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The use of the invariant Laplace wave equation on galaxies and planetary systems

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Abstract: *In this very paper a summary of several previous papers is shown – and an extended version of a new universal formula of the rotation velocity distribution of galaxies and also a new formula of the energy density distribution of galaxies are presented. The newest version of this formula is based on a relativistic invariant La Place wave equation where it has been possible to obtain expressions for the rotation velocity - and energy density distributions versus distance to the galactic centre. Several mathematical proofs of these new formulas are also given. These formulas are divided into a Keplerian (general relativity)-and a relativistic (special relativity) part. It has also been possible to derive these formulas for galaxies by using geodetic line expressions. The relativistic invariant La Place wave equation has also been shown to be applicable to planetary systems too and it has been still developed in a new and very dynamic version with several advantages. According to the rotation velocity distribution of the galaxies the rotation velocity increases very rapidly from the centre and reach a plateau which is constant out to large distance from the centre. This is in accordance with observations and is also in accordance with the main structure of rotation velocity versus distance from different galaxy measurements. It is also possible to determine the mass, radius and maximum rotation velocity of the galaxy from these rotation velocity-and energy density calculations. Newer and more advanced computer simulation programs have also been developed and used to establish and verify the rotation velocity and energy density distributions in the galaxy system, according to this paper. These computer simulations are in accordance with observations in two and three dimensions. It has also been possible to study the matching percentage in these calculations showing a very high matching percentage between theoretical and observational values with this new formula.*
Keywords: *Astrophysics, Spiral galaxies, Theory of Relativity and Universal Formula.*

I. Introduction

A. It is common in established physics that electron circular movement around atoms and planetary circular movement around the sun follow the usual Keplerian relationship V α (1 / √ R), where V is the rotation speed and R is the distance to the nucleus of the atom or the centre of the sun respectively.
The mass of a spiral galaxy can be determined from the dependence of its rotational velocity as a function of the distance from the center of the galaxy. Such a rotational curve has been determined from gas and stars in the distant parts of our galaxy, far beyond our distance to the centre. Unexpectedly, it does not follow the Keplerian decrease in which the circular
rotation velocity V decreases $\alpha\ R^{-1/2}$ where R is the distance to the center. According to the
the 3:rd law of Kepler the mass of a galaxy can be expressed as :

$$\mathbf{M = V^2 R / G} \quad \mathbf{(1)}$$

and the rotation velocity as :

$$\mathbf{V = (G M / R)^{1/2}} \quad \mathbf{(2)}$$

where G is the gravitation constant.

By using these formulas it is possible to determine the mass and rotation speed at a certain distance of the galaxy. According to these equations both mass and rotation velocity will
decrease with increasing distance, which is established today.

In the 1970s and 1980s radio astronomers discovered that the spiral rotation velocity remains constant with increasing radius Freeman (1) ,Rubin and Ford (2). They studied neutral hydrogen clouds at 21-cm radio wavelength and in the optical wavelength in spiral galaxies and found non-Keplerian rotation curves. These facts were not in accordance to the established views and came as a shock to the establishment.
This is illustrated in Combes et al.(3) (Figs 3.1-3.3) , where the velocities of many spiral galaxies increase the velocity very rapidly at small distances up to a constant plateau at larger distances from the galaxy centre. Astronomers discovered that many galaxies rotated at very high velocities.

To explain this most astronomers believe that this is caused by introducing dark matter in the Keplarian equations above and to keep the galaxies together. They believe that most matter in a galaxy consists of dark matter and only a minor part consists of ordinary matter which emits light.

This paper shows another aspect about this problem by presenting a summary of results from different papers about a new rotation velocity formula for galaxies. An important formula in these papers is the relativistic invariant La Place wave equation for evaluating the new rotation velocity formula. This relativistic invariant La Place wave equation has also shown to be useful for planetary orbit equations too.

B. Keplarian relationships of planetary systems

Planets follow a similar equation to equation 2 following the 3:rd law of Kepler. The exact formula between the planet rotation speed V and the masses m for the planet and M for the sun is

$$V = (G (m + M) / r)^{1/2} \qquad (3)$$

for circular motion according to Lang (4) p.542.

II. Galaxies. The Use Of Schwarzschild Metric

In Barrera and Thelin (5) we have presented a new formula about the formation of galaxies. It is based on the relativistic Schwarzschild/ Minkowski metric, Schwarzschild (6),Einstein (7), where it has been possible to obtain a formula for the rotation velocity and also a density distribution versus distance to the galactic centre. Similar rotation velocity profiles to our new formula have also been observed from data published in established books in this field. These profiles are in accordance to observations, as is seen in equations (1-10) in Barrera and Thelin (5). Computer simulations of equations 19 and 22 of Barrera and Thelin (5) were also performed to establish and verify the velocity and density distributions suggested in that paper.

According to Lang (8) p.146 a spherically symmetric gravitational field outside a massive non-rotating body in vacuum, can follow the Schwarzschild expression, where the line element ds becomes :

$$ds^2 = (1 - 2GM / c^2 r)c^2 dt^2 - (1 - 2GM /c^2 r)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta \, d \psi^2 \qquad (4)$$

Here r, θ and ψ are spherical coordinates whose origin is at the centre of the massive object with the mass M , G is here the gravitational constant and r is the distance. These things have earlier been studied and published by Barrera and Thelin (5) in detail for the rotation velocity studies in galaxies, where a new formula has been presented. A summary of these studies is presented here.

By using a polar coordinate system with $d\theta = 0$ the following expression from equation (4) is obtained :

$$ds^2 = - \gamma^{-1} dr^2 - r^2 d\theta^2 + \gamma dt^2 \qquad (5)$$

where $\gamma = (1 - 2M/r)$, which is the Schwarzschild term Eddington (9)(p.82-85)

From this formula it is possible to obtain an expression for the angular rotation speed of a galaxy :

$$d\theta = (1 / r) ((1 - 2M / r) dt^2 - (1 - 2M / r)^{-1} dr^2)^{1/2} \qquad (6)$$

The formula of the rotation speed (km/s) is obtained from the expression

$$v = r (d\theta/ dt) \qquad (7)$$

This means that the rotation speed in km/s will be the following expression :

$$v = k ((1 - 2M/r))^{1/2} \qquad (8)$$

III. The New Universal Formula

According to this rotation velocity formula, the rotation velocity increases very rapidly from the center and reach a plateau which is constant out to big distance from the center. This has also been observed in many papers, Sofue and Rubin.(10) and Combes et al.(3).

In paper by Barrera and Thelin (5) an improvement is made of the rotation formula for galaxies in equation(19) of Barrera and Thelin (5) and is seen in equation (33) in paper (11) and equations (14,20,22,24,52 and 59) in this summary paper. The approximate formula from Barrera and Thelin (5) is also seen in equation (15) of paper(11). The new improved rotation formula in equation 33 is divided into one Keplerian (general relativity) part and one relativistic (special relativity) part, which also makes it possible to use this formula for atoms, planets and galaxies. A mathematical proof of this new universal formula is also given. Computer simulations in 2 and 3 dimensions with this new formula in paper (11) are also achieved, giving it a strong support of the appearance of atoms, planets and galaxies. In Table 1 in paper (11), galaxy parameters and matching between theoretical and observational data are also studied for a number of galaxies.

In paper (11) an exact solution of complete relativity for polar coordinates is presented where Minkowsky/Schwartschild metric gives for non geodetic lines(Inverse lines)

$$\mathbf{ds^2 = - \gamma^{-1} dr^2 - r^2 d\theta^2 + \gamma dt^2} \quad \textbf{(9)}$$

According to the gravitation tensor of Einstein $\mathbf{G_\mu = 0}$ which is symmetric and gives possibility to the interchange between the t and s variables.

This interchange between the s and t variables gives the rotation velocity in arc-length parameters gives :

$$\mathbf{dt^2 = - \gamma^{-1} dr^2 - r^2 d\theta^2 + \gamma ds^2} \quad \textbf{(10)}$$

Rearranging the order between the parameters involved gives :

$$\mathbf{r^2 d\theta^2 = - \gamma^{-1} dr^2 + \gamma ds^2 - dt^2} \quad \textbf{(11)}$$

By dividing with ds we obtain :

$$\mathbf{r^2 (d\theta / ds)^2 = - \gamma^{-1} (dr / ds)^2 + \gamma - (dt / ds)^2} \quad \textbf{(12)}$$

From this we obtain the relativistic rotation speed $\mathbf{v_2 = r (d\theta / ds) = (- \gamma^{-1} (dr / ds)^2 + \gamma - (dt / ds)^2)^{1/2}}$ **(13)**

By assuming that we have circular orbit follows **(dr / ds) = 0** which gives :

$$\mathbf{v_2 = r (d\theta / ds) = (\gamma - (dt / ds)^2)^{1/2} = = ((1 - 2M/r - (dt / ds)^2))^{1/2}} \quad \textbf{(22)}$$

Now we weight the solution such that we obtain the new universal rotation formula, which is divided into one Keplarian and a relativistic part :

$$\mathbf{v_{tot} = w_1 (MG/r)^{1/2} + w_2 (v_{max}^2 - MG/r)^{1/2}} \quad \textbf{(14)}$$

IV. A. The Relativistic Invariant Laplace Wave Equation

To obtain such a formula in paper (12) we start with studying such a formula for a light wave

$$\mathbf{\nabla^2 \psi - (1 / c^2) \psi_{tt} = 0} \quad \textbf{(15)}$$

where a sinusiodal wave function is shown with c = speed of light and c = ω / k, where ω = angular frequency.

This formula (15) is an example of a relativistic invariant Laplacian wave equation Engström (13) p.129. It is invariant because it fulfills the Lorenz transformation Kay (14) p.172 and 167 and has proportionality in all coordinate systems. This equation is then transformed into spherical coordinates by using spherical harmonics explained in Råde and Westergren (15) p.264 with the "chain rule". After that we project the equation onto a plane in order to eliminate an unnecessary constant so it can represent a spiral galaxy.

This relativistic invariant Laplacian formula fully developed has the following appearance

$$\mathbf{(\partial^2\psi / \partial r^2) + (2/r) (\partial\psi / \partial r) + ((\partial^2\psi / \partial \theta^2) + (\cot \theta / r^2) (\partial\psi / \partial \theta)) +}$$
$$\mathbf{((1 / (r^2 \sin^2 \theta)) (\partial^2\psi / \partial \varphi^2)) - (1/ C^2) (\partial^2\psi / \partial t^2) = 0} \quad \textbf{(16)}$$

where r = distance to centre, C = wave phase constant , θ = spherical angle

and will be a basis equation for galaxies. This formula is also invariant according to the Lorenz transformation according to Kay (14) p.167 and has proportionality in all coordinate systems.

This equation can be rewritten as:

$$\mathbf{\psi_{rr} + (2/r) \psi_r - (l (l+1) / r^2) Y_{nl} - (1 / C^2) \psi_{tt} = 0} \quad \textbf{(17)}$$

In equation (17) Y_l is here a spherical harmonics function and ψ_{rr} and ψ_r are wave function deravities and **l** is the azimuthal quantum number. These spherical harmonics are compositions of orthogonal sines- and cosines functions and Legendre polynomials which are transformed into spherical coordinates according to Kay (14) p.167.

The general solution including time terms will be :

$$\mathbf{\psi(r, \theta, t) = (\cos (r + \theta) / r) (\exp (-i C t)} \quad \textbf{(18)}$$

The spiral galaxy model described is an energy density wave theory model and the resulting solution shall be powered by 2, i.e

$$\mathbf{\Psi (r, \theta, t) = | \psi (r, \theta, t) |^2 \approx (1 / r)} \quad \textbf{(19)}$$

Equation (19) shows a normal procedure in quantum mechanics used for atoms. Now we are using the same procedure for galaxies.

The energy density fall-off rate is therefore proportional to the inverted distance to the centre of the galaxy. The equation is invariant in coordinate system transformations, and invariant under a Lorenz transformation since the solution is produced by the relativistic invariant Laplace equation. It has the general and special relativity characteristics, such as rotation velocity where $\mathbf{\psi = v_{rot}^2}$ which gives :

$$\mathbf{v_{rot} = (M / r)^{1/2} (1-w^2)^{1/2} + (v^2_{max} - (M / r) w^2)^{1/2}} \quad \textbf{(20)}$$

where w is a weighing term used for balancing the equation , M is the mass of the galaxy and v_{max} is the maximum rotation velocity of the galaxy. Gravity units are applied in this equation. Equation (20) is similar to the rotation velocity formulas in papers (5) and (11)

B. General solution of relativistic invariant Laplace equation in spherical coordinates using Quaternions

The general equation to the galaxy relativistic Laplace equation in spherical coordinates from equation 28 of paper (12) in gravitational units projected onto the polar plane is using quaternions.

Trigonometry gives :

$$| \mathbf{v}_{rot} |^2 = A^2 (M/r) + B^2 (v^2_{max} - (M/r)) \quad (21)$$

Using a complex coefficient on B we obtain in gravitation units :

$$\mathbf{v}_{rot} = A (M/r)^{1/2} + i B (v^2_{max} - (M/r))^{1/2} \quad (22)$$

This is our general rotation velocity formula for galaxies similar to papers (5) and (11).

C. The complete galaxy equation in spherical coordinates

which can be rewritten in a shorter version under certain restrictions as :

$$\nabla^2 \psi - (1 / c^2)(\partial^2\psi / \partial t^2) = 0 \quad (23)$$

which is similar to the light wave equation (15).

The difference between the Laplace equation and the relativistic galaxy equation determines the amount of uniform rotation in its own coordinate system.

$$\mathbf{v}_{rtot} = A (M / r)^{1/2} + i B (v^2_{max} - (M / r))^{1/2} \quad (24)$$

Equation (24) is also similar to the rotation velocity formulas in papers (5) and (11) and equations (17) and (28) in paper (12). The first term of equation (24) concerns the galaxy body (Keplerpart) and the second part concerns the spiral arms(relativistic part) in gravitation units.

V. Planetary Orbits From Special Relativity.

This section shows that it is possible to use the relativistic invariant Laplacian equation (16) in spherical coordinates on planetary orbits. These results are in accordance with the planetary orbit calculations and formulas by Einstein (9) .

Letting the angle **θ** be a constant and by using a deflating procedure and eliminating one variable at a time the following expression is achieved.

$$(\partial^2\psi / \partial r^2) + (2/r) (\partial\psi / \partial r) + ((1 / (r^2 \sin^2 \theta)) (\partial^2\psi / \partial \varphi^2)) - (1/ C^2) (\partial^2\psi / \partial t^2) = 0 \quad (25)$$

By assuming a solution of the type :

$$\psi = R(r) \Phi(\varphi) T(t) = R \Phi T \quad (26)$$

where r = distance , φ = azimute angle, and t = time

and substituting this into the equation(25) and performing separation of variables gives :

$$(R'' \Phi T) + (2/r) (R' \Phi T) + ((1 / (r^2 \sin \theta)) (R \Phi'' T) - (1/ C^2) (R \Phi T'') = 0 \quad (27)$$

Dividing with R Φ T gives the following result :

$$((R'' + (2 / r) R') / R + ((1 / (r^2 \sin \theta)) (\Phi'' / \Phi) - (1/ C^2) (T'' / T) = 0 \quad (28)$$

In this chapter we analyze the sign of the radial part of equation (16) and use a deflating procedure. We first assume a trivial solution to equation (28)

$\mathbf{R = \psi = 1/ r - k}$ and its first and second derivatives :

$R' = \psi_r = - 1 / r^2$ and $R'' = \psi_{rr} = - 2 / r^3$

These derivatives were put in equation (28) and gave the following expression :

and $((R'' + (2 / r) R') / R = ((2 / r^3) + (2 / r)(- 1 / r^2)) / ((1 / r) + k) =$

$$r ((2 / r^3) + (2 / r)(- 1 / r^2)) / (1 + r k) = 0 / (1 + r k) = 0 \quad (29)$$

If r k ‡ -1 we can deflate our equation to :

$$(1 / (r^2 \sin \theta)) (\Phi'' / \Phi) - (1/ C^2) (T'' / T) = - (R'' + (2 / r) R') / R = 0 \quad (30)$$

or more clearly :

$$(1 / (r^2 \sin \theta)) (\Phi'' / \Phi) - (1/ C^2) (T'' / T) = 0 \quad (31)$$

If we now deflate our 4-dimentional wave function into the following two dimentional equations :

$$((1 / (r^2 \sin \theta)) (\Phi'' / \Phi) - (1/ C^2) (T'' / T) = 0 \quad (32)$$

which can also be written :

$$(1 / (r^2 \sin \theta)) (\Phi'' / \Phi) = (1/ C^2) (T'' / T) \quad (33)$$

If we assume complex exponential solutions on the left side (the Φ – variable) of this equation, then we have the second and zero (function) derivatives of Φ and T.

$$\Phi'' = - a^2 C \exp (i a \varphi) \quad (34)$$

$$\Phi = C \exp (i a \varphi) \quad (35)$$

$$T'' = b^2 C \exp (i b t) \text{ and} \quad (36)$$

$$T = C \exp (i b t) \quad (37)$$

Then the combined solution could be expressed as including a potential part, a φ-part and a T-part(time) in the following way :

$$\psi = (e / R_0) \exp (i n \varphi) \exp (i c t / \lambda_2) \quad (38)$$

We adjust the solution with a constant e for the eccentricity and set $r = R_0$ then we have :

$$\psi = (e / R_0) \exp (i n \varphi) \exp (i c t / \lambda) \quad (39)$$

We have also a trivial solution :

$$\psi = 1/ r - k \quad (40)$$

and the inverse

$$r = 1 / (\psi + k) \quad (41)$$

Now we substitute the wave equation into the trivial solution which gives the planetary orbits a precessing ellipse

$$r = 1 / ((1 / R_0) + (1 / R_0) e \cos (\varphi - \omega - \delta\omega)) = R_0 / (1 + e \cos (\varphi - \omega - \delta\omega)) \quad (42)$$

where n=1, $ct / \lambda = - 3m^2 \varphi / h^2$ and $R_0 = h^2 / m$ which is the same formula defined in General Relativity(Eddington)(9) p.88 where the radius is equal to

$$r = h^2 / m (1 + e \cos (\varphi - \omega - \delta\omega)) \quad (43)$$

This formula is planetary orbit formula by Einstein which we have shown to have its origin in the relativistic invariant Laplacian formula (16)

VI. Using The Geodetic Line Calculations For Deriving The New Rotation Velocity Formula For Galaxies

Set $\mathbf{Y = 1 - 2M / r}$ from the reference (Eddington/Einstein/Schwarzschild)(9) p.89 we use gravitation units for simplicity. **Y** is here the Schwarzschild term and φ is the rotation angle.

$$(d \varphi / dt)^2 = \| (1 / 2) Y' / r \| \quad (44)$$

$$r (d \varphi / dt)^2 = \| (1 / 2) Y' \| \quad (45)$$

Integrating gives :

$$\int r (d \varphi / dt)^2 = \| (1 / 2) \int Y' dr \| \quad (46)$$

Dividing by 2 then we have

$$r^2 (d \varphi / dt)^2 = \| +- C + Y \| \text{ or} \quad (47)$$

$$r^2 (d \varphi / dt)^2 = \| +- C + Y \| = \| 1 +- C - (2m / r) \| \quad (48)$$

Here C is the constant of integration letting $1 +- C = B^2 V_{lim}{}^2$ and splitting up the constant $2m = A^2 M - B^2 M$ we have

$$v^2_{rot} = r^2 (d \varphi / dt)^2 = \| +- C + Y dr = \| 1 +- C - (2m / r) \| =$$

$$= \| A^2 M / r + B^2 v^2_{lim} - B^2 M / r \| \quad (49)$$

$$v^2_{rot} = \| A^2 M / r + B^2 (v^2_{lim} - M / r) \| \quad (50)$$

$$v_{rot} = \| (A^2 M / r + B^2 (v^2_{lim} - M / r))^{1/2} \| \quad (51)$$

Now using complex numbers this solution becomes

$$v_{rot} = \| (A (M / r)^{1/2} + iB ((v^2_{lim} - M / r))^{1/2} \| \quad (52)$$

Equation (52) is also similar to the rotation velocity formulas in papers (5) and (11) and equations (17) and (28) in paper (12).

VII. Dynamic relativistic invariant La Place equation

We define the dynamic relativistic invariant La Place equation as the equation :

$$f (t)^2 \nabla^2\psi - (f (r)^2 / c^2) \psi_{tt} = 0 \quad (53)$$

where the two dynamic functions are $f(t) = V(t) / t$ and $f (r) = V(r) / r$. These functions make equation 53 to be a dynamic equation capable to regulate the length of arms and the rotation velocity of the galaxy. Inserting these functions in the equation 53 will lead to equation 54.

$$|(V(t) / t)|^2 \nabla^2\psi - (|(V(r) / r)|^2 / c^2) \psi_{tt} = 0 \quad (54)$$

This formula can be rewritten as :

$$\nabla^2\psi - (1/ c^2) (|(V(r) / r)|^2 / |(V(t) / t |^2) \psi_{tt} = 0 \quad (55)$$

with the solutions :

$$\psi_1 = (R / r) \exp \pm i ((2 \pi V(r) / \lambda_1 r) (r \pm c t)) + n \varphi / 2) \quad (56)$$

and

$$\psi_2 = \exp \pm ((2 \pi V(r) / \lambda_2 r) (r \pm c t)) \quad (57)$$

where λ_1 and λ_2 are the bandwidths of the galaxy. The combined solutions of equation 53 are

$$\psi = \psi_1 \psi_2 \quad (58)$$

which are the combined solutions of equation 53.

Letting $r = c t$ the differential equation 53 is reduced to :

$$\nabla^2 \psi - (1 / c^2) \psi_{tt} = 0 \quad (59)$$

which is the relativistic invariant La Place equation.

VIII. Results And Figure From The New Dynamic Invariant Relativistic La Place Equation Formula And The New Rotational Velocity Formula For Galaxies.

In this section "experimental" results are presented from computer simulations of the galaxy NGC 3200, where the dynamic equation (59) has been used. This is shown in Fig 1 and shows a galaxy with a very realistic appearance and is the result of further development of the new formula (59) and new computer programs. Fig 2 shows a rotation velocity distribution versus distance of the galaxy NGC 3200. These graphs do follow the observational velocity distributions with a Match of 95% and has followed equation (24).

Table 1 is a summary of calculations of mass and radius from different galaxies together with Match percentage of the rotation velocity curves. The equation (24) has been followed here giving a mean value of 97% of the Match percentages. These values of mass and radius are in accordance with values published in Combes (3).

IX. Discussion

We can se from section 2 and 3 from Barrera and Thelin (5) that the velocity formula between velocity and distance to the centre of the galaxy has a √x- structure. These facts are based on results presented in Combes (3) and Lang (4) and are not based on Kepler`s 3:rd law directly. These relationships are observed by the astronomers where the rotation velocity reach a constant speed at distances between 5 and 10 kpc from the centre of the galaxy. A similar structure of the rotation velocity versus distance is also obtained by using the theory of relativity and the Schwarzschild metric in equation 19 of Barrera and Thelin (5) and from the new expanded formula (33) of the Barrera and Thelin (11) paper and is seen in Figs (5-7) in that paper. In these papers a steep rising of the velocity (angular and circular in km/s) at low distances is observed. After that rising, a plateau is reached, which will be dominating up to large distances.

In paper Barrera and Thelin (12) an extended verification of the rotation velocity formula of galaxies from papers (5) and (11) together with a new energy density formula of galaxies are presented. These formulas have been derived with different mathematical methods together with a relativistic invariant Laplacian formula used in spherical coordinates and also using quaternions. This is also shown in paper (12) where such graphs are shown for the galaxies NGC7606, 3200, 801, 1417 and IC724 . All the observational graphs of the rotation velocity- distance curves are showing very good correlation between theoretical and observational values Figs (1b – 5b) in paper (12)

Our model is also in accordance to the energy density distribution in the galaxy where computer simulations were performed on these galaxies, where the energy density versus distance was studied giving a realistic structure (Figs 1a -5a) in paper (12), where also photographic pictures from space of these galaxies are shown. The similarity between experimental graphs and pictures is outstanding and gives a strong support of the results of this paper.

It was also possible to calculate the Radius , V_{max} , Estimated Mass of the galaxies studied, which are shown in Table 1 of paper (12). By using equation (28) in paper (12) a very good match percents are achieved. The approximated mass values follow a corresponding equation from Combes (3) p.83, which gives higher values.

The observations from Freeman (1) and Rubin (2) are also observed by many astronomers and have been a controversial discovery, because it contradict Kepler`3:rd law, which will follow a (1/ √r) - dependence according to equation (2) and is not observed in any galaxies. Therefore many astronomers claim that there must be a large amount of dark matter in the galaxy, which is the cause of this discrepancy and also hold the galaxies together at those high rotation speeds.

Similar √x- structures of the velocity curves as our curves have also been obtained in the so called Mond- project, where a modification of the Newton`s law is applied Sanders (16).

In paper (12) and in our earlier papers in (5), (11) and also the paper by Sanders (16) raise the question if there is so much dark matter in the universe ? Our rotation curves fit very well with observations anyhow.

An astonishing fact from this paper is the use of quantum mechanical (slightly corrected) calculations for galaxies which seem to be valid here. Such calculations are otherwise normally used for atoms. It is also interesting to note the similarity between the La Place equations (3) for a light wave and equation (36) for a galaxy in paper (12). It is fascinating how quantum mechanics and relativity theory can be unified here from the results of this paper(12).

This paper presents a summary of results from different papers about a new rotation velocity formula for galaxies in accordance to observations, without any influence of dark matter. An important formula in these papers is the relativistic invariant La Place wave equation for evaluating the new rotation velocity formula. This formula has been further developed into a dynamic version of the formula, which has shown to be still important here.

This relativistic invariant La Place wave equation has also shown to be applicable for planetary orbit equations too. These results agree well with planetary orbit calculations by Einstein which we have showed to have its origin in the relativistic invariant Laplacian formula (16). It has also been possible to derive the rotation velocity formula by using geodetic lines according to calculations by Eddington /Einstein/Schwarzschild)(9). This means that it is possible to use different methods to calculate the same rotational velocity formula, which means that this formula and relativistic invariant La Place wave equation seem to have a central role in this part of physics.

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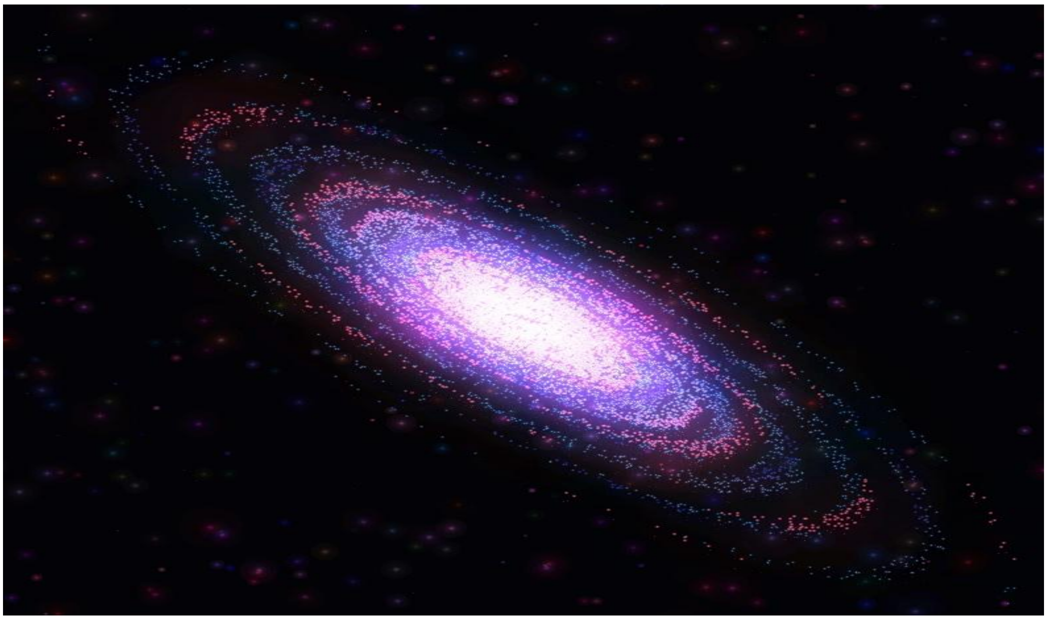

Fig 1 NGC 3200 Computer simulations of randomly distributed 30000 “stars” distributed like an ellipsoid in a galaxy. The dynamic relativistic invariant La Place equation (59) has been used here.

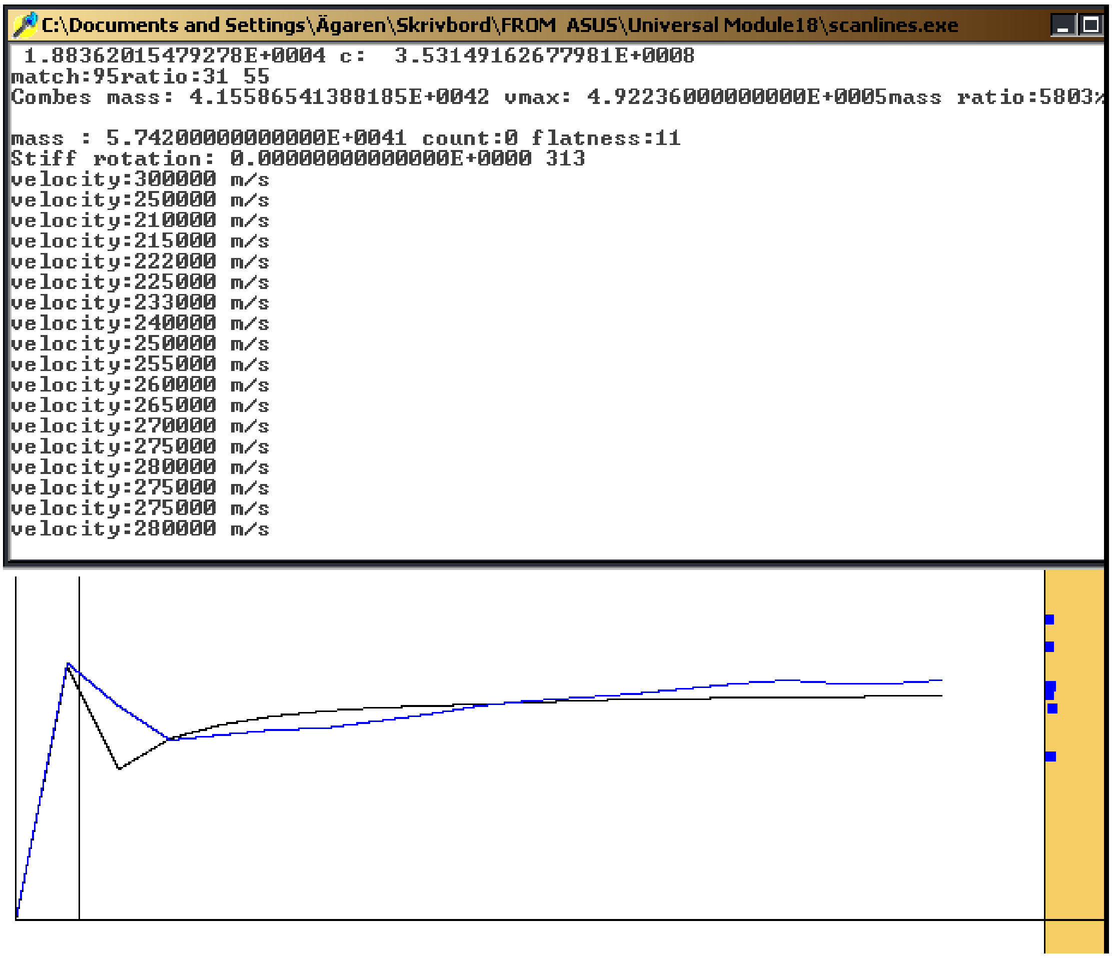

Fig 2 Rotation velocity distributions versus distance of the galaxy **NGC 3200**
These graphs do follow the observational velocity distributions.
(Computer simulations) Match 95 %. Equation (24) has been followed.
Reproduction from Barrera and Thelin (12)

Table 1

Match %	Galaxy	Mass (x 10^{42} kg)	Radius (x 1000ly)
98	NGC 3200	1.0	130
97	NGC 801	1.5	180
95	UGC 2885	5.4	350
97	Milky Way	0.4	60
97	NGC 7606	0.9	120
97	NGC 12810	1.3	180
99	NGC 1417	1.8	180
96	UGC 10205	0.6	90
96	IC 724	1.9	130
96	NGC 1024	1.4	180
97	NGC 4378	1.0	100
99	NGC 7083	0.7	130
98	M 33 Triangulus	0.2	100
95	NGC 3198	0.3	100
94	NGC 7164	0.3	80
96	NGC 3145	0.7	80
98	NGC 4984	0.8	80
95	NGC 2403	0.2	70
97	NGC 2841	1.6	140
97	NGC 2903	0.4	80
92	M 31 Andromeda	0.9	110
Mean value		Mean value	
97 %		1.1 x 10^{42} kg	