frac{1-x}{4t}\log\frac{1 - \frac{2t}{1-x}}{1 + \frac{2t}{1-x}}\right] \tag{41}$$

$$F_2^{(1)}(Q^2) = \frac{e^2}{8\pi^2}\int_0^1 dx \left\{x - x^2\left(1 + \frac{2m^2}{Q^2}\right)\right\}\frac{1-x}{4t}\log\frac{1 - \frac{2t}{1-x}}{1 + \frac{2t}{1-x}} \tag{42}$$

$$F_2^{(2)}(Q^2) = \frac{e^2}{8\pi^2}\frac{m^2}{Q^2}\int_0^1 dx \; x^2 \; \frac{1-x}{4t}\log\frac{1 - \frac{2t}{1-x}}{1 + \frac{2t}{1-x}} \tag{43}$$

$$F_2^{(3)}(Q^2) = \frac{e^2}{8\pi^2}\frac{m^2}{Q^2}\int_0^1 dx \; (x + x^2) \; \frac{1-x}{4t}\log\frac{1 - \frac{2t}{1-x}}{1 + \frac{2t}{1-x}} \tag{44}$$

If we can express the following term as

$$\frac{1-x}{4t}\log\frac{1-\frac{2t}{1-x}}{1+\frac{2t}{1-x}} = -\left\{a_0 - a_1\frac{4m^2}{Q^2}(x+x^2) + a_2\left(\frac{4m^2}{Q^2}\right)^2(x^2+2x^3)\right.$$

$$\left. - a_3\left(\frac{4m^2}{Q^2}\right)^3(x^3+3x^4) + ---+(-1)^n a_n\left(\frac{4m^2}{Q^2}\right)^n(x^n+nx^{n+1}) + ---\infty \right. \quad (45)$$

where, $a_0 = 1 + \frac{1}{3} + \frac{1}{5} + ----\infty$ (46)

$$a_1 = \frac{{}^1c_1}{3} + \frac{{}^2c_1}{5} + \frac{{}^3c_1}{7} ---\infty \quad (47)$$

$$a_2 = \frac{{}^2c_2}{5} + \frac{{}^3c_2}{7} + \frac{{}^4c_2}{9} ---\infty \quad (48)$$

$$a_3 = \frac{{}^3c_3}{7} + \frac{{}^4c_3}{9} + \frac{{}^5c_3}{11} ---\infty \quad (49)$$

....

$$a_n = \frac{{}^nc_n}{2n+1} + \frac{{}^{n+1}c_n}{2n+3} + \frac{{}^{n+2}c_n}{2n+5} ---\infty \quad (50)$$

Again, performing the Feynman parameter integrals of x in equations (39) - (44) and putting $\alpha = \frac{e^2}{4\pi}$ to yield

$$F_1^{(1)}(Q^2) = \frac{i\alpha}{6\pi}\left\{\Gamma(\frac{\varepsilon}{2}) + \frac{1}{3}\right\} + \frac{\alpha}{12\pi} + \frac{\alpha}{\pi}(C_1 - 2C_2 + C_3) \quad (51)$$

$$F_1^{(2)}(Q^2) = -\frac{i\alpha}{6\pi}\left\{2\Gamma(\frac{\varepsilon}{2}) - \frac{1}{3}\right\} + \frac{\alpha}{12\pi} + \frac{\alpha}{\pi}(-C_2 + 2C_3)\left(1 + \frac{2m^2}{Q^2}\right) \quad (52)$$

$$F_1^{(3)}(Q^2) = \frac{i\alpha}{6\pi}\left\{\Gamma(\frac{\varepsilon}{2}) + \frac{1}{3}\right\} + \frac{\alpha}{12\pi} + \frac{\alpha}{\pi}\left\{(-C_2 + 2C_3) + \frac{2m^2}{Q^2}(C_1 - 2C_2 + 2C_3)\right\} \quad (53)$$

$$F_2^{(1)}(Q^2) = \frac{\alpha}{2\pi}\left\{(-C_2 + 2C_3) + \frac{2m^2}{Q^2}(C_1 - 2C_2 + 2C_3)\right\} \quad (54)$$

$$F_2^{(2)}(Q^2) = \frac{\alpha}{2\pi}\frac{m^2}{Q^2}(-C_1 + 2C_2 - C_3) \quad (55)$$

$$F_2^{(3)}(Q^2) = \frac{\alpha}{2\pi}\frac{m^2}{Q^2}(-2C_1 + 3C_2 - 2C_3) \quad (56)$$

where, $C_1 = \sum_{n=0}^{\infty}(-1)^n a_n\left(\frac{4m^2}{Q^2}\right)^n\frac{(n+2)+n(n+1)}{(n+1)(n+2)}$ (57)

$$C_2 = \sum_{n=0}^{\infty}(-1)^n a_n\left(\frac{4m^2}{Q^2}\right)^n\frac{(n+3)+n(n+1)}{(n+1)(n+2)(n+3)} \quad (58)$$

$$C_3 = \sum_{n=0}^{\infty}(-1)^n a_n\left(\frac{4m^2}{Q^2}\right)^n\frac{(n+4)+n(n+1)}{(n+1)(n+2)(n+3)(n+4)} \quad (59)$$

IV. Behavior of Form Factors for Different Values of Q^2

Behavior of form factors for $Q^2 \to \infty$:

If we put $Q^2 \to \infty$ in equations (57) - (59), we get

$C_1 = a_0$, $C_2 = \frac{1}{2} a_0$ and $C_3 = \frac{1}{6} a_0$

Then the equations (51) - (56) become

$$F_1^{(1)}(Q^2) = \frac{i\alpha}{6\pi}\left\{\Gamma(\frac{\varepsilon}{2}) + \frac{1}{3}\right\} + \frac{\alpha}{12\pi^2} + \frac{\alpha}{6\pi} a_0 \tag{60}$$

$$F_1^{(2)}(Q^2) = -\frac{i\alpha}{6\pi}\left\{2\Gamma(\frac{\varepsilon}{2}) - \frac{1}{3}\right\} - \frac{\alpha}{12\pi} - \frac{\alpha}{6\pi} a_0 \tag{61}$$

$$F_1^{(3)}(Q^2) = \frac{i\alpha}{6\pi}\left\{\Gamma(\frac{\varepsilon}{2}) + \frac{1}{3}\right\} + \frac{\alpha}{12\pi} - \frac{\alpha}{6\pi} a_0 \tag{62}$$

and

$$F_2^{(1)}(Q^2) = \frac{\alpha}{12\pi}, \quad F_2^{(2)}(Q^2) = 0, \quad F_2^{(3)}(Q^2) = 0 \tag{63}$$

Behavior of form factors for $Q^2 \to 0$:

This behavior can not be found by putting $Q^2 \to 0$ in equations (51) - (56) as before. But this can be done by putting $Q^2 \to 0$ in equations (33) - (38) and after performing the Feynman parameters integrals, we get

$$F_1^{(1)}(Q^2) = \frac{i\alpha}{6\pi}\left\{\Gamma(\frac{\varepsilon}{2}) + \frac{1}{3}\right\} + \frac{\alpha}{12\pi} \tag{64}$$

$$F_1^{(2)}(Q^2) = -\frac{i\alpha}{6\pi}\left\{2\Gamma(\frac{\varepsilon}{2}) - \frac{1}{3}\right\} \tag{65}$$

$$F_1^{(3)}(Q^2) = \frac{i\alpha}{6\pi}\left\{\Gamma(\frac{\varepsilon}{2}) + \frac{1}{3}\right\} \tag{66}$$

and

$$F_2^{(1)}(Q^2) = -\frac{\alpha}{24\pi}, \quad F_2^{(2)}(Q^2) = \frac{\alpha}{48\pi}, \quad F_2^{(3)}(Q^2) = \frac{\alpha}{12\pi} \tag{67}$$

V. Discussion over the Result

The results obtained for VVA triangle diagram are displayed in equation (51) - (53) for $F_1^{(i)}(Q^2)$ and (54) - (56) for $F_2^{(i)}(Q^2)$. At this stage we are now able to calculate the form factors for different values of Q^2. Usually, the coefficient of $(p_1 - p_2)$ that is, $F_1^{(i)}(Q^2)$ is known as the Magnetic Form Factor [12] – [13] and the coefficient of $-(p_1 + p_2)$ is known as its Anomalous Magnetic Moment [12] – [13]. However, the first term of anomalous magnetic moment will vanish if we take the product of $(C' + C'')F_2^{(i)}(Q^2)$. Here the charge form factor is absent because there is no term with coefficient of γ^μ. In all these calculations the series for a 's that we have found and given in (46) - (50) are very important. Using these values we can find the most accurate values for magnetic form factors.

We have seen that in both cases, that is $Q^2 \to \infty$ and $Q^2 \to 0$, there are no divergent part in $F_2^{(i)}(Q^2)$. The divergent part lies only in $F_1^{(i)}(Q^2)$. The finite parts of $F_1^{(i)}(Q^2)$ and $F_2^{(i)}(Q^2)$ will give us magnetic form factors and its anomalous magnetic moment. Here we can see that the result is proportional to fine structure constant α. It should be noted here that we have already shown in section- 2 that the vector current is conserved while the axial vector current is not and now we have found the anomalous magnetic moment which comes from the axial vector current Ward identity. This is a quite new result that we have found using Modified Pre-regularization method.

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