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Quantum Radioactive Decay Law and Relaxation Time

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Abstract: *Relaxation time plays an important role in nuclear excitation .This is because excitation energy is related to the relaxation time according to the uncertainty principle. This relation between relaxation time and energy excitation was deduced from classical and quantum laws of harmonic oscillator. It resembles that of uncertainty principle , except that existence of imaginary term which is related to the energy lost by excitation and friction .This relation is used to find the wave function for frictional media .This wave function is used to derive radioactive decay law .*
Key wards:*Radioactive Decay Law, Quantum Tunneling, Frictional Force, Harmonic Oscillator , Relaxation Time , Nuclear Excitation ,quantum Tunnelling.*

I. Introduction

The basic building block of matter is the atom .Eachatom consists of protons, neutrons, and electrons.

The central portion of the atom is nucleus which it consist of protons and neutrons. Electrons orbit the nucleus. If the nucleus consists of excess mass or excess energy, it gets rid of them by emitting α or γ radiation.There are three radiation types , alpha(α) , Beta (β) and gamma radiation (γ) .[1,2]

Alpha (α) radiation is the Helium nucleus, Betaradiation is the electron or positron, while gamma (γ) radiation is a stream of photons.

When the nucleus has excess mass it get rid of it by emitting α radiation.

If the number of neutrons is not equal to the number of protons, the nucleus will not be stable. The unstable atom will try to become stable by converting the excess neutrons to protons or protons to neutrons respectively. This is done by emitting β radiation. The materials that emit radiation are called radioactive materials. A nucleus that have excess energy emits γ radiation.[3]

Radioactivity is widely used in medicine in curing some diseases like cancer, beside applications in diagnosis. It is also used in non-distractive testing and mineral exploration.[4,5,6]

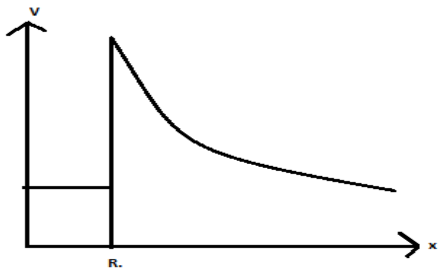
Radioactive nuclei are converted to other nuclei by a rate described by radioactive decay law. The radioactive decay law was explained quantum mechanically by tunneling effect .But. so far no new quantum model are widely known.[7,8]

The attempts made are few and no one of them utilizes harmonic oscillator model .

The aim of this work is to use harmonic oscillator model and relaxation time to derive radioactive decay law .This is done in section 3 .section 2 is devoted with the convention model . Section 4and 5 are devoted for discussion and conclusion.

II. Nuclear Quantum Tunnelling

The radioactive decay of nuclear particles can be considered as resulting from tunneling of them through the finite potential barrier.



Schrodinger equation takes the form

$$\frac{\partial^2\Psi}{\partial^2 x}+\frac{2m}{\hbar^2}(E-V)\Psi=0$$

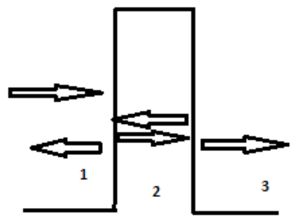
$$\frac{\partial^2\Psi}{\partial^2 x}+k^2\Psi=0 \qquad (2.1)$$

Where :

$$k^2=\frac{2m}{\hbar^2}(E-V) \qquad (2.2)$$

Consider the barrier as shown in fig (2). In the first and third region

$$V=0$$



And sch.eqn. becomes

$$\frac{\partial^2\Psi_1}{\partial^2 x}+k_0\Psi_1=0 \qquad (2.3)$$

$$k_0^2=\frac{2m}{\hbar^2}E_0 \qquad (2.4)$$

$$\frac{\partial^2\Psi_3}{\partial^2 x}+k_0\Psi_3=0 \qquad (2.5)$$

$$E_0=T_0=\text{Kinetic energy} \qquad (2.6)$$

Where Ψ_1 represents incident and reflected waves. Hence

$$\Psi_1=Ae^{ik_0x}+Be^{-ik_0x} \qquad (2.7)$$

While

Where Ψ_3 represents transmitted wave , thus

$$\Psi_3=Ge^{ik_0x} \qquad (2.8)$$

For particles inside the barrier

$$k_2^2=\frac{2m}{\hbar^2}(T-V)$$

$$k_b=-ik_2$$

The wave function becomes

$$\Psi_1=Ce^{-k_bx}+De^{k_bx} (2.9)$$

The boundary conditions at

$$x=0 \text{ and } x=L$$

requires

$$x=0$$

$$\Psi_1=\Psi_2$$

$$\Psi_1'=\Psi_2'$$

at

$$x=L$$

$$\Psi_2=\Psi_3$$

$$\Psi_2'=\Psi_3'$$

Which finally gives transmission probability P given by

$$P=\frac{|E|^2}{|A|^2}=e^{-2k_bL}$$

The decay constantλ is found to be equal

$$\lambda=fP=\frac{v}{2R}P$$

$$\mathbf{ln}\,\lambda=\ln\left(\frac{v}{2R_0}\right)+2.97Z^{1/2}R_0^{1/2}-3.95T^{-1/2} \qquad (2.10)$$

Where

R_0, Z and T are the nuclear radius , atomic number and kinetic energy of alpha particle respectively.

III. Relaxation time and friction

For any particle having mass m and velocity v the force Fexerted on it can be describe by the equation :

$$m\frac{dv}{dt} = F \qquad (3.1)$$

considering the particle as harmonic oscillator the velocity v is given by

$$v = v_0 e^{iw_0 t} \qquad (3.2)$$

Where

w_0 is the angular frequency

v_0 is the maximum velocity

Sub equation(3.2) in (3.1) yields

$$iw_0 v_0 = F \qquad (3.3)$$

If the particle moves in a resistive medium of coefficient γ the equation of motion becomes

$$m\frac{dv}{dt} = F - \gamma mv \ (3.4)$$

Assuming that the frictional force affect the frequency only ,one can assume

$$v = v_0 e^{iwt} \qquad (3.5)$$

This is obvious if we treat the particle as a harmonic oscillator , where :

$$E_0 = \hbar w_0, E = \hbar w \qquad (3.6)$$

Sub equation(2-1- 3) in equation(2-1-4)yields :

$$imwv_0 = imw_0 v_0 - \gamma m v_0$$

Cancelling similar terms and multiplying both sides by i yields

$$w - w_0 = i\gamma = \frac{i}{\tau} (3.7)$$

Thus the energy loss is given by

$$\Delta E = \hbar w_0 - \hbar w = -i\gamma\hbar = \frac{-i\hbar}{\tau} \qquad (3.8)$$

Thus the energy of the system is :

$$E = E_0 + \Delta E = E_0 - i\gamma \qquad (3.9)$$

Thus the wave equation can be written

$$\Psi = Ae^{\frac{i}{\hbar}[Px - (E_0 - i\hbar\gamma)t]}$$

$$= Ae^{\frac{i}{\hbar}(Px - E_0 t)} e^{-\frac{\gamma\hbar t}{\hbar}} \qquad (3.10)$$

$$\Psi = Ae^{-\gamma t} e^{\frac{i}{\hbar}(Px - E_0 t)} \qquad (3.11)$$

It is very striking to observe that the imaginary friction term in equations (3.7) and (3.8) appears in equation (3.11) to make the amplitude of Ψ decay with time.

Therefore the average energy $\overline{E}$ which is equal total classical value,i.e.

$$\overline{E} = \int \overline{\Psi}\, E\Psi dt = Ee^{-2\gamma t} \qquad (3.12)$$

Indicates that the energy decays with time .This agrees with the fact that friction causes particle energy to decrease

The relation time from uncertainty principle is given by

$$\Delta E \Delta t = \hbar (3.13)$$

With

$$\Delta E = \frac{\hbar}{\Delta t} = \frac{\hbar}{\tau} (3.14)$$

Where

$$\Delta t = \tau$$

It's very interesting to note that equations (3.8) and (3.14) give the same numerical values. But the expression (3.8) is more convenient, as far as it is insertion in the wave function predicts time decaying energy .This means that treating particles as harmonic oscillators gives quantum results similar to the classical one.

A simple expression for radioactive decay law can be obtained by using equation (3.11) . The number of particles is given by

$$N = \Psi\overline{\Psi} = Ae^{-2\gamma t} \qquad (3.15)$$

Since at

$$t = 0\, , N = N_0$$

Thus equation (3.15) gives

$$N_0 = A$$

Therefore equation (3.15) becomes

$$N = N_0 e^{-2\gamma t} \qquad (3.16)$$

By setting

$$2\gamma = \lambda$$

$$N = N_0 e^{-\lambda t} \qquad (3.17)$$

Which is the ordinary radioactive decay law
The radiation emitted by unstable nuclei is due to the fact that these nuclei are in an excited state .This can be shown with the aid of equations (3.8),(3.14)and (3.16), where

$$\Delta E = \frac{\hbar}{\tau} = \gamma\hbar \qquad (3.18)$$

Thus

$$N = N_0 e^{-2\frac{\Delta E}{\hbar}t} \qquad (3.19)$$

This shown that nuclear decay is due to nuclear excitation . i.e. the existence of the nucleus in an excited state.

IV. Discussion

According to the classical harmonic oscillator model, equation (3.8) shows that the energy loss due to frictional force is shown to be related to the reciprocal of the relaxation time . The relaxation time here measures the delay in particle motion .It is very striking to find that typical expression for energy loss by excited particle is obtained by using quantum uncertainty principle according to equation (3.14) .Here again τ represents time taken by a particle in an excited state . If a photon is absorbed by a particle it become excited for τ seconds , then it return back to the ground state after re-emitting a photon with time delay τseconds.

However the imaginary term in the classical expression (3.8) make it give a direct physical meaning of the role of friction in causing energy losses according to equation (3 .12).It is also very interesting to note that the wave function resulting from the energy expression for resistive media in equation (3.11) can be utilized to derive a simple expression for the radioactive decay law.

Fortunately this mew expression shows that decay of particles resultsfrom nuclear excitation .This is since the original energy does not appear, while excitation energy appears in decay expression as shown by equation (3.19).

V. Conclusion :

The harmonic oscillator model which is related to the string theory appears to be successful in describing the interaction of particles with bulk matter. The classical and quantum expression for the energy lost due to this interaction are numerically the same.This model succeeded in deriving very simple direct radioactive decay law.

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