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Representation of d'Alembertian Operator in Quantum Mechanics

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Abstract: *The quantum mechanical formulation of d'Alembertian operator $\square^2$ will be drawn in this paper.*
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I. Introduction

In classical mechanics, the d'Alembertian operator $\square^2=\Delta^2-1/c^2.\delta^2/\delta t^2$; where $\Delta^2= \delta^2/\delta x^2+ \delta^2/\delta y^2+ \delta^2/\delta z^2$. Now what will be the form of $\square^2$ in quantum representation?

II. d'Alembertian Operator

In quantum mechanics, the momentum operator $p=\hbar/i.\Delta$ and the energy operator $E=i\hbar.\delta/\delta t$.
Hence $p^2= \hbar^2/i^2.\Delta^2=-\hbar^2\Delta^2$ and $E^2=i^2\hbar^2.\delta^2/\delta t^2=- \hbar^2.\delta^2/\delta t^2$
Now $\Delta^2=-p^2/\hbar^2$ and $\delta^2/\delta t^2=-E^2/\hbar^2$
Thus $\Delta^2-1/c^2.\delta^2/\delta t^2= E^2/c^2\hbar^2-p^2/\hbar^2$
Therefore $\square^2=(E^2/c^2-p^2)/\hbar^2$

III. Energy and Momentum

It is known that $E=p^2/2m+V(r)$.
As $p=\hbar/i.\Delta$; so $E^2=\{-\hbar^2/2m.\Delta^2+V(r)\}^2$
$=\hbar^4/4m^2.\Delta^4-2.\ \hbar^2/2m.\Delta^2.V(r)+V^2(r)$
Hence $E^2/c^2= \hbar^4/4m^2c^2.\Delta^4-\hbar^2/mc^2.V(r).\ \Delta^2+V^2(r)/c^2$
Thus $E^2/c^2-p^2= \hbar^4/4m^2c^2.\Delta^4-\hbar^2/mc^2.V(r).\ \Delta^2+V^2(r)/c^2+\hbar^2\Delta^2$
$= \hbar^4/4m^2c^2.\Delta^4-\hbar^2\Delta^2\{V(r)/mc^2-1\}+V^2(r)/c^2$
Therefore $(E^2/c^2-p^2)/\hbar^2= \hbar^2/4m^2c^2.\Delta^4- \{V(r)/mc^2-1\}\ \Delta^2+V^2(r)/\hbar^2c^2$

IV. Wave Equation

The equation of motion of wave propagation is $\Delta^2\psi(r,t)=1/u^2.\delta^2/\delta t^2\ .\psi(r,t)$.
If u=c, then $\{\ \Delta^2-1/c^2.\delta^2/\delta t^2\ \}\psi(r,t)=0$.
Again from the above discussion, $(E^2/c^2-p^2)/\hbar^2= \square^2= \hbar^2/4m^2c^2.\Delta^4- \{V(r)/mc^2-1\}\ \Delta^2+V^2(r)/\hbar^2c^2$.
Therefore $\square^2\psi(r,t)= [\hbar^2/4m^2c^2.\Delta^4- \{V(r)/mc^2-1\}\ \Delta^2+V^2(r)/\hbar^2c^2]\psi(r,t)=0$.

V. Conclusion

Finally the d'Alembertian operator takes the form in quantum representation as
$\square^2= \hbar^2/4m^2c^2.\Delta^4- \{V(r)/mc^2-1\}\ \Delta^2+V^2(r)/\hbar^2c^2$.
As well as in this respect, the wave equation in quantum mechanical interpretation is
$[\hbar^2/4m^2c^2.\Delta^4- \{V(r)/mc^2-1\}\ \Delta^2+V^2(r)/\hbar^2c^2]\psi(r,t)=0$.

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