IOSR Journal of Applied Physics (IOSR-JAP)
e-ISSN: 2278-4861.Volume 7, Issue 5 Ver. I (Sep. - Oct. 2015), PP 28-33
www.iosrjournals

Four component electromagnetic waves and energy propagation

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Abstract: *Dirac-Maxwell equations are generalized by introducing electric scalar field and magnetic scalar field in order to obey second order differential equations by both scalar fields and vector fields without implementation of Lorentz condition on them. It makes the electric charges and magnetic charges time-dependent generating scalar field and magnetic scalar field respectively. These scalar fields further contribute to the electric vector field and magnetic vector field. They are further responsible to produce longitudinal components of electric vector field wave and magnetic vector field wave in addition to their transverse components as usual.*

Keywords: *Generalized Dirac-Maxwell's equations, Lorentz Gauge, Electric scalar field, Magnetic scalar field, Poynting vector*

I. Introduction

Symmetry is the ethical sense of nature and is always expected to be satisfied by theoretical models used to define performance of many phenomena observed. In Maxwell's equations, the symmetry is obtained by Dirac [1-6] by introducing magnetic monopoles as the source of static magnetic field. When the magnetic charge comes into motion produces electric field. Thus Dirac generalized the Maxwell's equations, called Dirac-Maxwell's equations (DME), and in vacuum they are

$$\nabla \cdot E = 4\pi\rho^{e} \tag{1a}$$

$$\nabla \cdot \boldsymbol{H} = 4\pi\rho^{m} \tag{1b}$$

$$\nabla \times \boldsymbol{E} + \frac{1}{c}\frac{\partial \boldsymbol{H}}{\partial t} = -\frac{4\pi}{c}\boldsymbol{j}^{m} \tag{1c}$$

$$\nabla \times \boldsymbol{H} - \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} = \frac{4\pi}{c}\boldsymbol{j}^{e} \tag{1d}$$

where ρ^{e} and ρ^{m} are the electric charge and magnetic monopole densities respectively, and je and jm are electric and magnetic current densities respectively. Solutions of these DME were given by Cabibbo & Ferrari [7], Epistein [8], Ferrari [9] as

$$\boldsymbol{E} = -\nabla\phi^{e} - \frac{1}{c}\frac{\partial A^{e}}{\partial t} - \nabla \times \boldsymbol{A^{m}} \tag{2a}$$

$$\boldsymbol{H} = -\nabla\phi^{m} - \frac{1}{c}\frac{\partial A^{m}}{\partial t} + \nabla \times \boldsymbol{A^{e}} \tag{2b}$$

These sources (of electric charges and magnetic monopoles), here, are subjected to the continuity equation, viz.

$$\nabla \cdot \boldsymbol{j}^{e,m} + \frac{\partial \rho^{e,m}}{\partial t} = 0 \tag{3}$$

and hence they are said to be conserved. This condition on the sources then leads to the Lorentz gauge on the four potentials, viz.

$$\nabla \cdot \boldsymbol{A}^{e,m} + \frac{1}{c}\frac{\partial \phi^{e,m}}{\partial t} = 0 \tag{4}$$

Thus the Lorentz condition on the potentials makes the electric charges and magnetic monopoles time independent. The equation (3) indicates that the electric charges or magnetic monopoles cannot be created or destroyed but they can be transferred from one position to other. However, in pair production electron and positron gets created from a gamma particle in which again charge is conserved but the equation (3) does not hold strictly. Hence one should allow the electric charges and magnetic monopoles to be function of time and to look their conservation in another way. If the sources are allowed to be functions of time then the above Dirac-Maxwell equations are not able to describe electromagnetic fields of such sources. One has to generalize these equations in such a way that the generalized set of equations should be able to explain electromagnetic fields of time dependent as well as time independent sources. Such an attempt has been done by [10 -13] and generalized the DME by introducing two scalar fields in the DM equation which are actually replacement of the Lorentz gauges on the electric and magnetic potentials. These Generalized Dirac-Maxwell's equations (GDME) are

$$\nabla \cdot \boldsymbol{E} = \frac{1}{c}\frac{\partial E_0}{\partial t} + 4\pi\rho^e \tag{5a}$$

$$\nabla \cdot \boldsymbol{H} = \frac{1}{c}\frac{\partial H_0}{\partial t} + 4\pi\rho^m \tag{5b}$$

$$\nabla \times \boldsymbol{E} + \frac{1}{c}\frac{\partial \boldsymbol{H}}{\partial t} + \nabla H_0 = -\frac{4\pi}{c}\boldsymbol{j}^m \tag{5c}$$

$$\nabla \times \boldsymbol{H} - \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} - \nabla E_0 = \frac{4\pi}{c}\boldsymbol{j}^e \tag{5d}$$

These have usual solutions for $\boldsymbol{E}$ and $\boldsymbol{H}$ given by equations (2) and in addition to that they have solutions for scalar fields as

$$E_0 = \nabla \cdot \boldsymbol{A}^e + \frac{1}{c}\frac{\partial \varphi^e}{\partial t} \tag{6a}$$

$$H_0 = \nabla \cdot \boldsymbol{A}^m + \frac{1}{c}\frac{\partial \varphi^m}{\partial t} \tag{6b}$$

where E_0 and H_0 are the electric and magnetic scalar fields respectively. Clearly these scalar fields are the removal of the Lorentz gauge on their respective potentials.

As the Lorentz gauge on the potentials is removed, one expect that the continuity equation need not be hold by the sources but again total amount of charge should be conserved. This means that at any position if charge on a particle is decreasing then the decreased amount of the charge should produce a field, which then should cause to rise the same amount of charge on other particle (say sink) to which it is interlinked. Such a created field should be a scalar field given by equations (6) and should be proportional to the rate of change of charge on the source. If the charges become time-independent then the scalar fields should vanish with satisfying the Lorentz condition by the potentials and the GDME then reduce to the original DME. Hence the set of GDME given by equations (5) becomes proper generalization of the Dirac-Maxwell equations.

Our aim is to investigate effect of the scalar fields (E_0 and H_0) on the electromagnetic vector field waves and their propagation in vacuum.

II. Wave equations for the four components of electromagnetic fields

The differential equations satisfied by the four components of the electromagnetic fields in vacuum are

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\boldsymbol{E} = 4\pi\left(\nabla\rho^{\boldsymbol{e}} + \frac{1}{c^2}\frac{\partial \boldsymbol{j}^{\boldsymbol{e}}}{\partial t} + \nabla \times \boldsymbol{j}^{\boldsymbol{m}}\right) \tag{7a}$$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\boldsymbol{H} = 4\pi\left(\nabla\rho^{\boldsymbol{m}} + \frac{1}{c^2}\frac{\partial \boldsymbol{j}^{\boldsymbol{m}}}{\partial t} + \nabla \times \boldsymbol{j}^{\boldsymbol{e}}\right) \tag{7b}$$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)E_0 = -\frac{4\pi}{c}\left(\nabla \cdot \boldsymbol{j}^{\boldsymbol{e}} + \frac{\partial \rho^e}{\partial t}\right) \tag{7c}$$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)H_0 = -\frac{4\pi}{c}\left(\nabla \cdot \boldsymbol{j}^{\boldsymbol{m}} + \frac{\partial \rho^m}{\partial t}\right) \tag{7d}$$

The first two equations are the same as satisfied by the vector fields in the usual DME and the last two are outcome of the generalization from the last two equations. We conclude that the continuity equation for the sources due to generalization gets modified.

In absence of the sources all the four components of the EM fields satisfy the following differential wave equation and all of them propagate with velocity c in vacuum.

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\psi = 0 \tag{8}$$

where $\psi = \boldsymbol{E}, \boldsymbol{H}, E_0, H_0$.

It follows from the wave equation one can write a monochromatic four component electromagnetic wave in the form

$$\boldsymbol{E}(r,t) = \boldsymbol{E}^{\boldsymbol{0}} e^{-\boldsymbol{i}(\omega \boldsymbol{t} - \boldsymbol{k}\cdot\boldsymbol{r})} \tag{9a}$$

$$\boldsymbol{H}(r,t)=\boldsymbol{H^0}e^{-i(\omega t-\boldsymbol{k}\cdot\boldsymbol{r})} \tag{9b}$$

$$E_0(r,t)=E_0^0e^{-i(\omega t-\boldsymbol{k}\cdot\boldsymbol{r})} \tag{9c}$$

$$H_0(r,t)=H_0^0e^{-i(\omega t-\boldsymbol{k}\cdot\boldsymbol{r})} \tag{9d}$$

where $\boldsymbol{k}$ is a propagation constant given by ω/c, the amplitudes $\boldsymbol{E^0}$ and $\boldsymbol{H^0}$ are constant vectors and the other two amplitudes of the scalar field waves E_0^0 and H_0^0 are constant scalars. Our interest is in the plane wave solutions as they are simplest one to describe.

Using these wave equations GDME give

$$k\cdot\boldsymbol{E}-kE_0=0 \tag{10a}$$

$$k\cdot\boldsymbol{H}-kH_0=0 \tag{10b}$$

$$k\times\boldsymbol{E}-k\boldsymbol{H}-kH_0=0 \tag{10c}$$

$$k\times\boldsymbol{H}-k\boldsymbol{E}-kE_0=0 \tag{10d}$$

Last two equations show that the wave field vectors $\boldsymbol{E}$ and $\boldsymbol{H}$ have components parallel to the propagation vector due to presence of the scalar components E_0 and H_0. Thus the electromagnetic vector field waves have, now, no longer only transverse character but in addition to the transverse components it has longitudinal vector components. If both the scalar field waves are absent then the electromagnetic vector field wave attains its transverse character again. If one of the scalar field waves is absent, say H_0, then the magnetic vector field wave has no longitudinal component though the electric scalar field wave is present. The electric scalar field wave produces longitudinal component of the electric vector field wave only. Similarly if electric scalar field wave is absent but magnetic scalar wave is present then it produces longitudinally polarized magnetic vector field wave in addition to its transverse nature. It has no longitudinal component of the electric field wave. Thus the presence of the scalar field waves is responsible for longitudinal polarization of the electromagnetic vector field waves in addition to their transverse nature.

III. Energy in the four component electromagnetic fields

Equations (5c and 5d) from the set of GDME give

$$\nabla\cdot(E\times H)=-\frac{1}{2c}\frac{\partial}{\partial t}\left[E^{\boldsymbol{2}}+H^{\boldsymbol{2}}\right]-\frac{4\pi}{c}\left[\boldsymbol{E}\cdot\boldsymbol{j^e}+\boldsymbol{H}\cdot\boldsymbol{j^m}\right]-\left[\boldsymbol{E}\cdot\nabla E_{\boldsymbol{0}}+\boldsymbol{H}\cdot\nabla H_{\boldsymbol{0}}\right] \tag{11}$$

Further the dot product obeys

$$E\cdot\nabla E_0=\nabla\cdot(EE_0)-\frac{1}{2c}\frac{\partial}{\partial t}\left[E_0{}^{\boldsymbol{2}}\right]-4\pi E_0\rho^{\boldsymbol{e}} \tag{12}$$

and

$$H\cdot\nabla H_0=\nabla\cdot(HH_0)-\frac{1}{2c}\frac{\partial}{\partial t}\left[H_0{}^{\boldsymbol{2}}\right]-4\pi H_0\rho^{\boldsymbol{m}} \tag{13}$$

Thus equation (11) takes the form

$$\frac{c}{4\pi}\nabla\cdot(E\times H+EE_0+HH_0)=$$
$$-\frac{1}{8\pi}\frac{\partial}{\partial t}\left[E^2+E^2+E_0{}^2+H_0{}^2\right]-\frac{4\pi}{c}\left[\boldsymbol{E}\cdot\boldsymbol{j^e}+\boldsymbol{H}\cdot\boldsymbol{j^m}-cE_0\rho^e-cH_0\rho^m\right] \tag{14}$$

Integrating over a volume V, one obtains

$$\frac{d}{dt}\int_V\frac{1}{8\pi}\left[E^2+H^2+E_0{}^2+H_0{}^2\right]dv=\oint_{\boldsymbol{a}}\boldsymbol{S}\,.\,d\boldsymbol{a}$$
$$-\int_V\left[\boldsymbol{E}\,.\,\boldsymbol{j^e}+\boldsymbol{H}\,.\,\boldsymbol{j^m}-cE_0\rho^e-cH_0\rho^m\right]dv \tag{15}$$

Thus the Poynting vector takes the form

$$S=\frac{c}{4\pi}\nabla\cdot(E\times H+EE_0+HH_0) \tag{16}$$

This the generalized form of the Poynting vector.

Now, the current distribution by the vector $\boldsymbol{j^e}$ can be considered as made up of various charges $q_\propto$ moving with velocity $u_\propto$. Therefore the volume integral of $\boldsymbol{j^e}\cdot\boldsymbol{E}$ may be replaced by

$$\int_V\boldsymbol{j^e}\cdot\boldsymbol{E}dv\rightarrow\sum_{\boldsymbol{a}}q_\alpha\,\boldsymbol{u}_\alpha\,.\,\boldsymbol{E}_\alpha \tag{17}$$

where $\boldsymbol{E}_{\propto}$ denotes the electric field at the position of the charge q_{α}^{e}. As usual the work done per unit time on the charge q_{α}^{e} by the electromagnetic field is

$$\frac{dT_{\alpha}^{e}}{dt} = q_{\alpha}^{e}\,\boldsymbol{u}_{\alpha} \cdot \boldsymbol{E}_{\alpha} \tag{18}$$

where T_{α}^{e} is the kinetic energy of the $\propto^{th}$ electric charged particle.
Similarly the work done per unit time on $\propto^{th}$ magnetic monopole by the electromagnetic field is given by

$$\frac{dT_{\alpha}^{m}}{dt} = q_{\alpha}^{m}\,\boldsymbol{u}_{\alpha}\,.\,\boldsymbol{H}_{\alpha} \tag{19}$$

where T_{α}^{e} is the kinetic energy of the $\propto^{th}$ magnetic monopole.
Thus considering the volume integral of the product of electric scalar field and the electric charge, one writes

$$\int_V cE_0 \rho^e \, dv \to \sum_a cE_{0\alpha}\, q_{\alpha}^{e} \tag{20}$$

We expect that it must be amount of work done by the electric scalar field on the electric charges. Let us denote this amount of work done per unit time is equivalent to a rate of change of kinetic energy like term denoting by $T_{0\alpha}^{e}$. Therefore,

$$\frac{dT_{0\alpha}^{e}}{dt} = cE_{0\alpha}\, q_{\alpha}^{e} \tag{21}$$

Similarly, the magnetic scalar field $H_{0\alpha}$ linked to a magnetic monopole q_{α}^{e} does work on magnetic monopoles as given by

$$\frac{dT_{0\alpha}^{m}}{dt} = cH_{0\alpha}\, q_{\alpha}^{m} \tag{22}$$

Obviously, the term $\int_V \frac{1}{8\pi}\left[E^2 + H^2\right] dv$ represents the electric and magnetic vector field energies in the volume. Therefore, the term $\int_V \frac{1}{8\pi}\left[E_0{}^2 + H_0{}^2\right] dv$ should represent the electric and magnetic scalar field energies in the given volume. Thus the net electromagnetic field energy density in the volume is then

$$\in = \frac{1}{8\pi}\left[E^2 + H^2 + E_0{}^2 + H_0{}^2\right] \tag{23}$$

Let us construct the surface 'a' of the integral in such a way that in the interval of time under consideration none of the particles (electric charges and magnetic monopoles) will cross the surface. Then the equation (16) gives

$$\frac{d}{dt}\left(\int_V \in dv + \sum_{\alpha}\left(T_{\alpha}^{e} + T_{0\alpha}^{e}\right) + \sum_{\beta}\left(T_{\beta}^{m} + T_{0\beta}^{m}\right)\right) = -\oint_a \boldsymbol{S}\,.\,\boldsymbol{n}\, da \tag{24}$$

where the sum over $\propto$ and β includes only those particles lying within the volume enclose by the surface ‘a’. In this equation the left-hand side is the time rate of change of energy of the field and particle contained within the volume *V*. Thus the surface integral of $\boldsymbol{S}\cdot\boldsymbol{n}$ must be considered as the energy flux of the electromagnetic field flowing out of the volume bounded by the surface 'a'. Clearly from equation (24) one concludes that there is an unavoidable contribution from the scalar fields to the net energy in *EM* fields in addition to the vector fields. If the scalar fields are absent then the equation (24) reduces to the usual form given by

$$\frac{d}{dt}\left(\int_V \in dv + \sum_{\alpha,\beta}\left(T_{\alpha}^{e} + T_{\beta}^{e}\right)\right) = -\oint_a \boldsymbol{S}\,.\,\boldsymbol{n}\, da \tag{25}$$

where

$$\boldsymbol{S} = \frac{c}{4\pi}\left(\boldsymbol{E} \times \boldsymbol{H}\right) \tag{26}$$

and

$$\in = \frac{1}{8\pi}\left[E^2 + H^2\right] \tag{27}$$

IV. Energy propagation due to a four component electromagnetic waves

We know that when three component electromagnetic waves propagate through space from their source to distant receiving point, there is a transfer of energy from source to the receiver. A relation between rate of this energy transfer and the amplitude of electric and magnetic field strength (***E*** and ***H***) is called Poynting theorem (equation 24). The direction of energy flow is along the Poynting vector. In free space a three component electromagnetic wave has purely transverse nature (i.e. ***E*** and ***H*** are both perpendicular to the direction of propagation) and as the direction of the Poynting vector ($\boldsymbol{S} = \frac{c}{4\pi}(\boldsymbol{E}\times\boldsymbol{H})$) is in the same direction as that of the propagation vector, there occurs a transfer of energy at that time. In case of the four component electromagnetic wave the vector fields ***E*** and ***H*** have no longer transverse nature and the Poynting vector also gets modified. Let us see whether the Poynting vector has the same direction as that of the propagation vector and there is any possibility of energy flow.

Let us consider the scalar field wave in free space as

$$E_0 = E_0^0 e^{-i(\omega(-kz)} \tag{28a}$$

$$H_0 = H_0^0 e^{-i(\omega(-kz)} \tag{28b}$$

with constant amplitudes.

The first equation of **GDM**E, gives

$$E_z = nE_0 \tag{29}$$

with $\boldsymbol{n} = \boldsymbol{k}/k$.

Thus the presence of the electric scalar field wave produces longitudinal component of the electric vector field wave. Now the electric vector field wave has longitudinal component in addition to its transverse component as usual. Therefore we can have an electric vector field wave as

$$\boldsymbol{E_0} = \boldsymbol{E^0} e^{-\boldsymbol{i(\omega(-kz)}} \tag{30}$$

with $\boldsymbol{E^0} = \boldsymbol{E_x^0} + \boldsymbol{E_z^0}$ without any loss of generality. Then the fourth equation of GDME gives

$$\boldsymbol{H}_x = 0 \tag{31}$$

With considering the second equation of GDME, we get

$$\boldsymbol{H}_z = \boldsymbol{n}H_0 \tag{32}$$

In similar way that of the electric scalar fields the presence of the magnetic scalar field wave also produces longitudinal component of the magnetic vector field wave. The magnetic vector field wave has longitudinal component in addition to its transverse component as usual. Therefore we are again restricted to consider the magnetic vector field wave as

$$\boldsymbol{H} = \boldsymbol{H}^0 e^{-\boldsymbol{i(\omega(-kz)}} \tag{33}$$

with $\boldsymbol{H^0} = \boldsymbol{H}_y^0 + \boldsymbol{H}_z^0$.

The Poynting vector for such a four component EM wave with use of equations (29 to 33), gives

$$\boldsymbol{S} = \frac{c}{4\pi}\boldsymbol{n}\left(E_x H_y + {E_0}^2 + {H_0}^2\right) \tag{34}$$

Thus the direction of the Poynting vector is the same as that of the wave propagation vector ***k*** and hence there must be transfer of energy from source of the four component EM waves to distant receiving point. Contribution to the energy is both from the vector as well as scalar fields.

V. Discussion

We have established that generalization of the Dirac-Maxwell equations lifts the Lorentz condition on the potentials. Further the potentials satisfy the d'Almberts equation without restriction of any gauge making the potentials gauge free. Therefore the sources not satisfy the continuity equation. In being away of the sources the vector fields (***E***, ***H***) as well as the scalar fields (E_0, H_0) satisfy differential wave equation and all of them propagate with velocity *c* in vacuum. Scalar field wave has some interesting feature that it produces longitudinal polarization of the vector field wave. In the presence of plane electric scalar field wave produces longitudinal polarization of the electric vector and the magnetic plane scalar field wave produces longitudinal polarization in the magnetic vector field wave. These longitudinal polarizations are not possible in the usual Maxwellian theory. The generalized form of the Lorentz force indicates that the scalar field wave takes active part in the force and it produces change in the velocity of the charge. The electric scalar field produces deceleration in the electric charge and the magnetic scalar field produces deceleration in the magnetic monopole. If the velocity of the charged particle becomes zero then the force on it due to scalar field disappears. The generalization of the Poynting theorem discovers the contribution of the scalar field in addition to the vector field in the four component electromagnetic wave in the vacuum is in the direction of propagation of the wave and hence making possible propagation of the electromagnetic energy along the propagation of wave.

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