

Properties of Some Superdeformed Bands in Hg-Tl-Pb Nuclei Using Nuclear Softness Model with Weaken Parameter

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Abstract: A nuclear softness model with weaken parameter is introduced for the first time to describe the structure of superdeformed (SD) bands in $A \sim 190$ mass region. The data set includes fourteen SD bands in Hg-Tl-Pb nuclei. For each band the model parameters and the unknown bandhead spin are deduced by fitting the calculated transition energies E_{γ}^{cal} with the recent experimental ones E_{γ}^{exp} . A computer simulated search program depending on iteration procedure has been used in the fitting. The excellent agreement between E_{γ}^{cal} and E_{γ}^{exp} presents reasonable support of the model. The calculated level spins I , rotational frequencies $\hbar\omega$, the kinematic $J^{(1)}$ and dynamic $J^{(2)}$ moments of inertia are examined. For all the studied SD bands, $J^{(2)}$ are found to rise with $\hbar\omega$, due to rotation alignment of pairs of valence nucleons occupy a specific high- j intruder orbitals and also due to gradually decrease in pairing. The energies of the yrast SD bands in the two isotones ^{194}Pb and ^{192}Hg are identical to within an average of about 3keV up to $\hbar\omega \sim 0.250$ MeV. This similarity suggests that the excess two protons in ^{194}Pb do not change the SD rotational properties. Five pairs of our selected SD bands in odd nuclei are signature partners. Their staggering parameters which represent the difference between the averaged transition $I+2 \rightarrow I \rightarrow I-2$ energies in one band and the transition $I+1 \rightarrow I-1$ energies in its signature partner are extracted and examined. These signature partner pairs reveal large amplitude $\Delta I=1$ staggering.

I. Introduction

Since the initial discovery of the high spin discrete superdeformed (SD) rotational bands in the nucleus ^{152}Dy [1] in 1986 a great number of SD bands were observed in several mass regions $A \sim 150, 190, 130$ and 80 [2]. These SD bands were studied with the aid of large, high-parity Ge detector arrays. The increased sensitivity obtained by combining the latest generation of gamma-ray spectrometers with charged particle detectors made it possible to identify other regions of superdeformation. Several SD bands have been found in the mass regions $A \sim 90, 70, 60, 40$ [3,4]. Every SD region has their own characteristics so that we can significantly enlarge and deepen our understanding of nuclear structure by systematically investigating similarities and differences between the SD bands in different mass regions.

In this respect the $A \sim 190$ region is of special interest since SD states have been observed down to quite low spin. Superdeformation in this mass region was first observed in ^{191}Hg [5], and since then more than 85 SD bands were found in this region. In this region ^{192}Hg can be considered as a doubly magic SD nucleus, this is due to the presence of large shell gaps at proton number $Z=80$ and neutron number $N=112$ for values of the quadrupole deformation parameter $\beta_2 = 0.5$. An exciting difference between the SD bands in the $A \sim 190$ region is the behaviour of the dynamical moment of inertia $J^{(2)}$ as a function of rotational frequency $\hbar\omega$. The majority of bands in the $A \sim 190$ region shows the same smooth rise in $J^{(2)}$ with increasing $\hbar\omega$. This behaviour of $J^{(2)}$ was interpreted [6,7] as resulting from the gradual alignment of a pair of nucleons occupying specific high- N intruder orbitals originating from the $j_{15/2}$ neutrons and $i_{13/2}$ protons subshells in the presence of dynamical pairing correlations. In odd- A SD nuclei, the unpaired nucleons close to the Fermi surface are expected to reduce or block the pairing strength and hence increase the moment of inertia.

Level spins in SD bands were not determined experimentally (with the exceptions of $^{194}\text{Hg}(\text{SD1}, \text{SD3})$, $^{193}\text{Tl}(\text{SD1}, \text{SD2})$ and $^{194}\text{Pb}(\text{SD1})$ [8]) because linking transitions between the SD states and the normal deformed (ND) states with known spins were not observed. Therefore, the exact excitation energies, spins and parities of SD bands remain undetermined. Experimentally the only determined quantities are the gamma-ray transition energies between levels differing by two units of angular momentum (stretched quadrupole $\Delta I = 2$). Several related approaches were proposed for the spin assignments in SD bands [9–18]. For all approaches an extrapolation fitting procedure was used to fit the experimentally measured gamma-ray transition energies or the dynamical moment of inertia as a power series expansion in the square of rotational frequency which is then integrated to give the spin.

An exciting aspects of SD studies were the discovery of identical bands (IB's) phenomenon [19], whereby a pair of SD bands with very nearly identical transition energies were observed in nuclei having a different mass number. This requires that the dynamical moment of inertia in the two bands is identical. In the A~190 mass regions Stephens et al [20] used the near-integer alignment spin of these bands relative to one another as evidence from quantized pseudospin alignment and also introduced the spin independent incremental alignment which depends only on gamma-ray energies. Several explanations [15,17,21-24] were put forward to understand the origin of the IB's phenomenon but none are fully satisfactory.

It was found that the majority of SD bands observed in odd-A, odd-odd and excited SD bands in even-even nuclei are signature partner SD bands, and it was interesting that the bandhead moments of each signature partner SD band are almost identical. In A~190 mass region most signature partner pairs of SD odd-A nuclei exhibit $\Delta I=1$ staggering with large amplitude splitting [13,16,25-27].

The main purpose of the present work is twofold. The first is to propose an energy formula for SD bands and to assign the unknown level spins for SD bands in A~190 region and then calculate transition energies, rotational frequencies, kinematic and dynamic moments of inertia. The second objective is to analyse the theoretical calculations to investigate some properties like the behaviour of dynamical moment of inertia, the identical bands and the $\Delta I=1$ staggering in odd-A SD nuclei. The paper originates as follows.

II. Outline of the Model

The excitation energy of an axially symmetric nucleus can be described by the rigid rotor model

$$E(I) = \frac{\hbar^2}{2J} I(I + 1) \tag{1}$$

Here J is the moment of inertia for an axially symmetric even mass nucleus for the angular momentum I. An extension of this rotational model was chosen in order to take into account the nuclear softness parameter σ , which measures the relative initial variation of J with respect to I.

$$\sigma = [J^{-1} dJ/dI]_{I=I_0} \tag{2}$$

The moment of inertia J can be written in terms of the softness parameter σ as:

$$J = J_0(1 + \sigma I) \tag{3}$$

Substituting in equation (1) yield:

$$E(I) = \frac{\hbar^2}{2J_0} \left[\frac{1}{1 + \sigma I} I(I + 1) \right] \tag{4}$$

This formula contains only two parameters J_0 and σ and denoted as NS2.

In this paper an extension is chosen in order to take into account the pairing correlations.

A weak parameter (a) is introduced for the first time to reduce the pairing correlations and describe the SD bands. The excitation energy become:

$$E(I) = \frac{\hbar^2}{2J_0} \left[\frac{1}{(1 - a) + \sigma I} I(I + 1) \right] \tag{5}$$

This new formula contains three parameters and is denoted by NS3. For the SD bands gamma-ray transition energies are the only spectroscopic information universally available. Equation (5) leading to a form for the transition energies

$$\begin{aligned} E_\gamma(I) &= E(I) - E(I - 2) \\ &= A \left[\frac{I(I + 1)}{(1 - a) + \sigma I} - \frac{(I - 2)(I - 1)}{(1 - a) + \sigma(I - 2)} \right] \end{aligned} \tag{6}$$

with $A = \hbar^2/2J_0$

III. Theoretical Aspects

In the framework of nuclear collective rotational model with $K = 0$, where K is the projection of angular momentum I along the symmetry axis, the rotational frequency $\hbar\omega$ is defined as the derivative of the excitation energy $E(I)$ with respect to the angular momentum $\hat{I} = \sqrt{I(I+1)}$

$$\hbar\omega = \frac{dE(I)}{d\hat{I}} \quad (7)$$

That is $\hbar\omega$ is not directly measurable but it is related to the observed excitation energy and angular momentum by the canonical relation (7).

Two possible types of moments of inertia which reflects two different aspects of nuclear dynamics, are usually considered. The kinematic $J^{(1)}$ and the dynamic $J^{(2)}$ moments of inertia defined by

$$\frac{J^{(2)}}{\hbar^2} = \hat{I} \left(\frac{dE}{d\hat{I}} \right)^{-1} = \frac{\hat{I}}{\hbar\omega} \quad (8)$$

$$\frac{J^{(2)}}{\hbar^2} = \left(\frac{d^2E}{d\hat{I}^2} \right)^{-1} = \frac{1}{\hbar^2} \frac{1}{\omega} \frac{dE}{d\omega} = \frac{1}{\hbar} \frac{d\hat{I}}{\hbar\omega} \quad (9)$$

The two moments of inertia are obviously dependent. One has

$$J^{(2)} = J^{(1)} + \omega \frac{dJ^{(2)}}{d\omega} \quad (10)$$

In particular for a rigid nuclear rotor, we should obtain

$$J^{(2)} = J^{(1)} = J_{rigid} \quad (11)$$

Experimentally, for the SD bands, gamma-ray transition energies are the only spectroscopic information available. To compare the structure of these SD bands, information about their gamma-ray transition energies are commonly translated into values of rotational frequency $\hbar\omega$ and dynamical moment of inertia $J^{(2)}$ as:

$$\hbar\omega = \frac{1}{4} [E_\gamma(I+2) + E_\gamma(I)] (MeV) \quad (12)$$

$$J^{(2)} = \frac{4}{E_\gamma(I+2) - E_\gamma(I)} (\hbar^2 MeV^{-1}) \quad (13)$$

where E_γ in units of MeV

After extracting the level spins, the kinematic moment of inertia $J^{(1)}$ can be given by

$$J^{(1)} = \frac{2I-1}{E_\gamma(I)} (\hbar^2 MeV^{-1}) \quad (14)$$

It is seen that whereas $J^{(1)}$ depends on I proposition, $J^{(2)}$ does not.

The incremental alignment Δi is angular momentum between two nuclei A and B, where A represents the nucleus of interest and B is the reference nucleus. It is defined as [20]:

$$\Delta i_{AB} = 2 \frac{\Delta E_\gamma}{\Delta E_\gamma^{ref}} \quad (15)$$

ΔE_γ is obtained by substituting the transition energy in band A of interest from the closest transition energy in the reference band B

$$\Delta E_\gamma = E_\gamma^A(I+2) - E_\gamma^B(I) \quad (16)$$

and ΔE_{γ}^{ref} is calculated as the energy difference between the two closest transitions in the reference SD band

$$\Delta E_{\gamma}^{ref} = E_{\gamma}^B(I + 2) - E_{\gamma}^B(I) \quad (17)$$

The incremental alignment Δi value is spin independent and only dependent on the relationship between the gamma-rays in the SD band of interest and that of the reference band. The constant value of Δi implies similar moments of inertia, which is a strong sign that the high N intruder content is identical. For signature partners $\Delta i = 1\hbar$.

IV. Superdeformed Identical Bands

The discovery of SD rotational bands in nuclei differing in mass number having almost identical gamma transition energies to within an average of about 1-3 keV has generated considerable interest [19]. Although many explanations were proposed [15,17,21-24] to interpret the existence of identical bands IB's (assuming the occurrence of such IB's to be a specific property of SD nuclei), a satisfactory explanation is still lacking. Since the transition energy E_{γ} is very nearly twice the rotational frequency $\hbar\omega$ this means that $\hbar\omega$ for the IB's are very similar and also implies that the dynamical moment of inertia $J^{(2)}$ are almost equal. There are even more examples of IB's in A~190 region as an example the yrast SD bands in ^{194}Pb and ^{192}Hg are almost identical.

To determine whether a pair of SD bands is identical or not, one can extract the difference between transition energies ΔE_{γ} and plotted it versus $\hbar\omega$ or E_{γ} . On average the deviation is less than 2keV. Also one can compare their dynamical moments of inertia $J^{(2)}$. The incremental alignment Δi is a useful tool to select IB's bands and predict γ -ray energies of SD bands.

V. The $\Delta I = 1$ Staggering in Transition Energies of Signature Partners Pairs in Odd-SD Nuclei

Most of single-particle orbitals which lie near Fermi surface at $Z=80$ ($5/2 + [642]$, $7/2 - [514]$) and $N=112$ ($5/2 - [752]$, $9/2 + [624]$, $6/2 - [512]$) have large projection quantum number Ω . In SD nuclei, the coupling of the valence particles to the deformed core is much stronger than the Coriolisforce and the valence particles adiabatically follow the rotational motion of the core and the SD nucleus obey the strong coupling limit.

To explore more clearly the $\Delta I=1$ staggering in signature partner pairs of odd SD bands, one must extract the difference between the average transitions $I+1 \rightarrow I-1$ and $I-1 \rightarrow I-3$ energies in one band and the transition $I \rightarrow I-2$ energy in the signature partner

$$\Delta^2 E_{\gamma}(I - 1) = \frac{1}{2} [E_{\gamma}(I + 1) + E_{\gamma}(I - 1)] - E_{\gamma}(I) \quad (18)$$

where

$$E_{\gamma}(I) = E(I) - E(I - 2) \quad (19)$$

VI. Numerical Calculations and Discussion

In this section, fourteen SD bands in Hg, Tl, Pb nuclei in the region $A \sim 190$ mass region are considered. These SD bands are namely: $^{191}Hg(SD2, SD3)$, $^{192}Hg(SD1)$, $^{193}Tl(SD1, SD2)$, $^{192}Pb(SD1)$, $^{193}Pb(SD3, SD4, SD5, SD6)$, $^{194}Pb(SD1)$, $^{195}Pb(SD3, SD4)$, and $^{196}Pb(SD1)$. In our calculations, all gamma-ray transition energies are assumed from spin $I+2$ to spin I ($E_{\gamma}(I+2 \rightarrow I)$). For each SD band the optimized best model parameters A, a, σ and the bandhead spin I_0 have been calculated from adopted best iteration procedure [15,18], by fitting the calculated with the experimental transition energies using a computer simulated search program. The quality of the fitting is indicated by standard root mean square deviation χ

$$\chi = \left[\frac{1}{N} \sum_{i=1}^N \left(\frac{E_{\gamma}^{exp}(I_i) - E_{\gamma}^{cal}(I_i)}{\Delta E_{\gamma}^{exp}(I_i)} \right)^2 \right]^{1/2} \quad (20)$$

where N is the number of the data points entering into the fitting procedure and $\Delta E_{\gamma}^{exp}(I_i)$ are the experimental errors in transition energies.

The deviation χ for each SD band is around 1 keV or smaller. The bandhead spin I_0 is adjusted to the nearest half integer. Table (1) gives the model parameters A , a , σ obtained by the best fitting procedure, the correct bandhead level spin I_0 and also the lowest experimental transition energy $E_\gamma(I_0 + 2 \rightarrow I_0)$ (is listed in the first column) for all our selected SD bands. Using the parameters adopted in Table (1) the transition energies $E_\gamma(I)$, rotational frequencies $\hbar\omega$, the kinematic $J^{(1)}$ and dynamic $J^{(2)}$ moments of inertia are calculated. The calculated $E_\gamma(I)$ are consistent very well with the experimental ones. The systematic behaviour of $J^{(1)}$ and $J^{(2)}$ seems to be very useful to understand the properties of SD bands especially $J^{(2)}$. The moments of inertia $J^{(1)}$ and $J^{(2)}$ are plotted as a function of $\hbar\omega$ in Figures (1,2).

Table (1). The adopted best fit model parameters A, a, σ and the suggested bandhead level spin I_0 for our selected SD bands in $A \sim 190$ region. The experimental lowest transition energy $E_{\gamma(I_0+2 \rightarrow I_0)}$ for each SD band is also shown.

SD Bands	$E_{\gamma(I_0+2 \rightarrow I_0)}$ (keV)	A (keV)	a	σ	I_0
$^{191}\text{Hg}(SD2)$	252.4	4.03988	0.25367	1.23080×10^{-3}	10.5
$(SD3)$	272.0	3.89957	0.27986	1.31380×10^{-3}	10.5
$^{193}\text{Tl}(SD1)$	206.6	3.90015	0.27042	1.28503×10^{-3}	8.5
$(SD2)$	227.3	4.08216	0.23353	1.15934×10^{-3}	9.5
$^{193}\text{Pb}(SD3)$	251.5	4.20422	0.21786	1.09947×10^{-3}	10.5
$(SD4)$	273.0	3.86130	0.29044	1.34280×10^{-3}	11.5
$^{193}\text{Pb}(SD5)$	213.2	4.00128	0.26634	1.27229×10^{-3}	8.5
$(SD6)$	234.6	3.91432	0.28455	1.32664×10^{-3}	9.5
$^{195}\text{Pb}(SD3)$	198.2	3.89056	0.29353	1.35097×10^{-3}	7.5
$(SD4)$	213.5	3.88781	0.29497	1.35470×10^{-3}	8.5
$^{192}\text{Hg}(SD1)$	214.4	3.77663	0.33938	1.44435×10^{-3}	8
$^{192}\text{Pb}(SD1)$	214.8	3.47910	0.39877	1.45584×10^{-3}	8
$^{194}\text{Pb}(SD1)$	124.9	3.83197	0.32848	1.42740×10^{-3}	4
$^{196}\text{Pb}(SD1)$	171.4	3.93700	0.32448	1.42028×10^{-3}	6

The experimental data are taken from Ref. [4]. From the figures we notice a gradual tendency of increasing of $J^{(1)}$ and $J^{(2)}$ versus variation of $\hbar\omega$ can be reproduced in all studied SD bands. This rise is due to the alignment of angular momentum of a pair of nucleons occupied high intruder orbital and from the gradual disappearance of pairing correlations with increasing $\hbar\omega$. The kinematic moment of inertia $J^{(1)}$ is found to be smaller than that of the dynamic moment of inertia $J^{(2)}$. The $J^{(2)}$ moment of inertia for ^{194}Pb and ^{192}Hg are very similar and higher than for ^{196}Pb for similar frequencies. We also notice that the calculated $J^{(2)}$ in bands 2,3 of ^{191}Hg are similar to that of the lowest SD band in ^{192}Hg since all these bands have the same content in intruder orbitals, four $N=6$ ($i_{13/2}$) protons and four $N=7$ ($j_{15/2}$) neutrons (the average value of $J^{(2)}$ for $^{191}\text{Hg}(SD2,SD3)$ are $110 \hbar^2 \text{MeV}^{-1}$, $113 \hbar^2 \text{MeV}^{-1}$ respectively are close to the value reported for $^{192}\text{Hg}(SD1)$ $113 \hbar^2 \text{MeV}^{-1}$.

At low frequencies, the signature partner SD bands in $^{193}\text{Tl}(SD1,SD2)$ have higher $J^{(2)}$ values compared to the isotones $N = 112$ $^{192}\text{Hg}(SD1)$ and $^{194}\text{Pb}(SD1)$ due to the Pauli blocking effect that reduces superfluidity and hence increases $J^{(2)}$ for higher $\hbar\omega$, $J^{(2)}$ exhibit in $^{193}\text{Tl}(SD1,SD2)$ saturation and then a smooth turnover of the $J^{(2)}$ can be seen around $\hbar\omega = 0.36 \text{ MeV}$, reflecting the combined effects of the proton pairing blocking and the complete $j_{15/2}$ neutron alignment. The yrast SD bands in the two isotones ($N = 112$) ^{192}Hg and ^{194}Pb are populated at lower spins $I_0 = 8\hbar$ and their calculated gamma-ray transition energies are very close which reproduce a pair of identical bands. This similarity suggests that the added two protons in ^{194}Pb do not change the SD rotational properties in the observed frequency range.

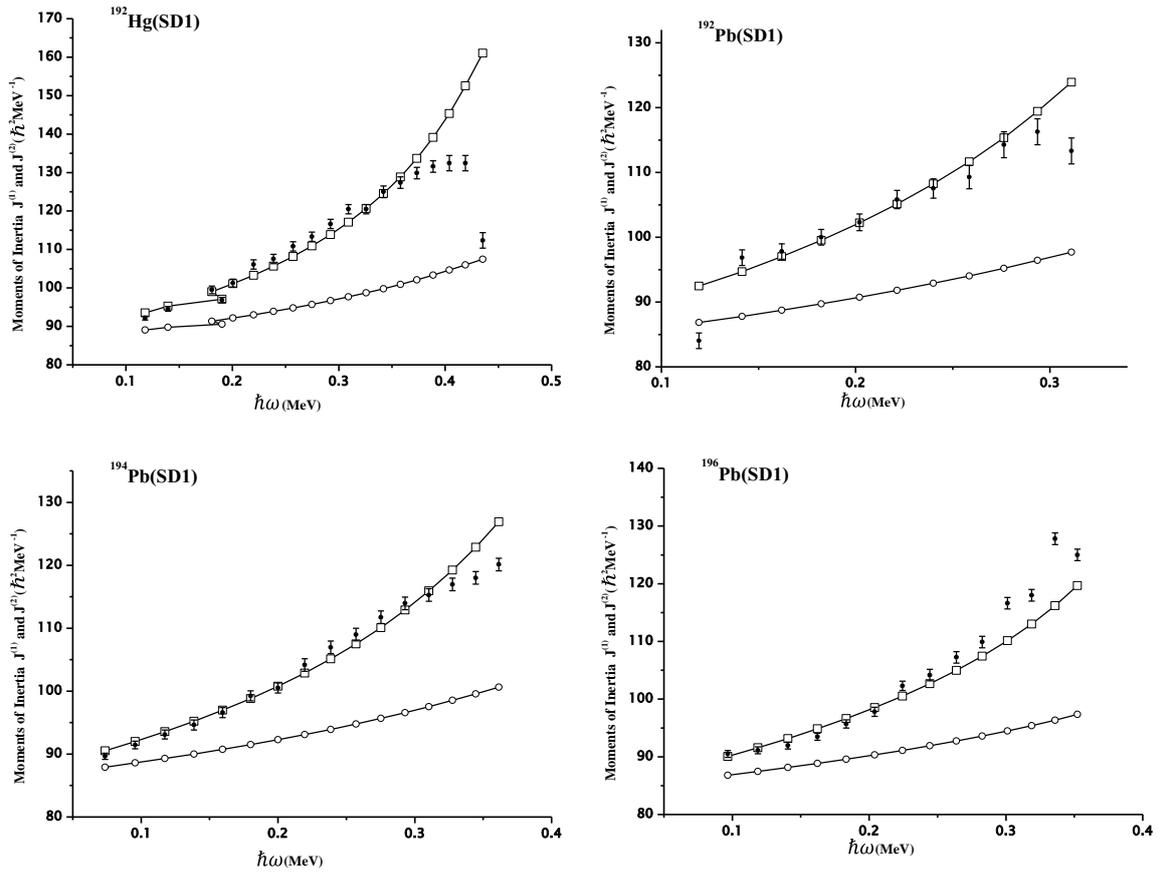
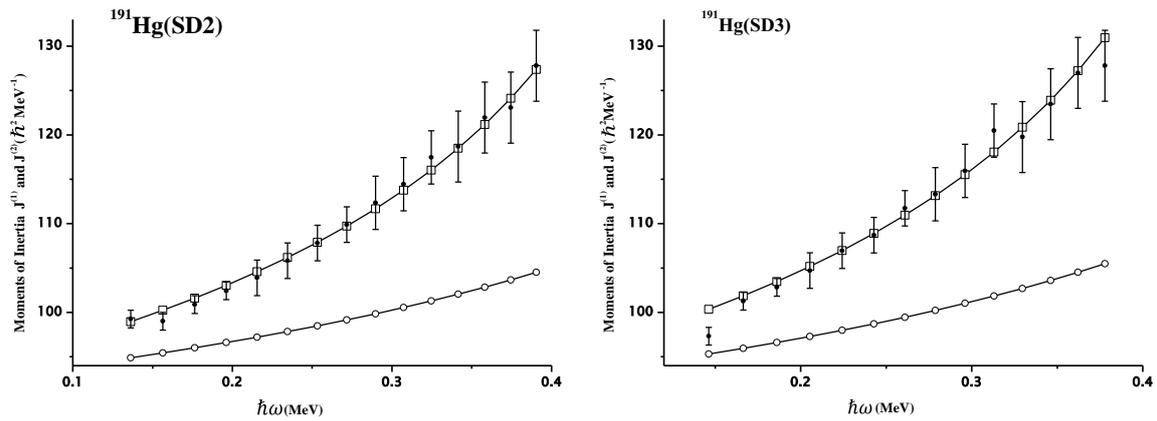
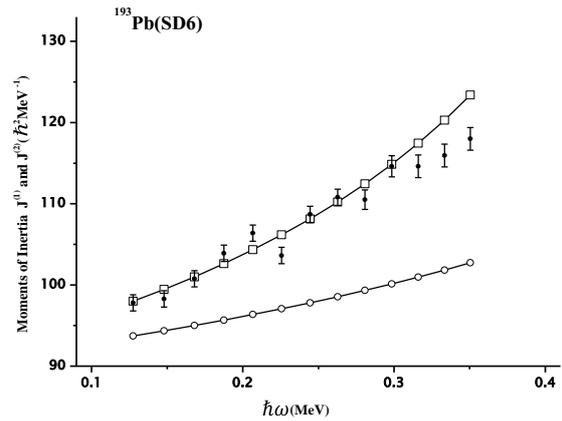
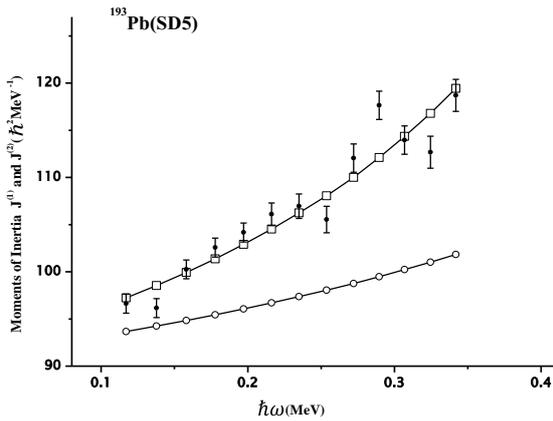
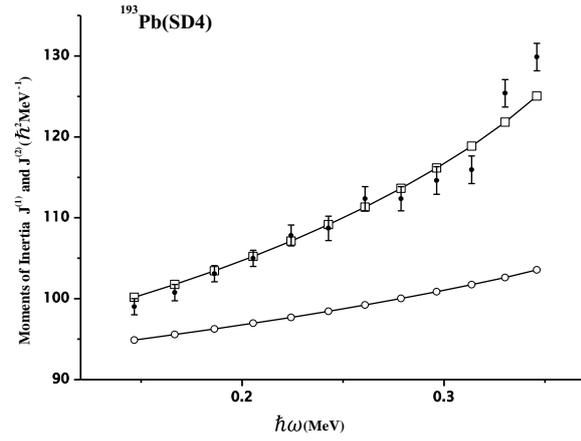
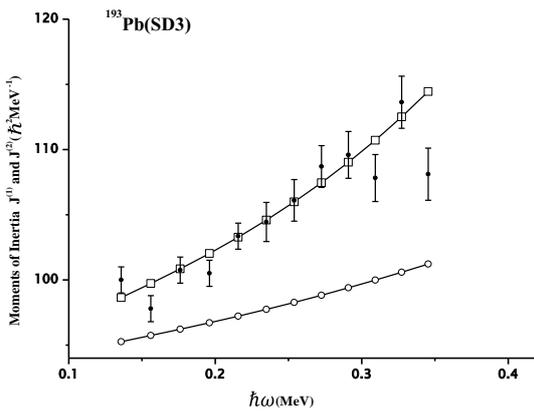
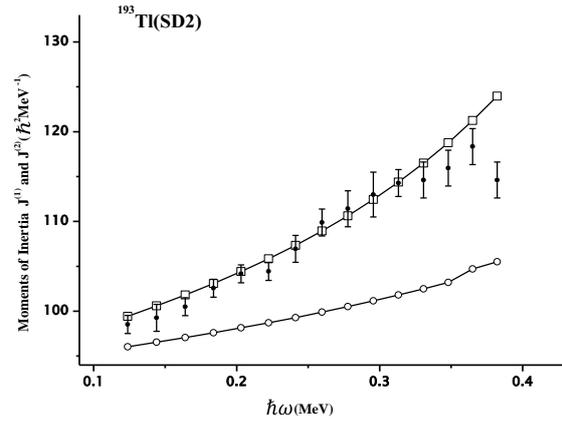
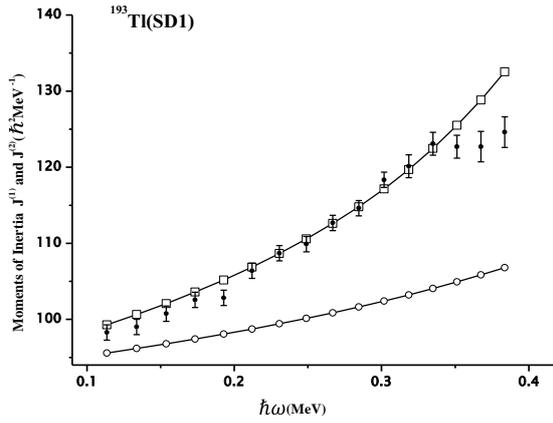


Fig. (1) The Calculated results of kinematics $J^{(1)}$ (open circles) and the dynamic $J^{(2)}$ (open squares) moments of inertia as a function of rotational frequency $\hbar\omega$ for our SD bands and comparison with experimental $J^{(2)}$ (closed circles with error bars). The experimental data are taken from Ref.[4].





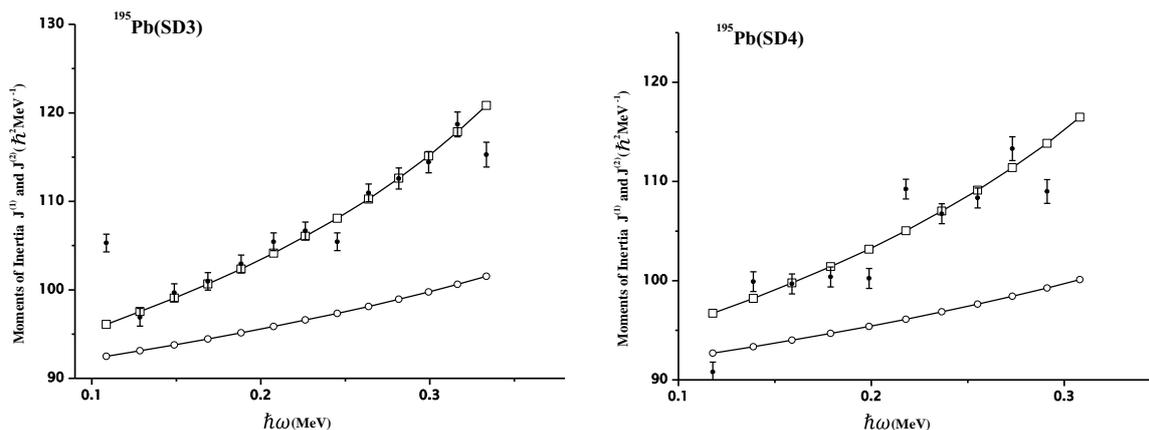


Fig. (2) The same as in Figure (1) but for theselectedsignature partners SD bands.

The experimental data are taken from Ref.[4]. From the figures we notice a gradual tendency of increasing of $J^{(1)}$ and $J^{(2)}$ versus variation of $\hbar\omega$ can be reproduced in all studied SD bands. This rise is due to the alignment of angular momentum of a pair of nucleons occupied high intruder orbital and from the gradual disappearance of pairing correlations with increasing $\hbar\omega$. The kinematic moment of inertia $J^{(1)}$ is found to be smaller than that of the dynamic moment of inertia $J^{(2)}$. The $J^{(2)}$ moment of inertia for ^{194}Pb and ^{192}Hg are very similar and higher than for ^{196}Pb for similar frequencies. We also notice that the calculated $J^{(2)}$ in bands 2,3 of ^{191}Hg are similar to that of the lowest SD band in ^{192}Hg since all these bands have the same content in intruder orbitals, four $N=6$ ($i_{13/2}$) protons and four $N=7$ ($j_{15/2}$) neutrons (the average value of $J^{(2)}$ for $^{191}\text{Hg}(SD2,SD3)$ are $110 \hbar^2\text{MeV}^{-1}$, $113\hbar^2\text{MeV}^{-1}$ respectively are close to the value reported for $^{192}\text{Hg}(SD1)$ $113 \hbar^2\text{MeV}^{-1}$. At low frequencies, the signature partner SD bands in $^{193}\text{Tl}(SD1,SD2)$ have higher $J^{(2)}$ values compared to the isotones $N = 112$ $^{192}\text{Hg}(SD1)$ and $^{194}\text{Pb}(SD1)$ due to the Pauli blocking effect that reduces superfluidity and hence increases $J^{(2)}$ for higher $\hbar\omega$, $J^{(2)}$ exhibit in $^{193}\text{Tl}(SD1,SD2)$ saturation and then a smooth turnover of the $J^{(2)}$ can be seen around $\hbar\omega = 0.36 \text{ MeV}$, reflecting the combined effects of the proton pairing blocking and the complete $j_{15/2}$ neutron alignment. The yrast SD bands in the two isotones ($N = 112$) ^{192}Hg and ^{194}Pb are populated at lower spins $I_0 = 8\hbar$ and their calculated gamma-ray transition energies are very close which reproduce a pair of identical bands. This similarity suggests that the added two protons in ^{194}Pb do not change the SD rotational properties in the observed frequency range. That is the high N intruder configuration ($\pi 6^4, 7^4$) determine the properties of the SD core ^{192}Hg also is in ^{194}Pb . The differences in gamma-ray energies ΔE_γ between transitions in the two isotones are listed in Table (2) as a function of rotational frequency $\hbar\omega$.

Up to $\hbar\omega \sim 0.30 \text{ MeV}$, the ΔE_γ values are small and constant (0.5keV). However, they diverge for higher $\hbar\omega$. Also the behaviour of the identical bands $^{194}\text{Hg}(SD1)$ and $^{194}\text{Pb}(SD1)$ can be seen in the plot of dynamical moment of inertia $J^{(2)}$ against rotational frequency $\hbar\omega$. A very similar $J^{(2)}$ is observed for the two SD bands (see Figure(1)).

The ^{192}Hg is a doubly magic SD nucleus due to the presence of a large shell gap in the Woods – Saxon single particle diagram at deformation in the range $\beta_2 = 0.45 - 0.55$ for proton number $Z = 80$ and neutron number $N = 112$. So that to compare SD bands in $A \sim 190$ region, we take ^{192}Hg as a reference SD band.

The incremental alignment Δi is plotted in Figure (3) relative to the yrast band in ^{192}Hg against rotational frequency $\hbar\omega$ for $^{194}\text{Pb}(SD1)$ and the two signature partner pairs $^{191}\text{Hg}(SD2,SD3)$ and $^{193}\text{Pb}(SD3,SD4)$.

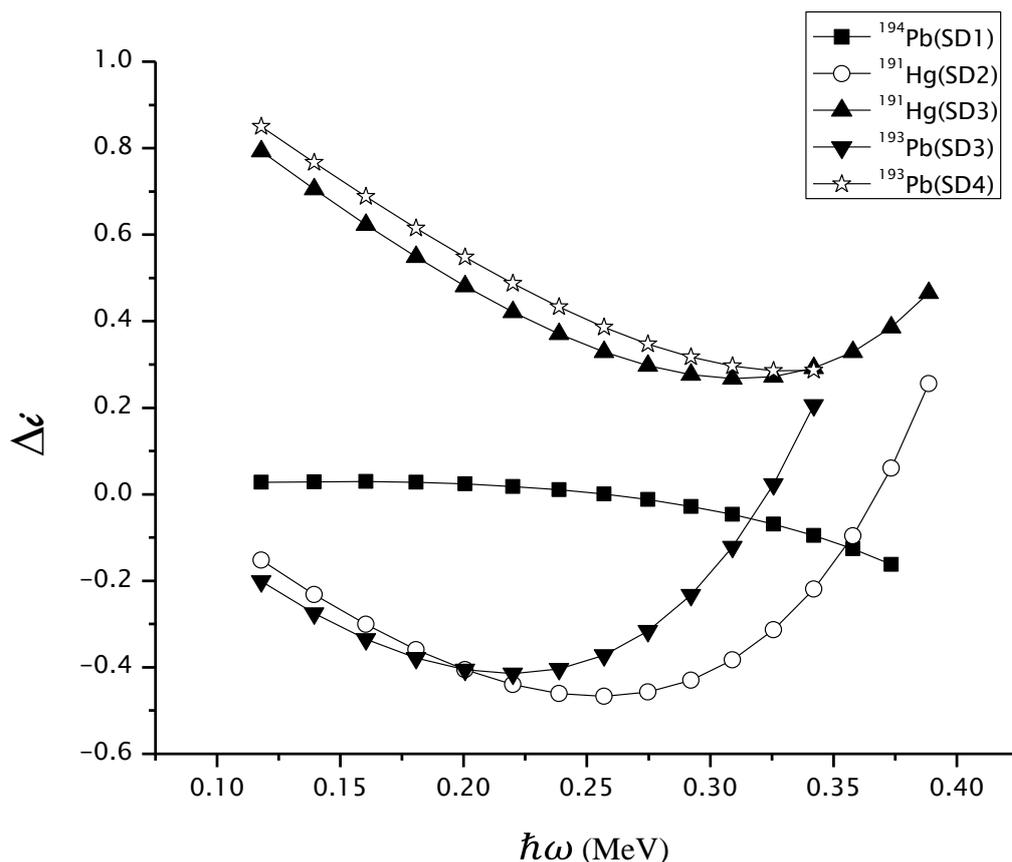


Fig. (3) incremental alignment Δi as a function of rotational frequency $\hbar\omega$ for $^{194}\text{Pb}(\text{SD1})$ and the two signature partner pairs $^{191}\text{Hg}(\text{SD2}, \text{SD3})$, $^{193}\text{Pb}(\text{SD3}, \text{SD4})$ with $^{192}\text{Hg}(\text{SD1})$ as a reference.

In order to investigate $\Delta I = 1$ staggering in our signature partner pairs $^{191}\text{Hg}(\text{SD2}, \text{SD3})$, $^{193}\text{Tl}(\text{SD1}, \text{SD2})$, $^{193}\text{Pb}(\text{SD3}, \text{SD4})$, $^{193}\text{Pb}(\text{SD5}, \text{SD6})$, $^{195}\text{Pb}(\text{SD3}, \text{SD4})$ the staggering parameters $\Delta^2 E_\gamma(I - 1)$ equation is determined for each signature partner pair and plotted as a function of spin I in Figure (4). Most of these signature partners show large amplitude staggering. In ^{191}Hg , band 2 and band 3 have been interpreted as signature partners built on the $3/2 + [642]$ orbital. The signature partner pair $^{195}\text{Pb}(\text{SD3}, \text{SD4})$ shows little signature splitting. It is interesting to note that the bandhead moments of inertia of each signature partner SD band are almost identical.

Table(2). Properties of the two identical bands $^{192}\text{Hg}(\text{SD1})$ and $^{194}\text{Pb}(\text{SD1})$. Calculated rotational frequencies $\hbar\omega$, transition energies E_γ and the differences in E_γ between the two isotones.

$\hbar\omega$ (MeV)	$E_\gamma(\text{keV})$		$J^{(2)}(\hbar^2\text{MeV}^{-1})$		ΔE_γ (keV)
	$^{192}\text{Hg}(\text{SD1})$	$^{194}\text{Pb}(\text{SD1})$	$^{192}\text{Hg}(\text{SD1})$	$^{194}\text{Pb}(\text{SD1})$	
0.1180	213.379	212.792	93.519	93.571	0.587
0.1394	256.151	255.540	95.247	95.222	0.611
0.1603	298.147	297.547	97.080	96.972	0.600
0.1807	339.350	338.796	99.022	98.819	0.554
0.2006	379.745	379.274	101.089	100.786	0.471
0.2199	419.314	418.962	103.289	102.878	0.352
0.2377	458.040	457.843	105.649	105.113	0.197
0.2570	495.901	495.897	108.178	107.512	0.004
0.2748	532.877	533.102	110.913	110.092	-0.225
0.2922	568.941	569.435	113.872	112.885	-0.494
0.3091	604.068	604.863	117.092	115.918	-0.795
0.3257	638.229	639.376	120.627	119.239	-1.147
0.3420	671.389	672.922	124.513	122.872	-1.533
0.3579	703.514	705.476	128.832	126.915	-1.962
0.3733	734.562	736.993	-	-	-2.431

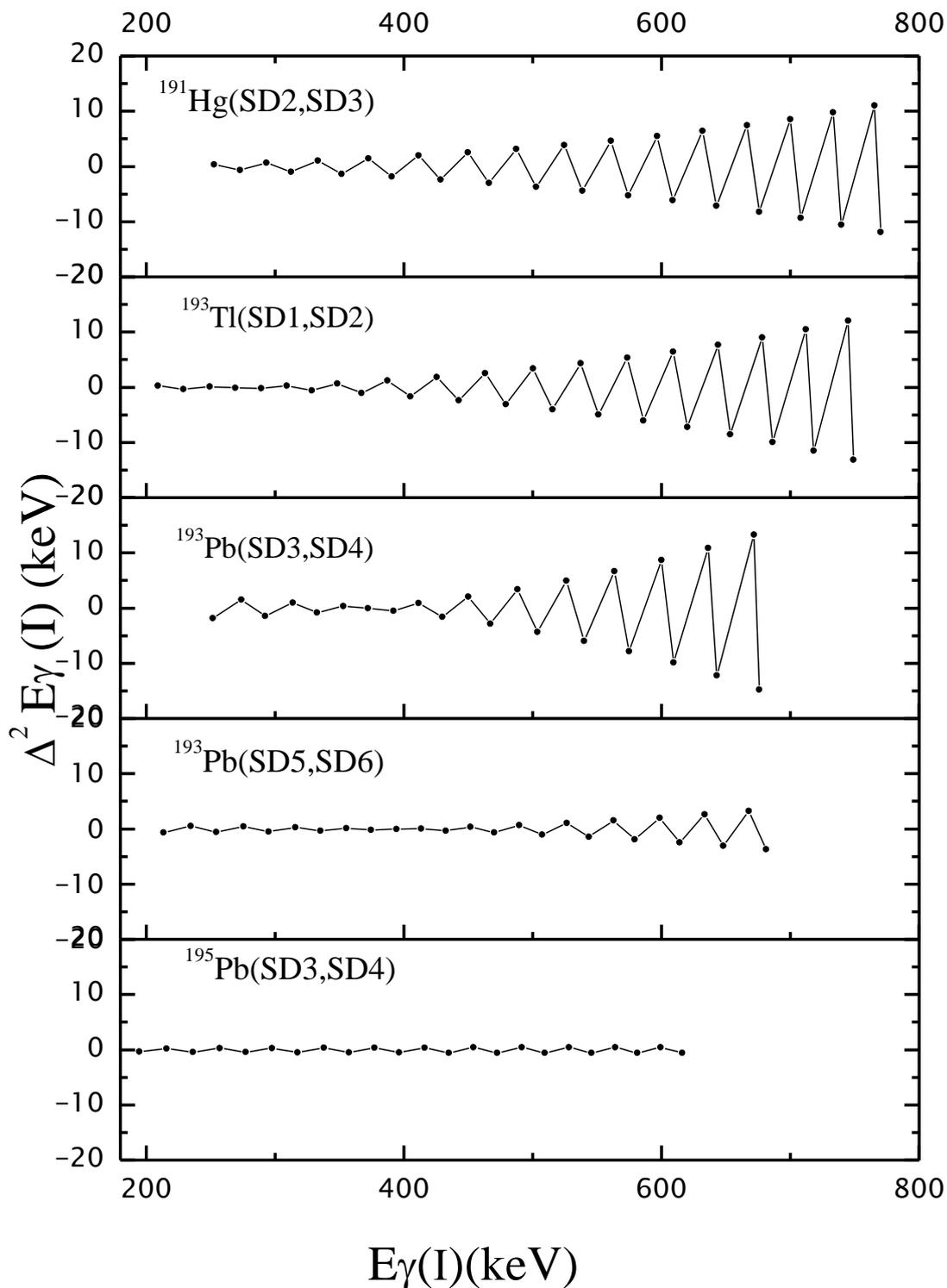


Fig.(4) The Calculated $\Delta I=1$ Staggering Parameter $\Delta^2 E_\gamma$ versus E_γ for a signature partner five pairs of odd-A SD nuclei in the mass region $A \sim 190$.

VII. Conclusion

In this paper the yrast SD bands in even-even nuclei ^{192}Hg and $^{192,194,196}\text{Pb}$ and the signature partners in odd-A nuclei $^{191}\text{Hg}(SD2, SD3)$, $^{193}\text{Tl}(SD1, SD2)$, $^{193}\text{Pb}(SD3, SD4)$, $^{193}\text{Pb}(SD5, SD6)$, $^{196}\text{Pb}(SD1, SD2)$ have been studied in framework of proposed three parameter nuclear softness model. A weakened parameter to reduce the pairing is introduced for the first time. The bandhead spin I_0 has been extracted for each SD band by assuming various values nearest to integer in even-even nuclei or half integer in odd-A nuclei, then a fitting procedure has been done to adopt I_0 and adopt also the model parameters to make the root mean square deviation between calculated transition energies and the measured ones minimum. The calculated results agree with experimental data very well. The calculated transition energies E_γ , level spins I , rotational frequencies $\hbar\omega$, kinematic $J^{(1)}$ and dynamic $J^{(2)}$ moments of inertia are examined. $J^{(2)}$ is found to increase steadily with $\hbar\omega$ this rise can be ascribed to the gradual alignments of a pair of nucleons occupying high N intruder orbitals. The identical bands in ^{192}Hg , ^{194}Pb are investigated, the difference between the calculated relative transition energies for the yrast bands in the two nuclei are within 1-2keV. The similarities can be understood in terms of intruder orbitals. The bandhead moments of inertia for our signature partner pairs in odd SD bands are identical. To investigate the $\Delta I=1$ staggering, we extracted the difference between the averaged transitions $I+2 \rightarrow I$ and $I \rightarrow I-2$ energies in one band and the transition $I+1 \rightarrow I-1$ and energies in the signature partner.

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