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Calculation the Energy Levels and Energy Bands and Energy Ratios for even-even$^{154, 156,158}$Dy Isotopes

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Summary: *Interacting boson model (IBM-1) was used in the present work to study some of nuclear structures for selected Dysprosium isotope of even mass number Dy(A=154-158). The energy levels and energy ratios of these isotopes were investigated for there are experimental. The calculated results were compared with the available experimental data, the results were in general good agreement.*

I. Introduction

The Interacting Boson Model (IBM), initially introduced by Arima and Iachello has been rather successful in describing the collective properties of several medium and heavy nuclei .We note that in the interacting boson model-1 (IBM-1)one describes an even-even nucleus as a system of N bosons able to occupy two levels, one with angular momentum re- stricted to zero (s boson) and one with angular momentum 2 (d boson). (1)

II. The interacting Boson model

In the simplest version of the interacting boson model (IBM-1), its assumed that low-lying collective states in even-even nuclei away from closed shells are dominated by excitation of the valence protons and the valence neutrons (particles outside the major closed shell) while the closed shell core is inert. Furthermore, its assumed that the particle configurations which are most important in shaping the properties of the low-lying states are these in which identical particles are coupled together forming pairs of angular momentum 0 and 2.(1)

The interacting Boson model (IBM-1) is used in the present work, this model represents very important step formed in the description of collective nuclear excitations. The underlying U(6) group structure of model basis leads to a simple Hamiltonian which is capable of describing the three specific limits of collective structure vibrational U(5), rotational SU(3) and gamma unstable O(6). (2)

The interacting boson model offers a simple Hamiltonian, capable of describing collective nuclear properties across a wide range of nuclei, based on general algebraic group theoretical techniques which have also recently found application in problems in atomic, molecular, and high- energy physics (3,4).

III. Theoretical Basis:

Hamiltonian operator function according to IBM-1 is written in terms of creation and annihilation operators as follows ;(5,6)

$$\hat{H} = \in \hat{n}_d + a_0(\hat{p}.\hat{p}) + a_1(\hat{I}.\hat{I}) + a_2(\hat{Q}.\hat{Q}) + a_3(\hat{T}_3.\hat{T}_3) + a_4(\hat{T}_4.\hat{T}_4) \quad \text{-----1}$$

Where E=Ed - Es is the boson energy.

Where ε, a_0, a_1, a_2, a_3 and a_4 are parameters used in IBM-1 to determine the Hamiltonian function .

The operators :

$\hat{n}_d = (\hat{d}^\dagger.\tilde{\hat{d}})$ the boson number operator

$\hat{p} = \frac{1}{2}(\tilde{\hat{d}}.\tilde{\hat{d}}) - \frac{1}{2}(\tilde{\hat{s}}.\tilde{\hat{s}})$ the pairing bosons operator

$\hat{I} = \sqrt{10}\left[\hat{d}^\dagger.\tilde{\hat{d}}\right]^{(1)}$ the angular momentum operator --------2

$\hat{Q} = \left[\hat{d}^\dagger \times \tilde{\hat{s}}) + (\hat{s}^\dagger \times \tilde{\hat{d}})\right]^{(2)} - \frac{1}{2}\sqrt{7}\left[\hat{d}^\dagger \times \tilde{\hat{d}}\right]$ the quadrupole operator

$\hat{T}_3 = \left[\hat{d}^{\dagger} \times \tilde{\hat{d}}\right]^{(3)}$ the octupole operator

$\hat{T}_4 = \left[\hat{d}^{\dagger} \times \tilde{\hat{d}}\right]^{(4)}$ the hexadecaploe operator

and a_0 ,a_1,a_2,a_3,a_4 are the phenomenological parameter .

3.1.1 Rotational Limit SU(3):

The electromagnetic transition rates, B(E2) values of this chain and the quadrupole moments (Q_I) are described by [6];

$$B(E2; I+2 \rightarrow I) = \alpha_2^2 \left[\frac{I+2}{I}\right]\left[\frac{2N-I}{2}\right] \quad \text{----- (3)}$$

$$Q_I = \beta_2 \left[\left(\frac{16\pi}{70}\right)^{\frac{1}{2}} I\right]$$

In particular, for I=0, or 2

$$B(E2; 2_1^+ - 0_1^+) = \alpha_2^2 N \quad \text{-----(4)}$$

$$Q_{2_1^+} = \beta_2 \left[\frac{32\pi}{35}\right]^{1/2}$$

The basic condition for the observation of a SU(5) symmetry in the electromagnetic transition is[7]:

$$\frac{B(E2;4_1^+ - 2_1^+)}{B(E2;2_1^+ - 0_1^+)} = \frac{B(E2;2_2^+ - 2_1^+)}{B(E2;2_1^+ - 0_1^+)} = \frac{B(E2;0_2^+ - 2_1^+)}{B(E2;2_1^+ - 0_1^+)} = 2\left[\frac{(N-1)}{N}\right] < 2 \quad \text{--(5)}$$

Where the necessary conditions for the observation of the SU(3) symmetry are[8];

$$\frac{B(E2;4_1^+ \rightarrow 2_1^+)}{B(E2;2_1^+ \rightarrow 0_1^+)} = \frac{10(N-1)(2N+5)}{7 \quad N(2N+3)} \langle \frac{10}{7} \quad \text{----- (6)}$$

$$\frac{B(E2;2_2^+ \rightarrow 2_1^+)}{B(E2;2_1^+ \rightarrow 0_1^+)} = \frac{B(E2;0_2^+ \rightarrow 2_1^+)}{B(E2;2_1^+ \rightarrow 0_1^+)} = 0 \quad \text{------ (7)}$$

3.1.2.The Vibrational SU(5) Limit)

The electromagnetic transition rates, B(E2) values of this chain and the quadrupole moments (Q_I) are described by (8).

$$B(E2; I+2 \rightarrow I) = \alpha_2^2 \left[\frac{I+2}{I}\right]\left[\frac{2N-I}{2}\right] \quad \text{----- (8)}$$

$$Q_I = \beta_2 \left[\left(\frac{16\pi}{70}\right)^{\frac{1}{2}} I\right] \quad \text{----- (9)}$$

In particular, for I=0, or 2

$$\mathbf{B(E2;} 2_1^+ - 0_1^+ \mathbf{)} = \alpha_2^2 N \quad \text{-----(10)}$$

$$Q_{2_1^+} = \beta_2 \left[\frac{32\pi}{35} \right]^{1/2} \quad \text{----- (11)}$$

The basic condition for the observation of a SU(5) symmetry in the electromagnetic transition is[8]:

$$\frac{B(E2;4_1^+ - 2_1^+)}{B(E2;2_1^+ - 0_1^+)} = \frac{B(E2;2_2^+ - 2_1^+)}{B(E2;2_1^+ - 0_1^+)} = \frac{B(E2;0_2^+ - 2_1^+)}{B(E2;2_1^+ - 0_1^+)} = 2\left[\frac{(N-1)}{N} \right] < 2 \text{ --(12)}$$

IV. Transitional Regions in IBM-1

4.1.1 SU(3)-SU(5) transitional dynamical symmetry

This transitional region includes the two groups, SU(3) and SU(5). The SU(3) has to be broken with $\in n_d$ term. The general form of Hamiltonian operator of this region can be given as(9,10).

$$\hat{H} = \in \hat{n}_d + a_1 \hat{I}.\hat{I} + a_2 \hat{Q}.\hat{Q} \text{ ----------(13)}$$

The solution of the equation (13) depends on the ratio of ($\in$/a_2), when the ratio ($\in$/a_2) is large the eigenfunction of $\hat{H}$ are those appropriate to the limiting SU(5). Also the B(E2) values are affected by the ratio ($\in$/a_2). The B(E2) ratios (Branching Ratios) R can be given by (10).

$$R = \frac{B(E2;2_2^+ \to 0_1^+)}{B(E2;2_2^+ \to 2_1^+)} \quad \text{----- (14)}$$

Where :
R= 0 in SU(5) region
R=7/10 in SU(3) region

4.1.2 SU(3)-O(6) transitional dynamical symmetry

The breaking of SU(3) symmetry in the direction of O(6) symmetry can be treated in this transitional region by adding the term $\hat{P}^\dagger.\hat{P}$, so that the Hamiltonian form can be written as (11).

$$\hat{H} = a_o \hat{P}^\dagger.\hat{P} + a_1 \hat{I}.\hat{I} + a_2 \hat{Q}.\hat{Q} + a_3 \hat{T}_3.\hat{T} \text{ ----------(15)}$$

The solutions of equation (14) depends on the ratio (a_o/a_2), when it is large, the eigenfunction of the Hamiltonian $\hat{H}$ are appropriate to O(6)
Symmetry, but if it is small, the eigenfunction are appropriate to SU(3) symmetry. In this region the change in the electromagnetic rates can be seen from the branching ratios R which takes values
R= 7/10 in SU(3) symmetry
R= 0 in O(6) symmetry

4.1.3 .SU(5)-O(6) transitional dynamical symmetry

The form of Hamiltonian in this region can be written as (12).

$$\hat{H} = \in \hat{n}_d + a_o \hat{P}^\dagger.\hat{P} + a_1 \hat{I}.\hat{I} \quad \text{----------(16)}$$

The solution of the equation (15) depends on the ratio ($\in$/a_o)
The B(E2) values show a smooth transition towards typical O(6) vales, the branching ratios takes constant value (12,13).
R= 0 in SU(5) symmetry
R=0 in O(6) symmetry

Also, when the ratio ($\in$/a_o) is large, the eigenfunction of $\hat{H}$ are those appropriate to the limiting SU(5), while when it is small, the eigenfunctions are appropriate to symmetry O(6).

V. Results & Discussion

In this work we have studied the energy levels of even-even Dy (A =154 -158) isotope transitions with change in number of neutrons observed when moving from the lighter to heavier isotopes, i.e. SU(5) - SU(3) transitional regions (Table 1).

The even-even$^{154, 156,158}$Dy isotopes have (66) protons and (88,89,90) neutrons respectively. The core is taken at major closed shell (82) for protons and neutrons. Therefore, the number of bosons were determined for ^{154}Dy ,^{156}Dy and ^{158}Dy, is equal (11),(12) and (13) bosons respectively. The nuclear deformation increases with

the increasing of valance boson number. The interacting boson model version one (IBM-1) gives us a very closing value with the experiment.

5.1.2 .Energy bands (g, β, γ) ;

Figures (1- 3) show the energy levels and the energy bands arrangement compared with identical bands for the three limits SU(5) - SU(3) and O(6).The IBM-1 model in its group theoretical formulation, exhibits three dynamical symmetries, each corresponding to a particular a way of breaking the degeneracy of the parent U(6) group.

The behavior of the structure of each nucleus considered in this work is deduced by studying the dynamical symmetry of deformed Dy (A = 154-158) and the energy spectrum according to the sequences of energy bands g, β, γ.The parameters of Eq.[1] fitted to the experimental data are used to calculate the eigenvalues and eigenvectors of Dy (A = 154-158) isotopes, which are tabulated in Table 1.

To carry out the calculation of $^{154\text{-}156\text{-}158}$ Dy using interacting boson model-1(IBM-1), which does not distinguish between the neutron- and proton- boson, we must evaluate the total number of bosons N (Table 2) and the dynamical symmetry. Levels energy belonging to the(g,β,γ) bands,energy Levels, and energy ratios are calculated. The calculated values are compared with the available theoretical and experimental data and show reasonable agreement.

the energy spectrum accordingto the sequences energy bands (g, β, γ) give agood agreement for the level sequence of each band with the typical sequence of ground band (0+, 2+, 4+,……), (β – band) (0+, 2+, 4+,……), and (γ – band) (2+, 3+,4+, 5+,……).There are good agreement between the present results and the experimental

5.1.1 .Energy ratios

Table(3)show the theoretical energy ratios compared with the experimental data for chosen even-even IsotopesDy.The energy ratios of Dy(A=154-158)is the $\frac{E(4_1^+)}{E(2_1^+)}$, $\frac{E(6_1^+)}{E(2_1^+)}$ and $\frac{E(8_1^+)}{E(2_1^+)}$ comparing with the ideal values,it has beenpointedout by Zhang et al [7] that the ratio $E4_1^+/E2_1^+$ is an important parameter for determining the shape of a nucleus.The value $E4_1^+/E2_1^+$increases from 2.33 for ^{154}Dy to 2.93 for ^{156}Dy and after that, there is increase in the value 3.2 of E_4^+/E_2^+ ratio from ^{158} Dy.

Figures(A, B and C) showThe relation between the energy ratios as a function of number of neutron N for the even-even Dy(A=154-158) isotopes.. The dynamical symmetries for the isotopes under study have been determined.Which is found to be ^{154}Dy which has transitional motion of the SU(5)-O(6) translation region ,^{156}Dy which hastransitional motion of theO(6)-SU(3) dynamical symmetryand^{158}Dy which has rotational motion of theSU(3) dynamical symmetry.

Table(1) Theoretical energy ratios compared with the experimental data for chosen even-even isotopes

Isotope	I^n	Energy level (MeV)		Isotope	I^n	Energy level (MeV)		Isotope	I^n	Energy level (MeV)	
		Exp[14]	IBM-1 (pw)			Exp[14]	IBM-1 (pw)			Exp[14]	IBM-1 (pw)
$^{154}_{66}Dy_{88}$ SU(5) – O(6)	0_1^+	0.0000	0.0000	$^{156}_{66}Dy_{90}$ SU(5)-SU(3)-O(6)	0_1^+	0.000	0.000	$^{158}_{66}Dy_{92}$ SU(3)	0_1^+	0.000	0.000
	2_1^+	0.3346	0.33		2_1^+	0.137	0.129		2_1^+	0.0989	0.096
	4_1^+	0.7467	0.842		4_1^+	0.4041	0.425		4_1^+	0.3172	0.32
	6_1^+	1.2237	1.537		6_1^+	0.7703	0.884		6_1^+	0.6378	0.672
	8_1^+	1.7473	2.415		8_1^+	1.2156	1.501		8_1^+	1.0441	1.151
	0_2^+	0.6608	0.66		0_2^+	0.6756	0.673		0_2^+	0.9906	0.975
	2_2^+	0.905	0.792		2_2^+	0.8283	0.808		2_2^+	1.0856	1.073
	4_2^+	1.2519	1.458		4_2^+	1.0883	1.112		4_2^+	1.28	1.3
	6_2^+	1.659	2.197		6_2^+	1.4371	1.577		6_2^+	1.5473	1.656
	8_2^+	2.1632	3.075		8_2^+	1.8586	2.2		8_2^+	1.89	2.139
	0_3^+	1.0577	1.21		0_3^+	1.226	1.485		2_3^+	0.9462	1.091
	2_4^+	1.3904	1.452		2_4^+	1.4478	1.377		3_1^+	1.0445	1.189
	4_4^+		2.052		4_4^+	1.6272	1.69		4_3^+	1.1637	1.32
	6_4^+		2.747		6_4^+	1.8956	2.168		5_1^+	1.3114	1.481
	8_4^+		3.625		8_4^+	2.2 016	2.809		6_3^+	1.4864	1.679
	2_3^+	1.027	0.99		2_3^+	0.8907	0.934		7_1^+	1.6758	1.903
	3_1^+	1.3343	1.429		3_1^+	1.022	1.088		8_3^+	1.8909	2.171
	4_3^+	1.4426	1.502		4_3^+	1.1684	1.273		2_4^+	1.8522	1.905
	5_1^+	1.74	2.26		5_1^+	1.3352	1.515		3_2^+	1.9407	2.005
	6_3^+	1.8856	2.34		6_3^+	1.5252	1.774		4_4^+	2.0553	2.145
	7_1^+		3.282		7_1^+	1.7287	2.107		5_2^+	2.2111	2.304
	8_3^+		3.339		8_3^+	1.9572	2.429		6_4^+	2.3866	2.512
	2_5^+	1.5076	1.54		2_5^+	1.5259	1.464		7_2^+		2.731
	3_2^+	1.782	2.089		3_2^+		1.654		8_4^+		3.425
	4_5^+		2.218		4_5^+		1.8				

	5_2^+		2.924		5_2^+		2.094			
	6_5^+		3		6_5^+		2.329			
	7_2^+		3.9416		7_2^+		2.71			
	8_5^+		4		8_5^+		3			

Table(2) The parameters of Hamiltonian function operator for Dy (A = 154-158) isotopes.

Isotopes	N_π	N_ν	N_{Tot}	ESP MeV	$\hat{p}.\hat{p}$ MeV	$\hat{I}.\hat{I}$ MeV	$\hat{Q}.\hat{Q}$ MeV	$(\hat{T}_3.\hat{T}_3)$ MeV	$(\hat{T}_4.\hat{T}_4)$ MeV	CHI
$^{154}_{66}Dy_{88}$	8	3	11	0.0000	0.0550	0.0190	0.0000	0.1540	0.0000	1.0000
$^{156}_{66}Dy_{90}$	8	4	12	0.3250	0.0100	-0.0090	-0.0142	0.1691	0.0312	-1.2400
$^{158}_{66}Dy_{92}$	8	5	13	0.0000	0.0000	0.0195	-0.0089	0.0019	0.0000	1.6500

Table(3) Theoretical energy ratios compared with the experimental data for chosen even-even isotopes

Isotopes	$E(4_1^+)/E(2_1^+)$		$E(6_1^+)/E(2_1^+)$		$E(8_1^+)/E(2_1^+)$	
	EXP.	IBM-1 (pw)	EXP.	IBM-1 (pw)	EXP.	IBM-1 (pw)
$^{154}_{66}Dy_{88}$	2.2356	2.5515	3.6637	4.6575	5.2314	7.3181
$^{156}_{66}Dy_{90}$	2.9325	3.2945	5.5899	6.8527	8.8150	11.6356
$^{158}_{66}Dy_{92}$	3.2072	3.3333	6.4469	7.000	10.5571	11.9895

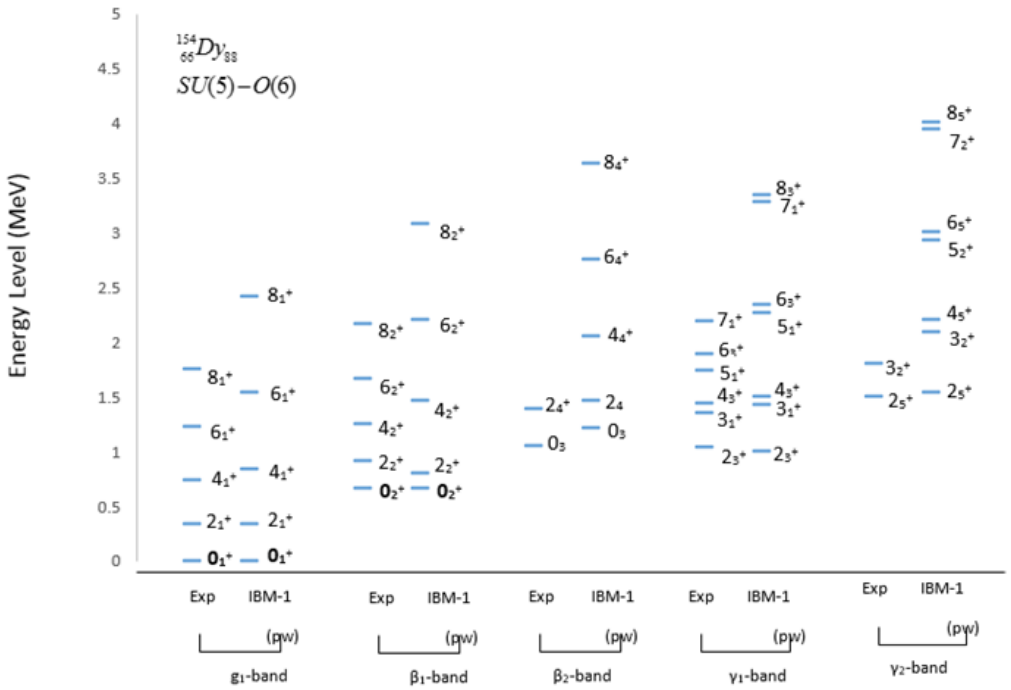


Fig (1) Comparison between calculated IBM (pw) and experimental energy bands states g, β, γin isotope $^{154}_{66}Dy_{90}$ of the dynamical symmetry SU(5) – O(6).

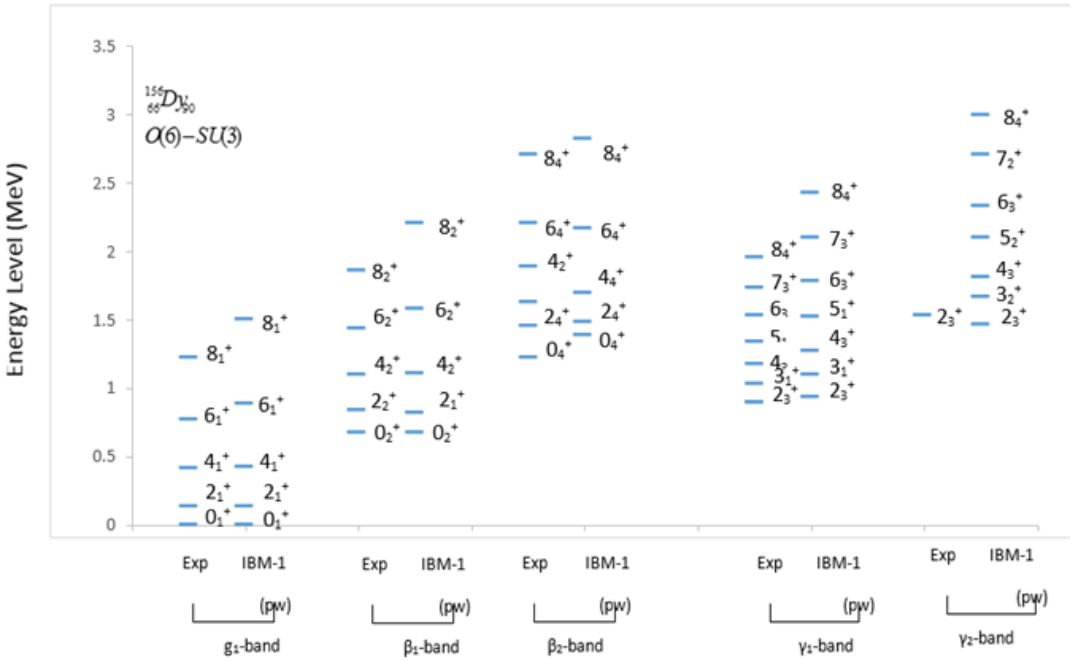


Fig 2 Comparison between calculated IBM (pw) and experimental energy bands states(g, β, γ) in isotope $^{156}_{66}Dy_{92}$ of the dynamical symmetry SU(3) – O(6).

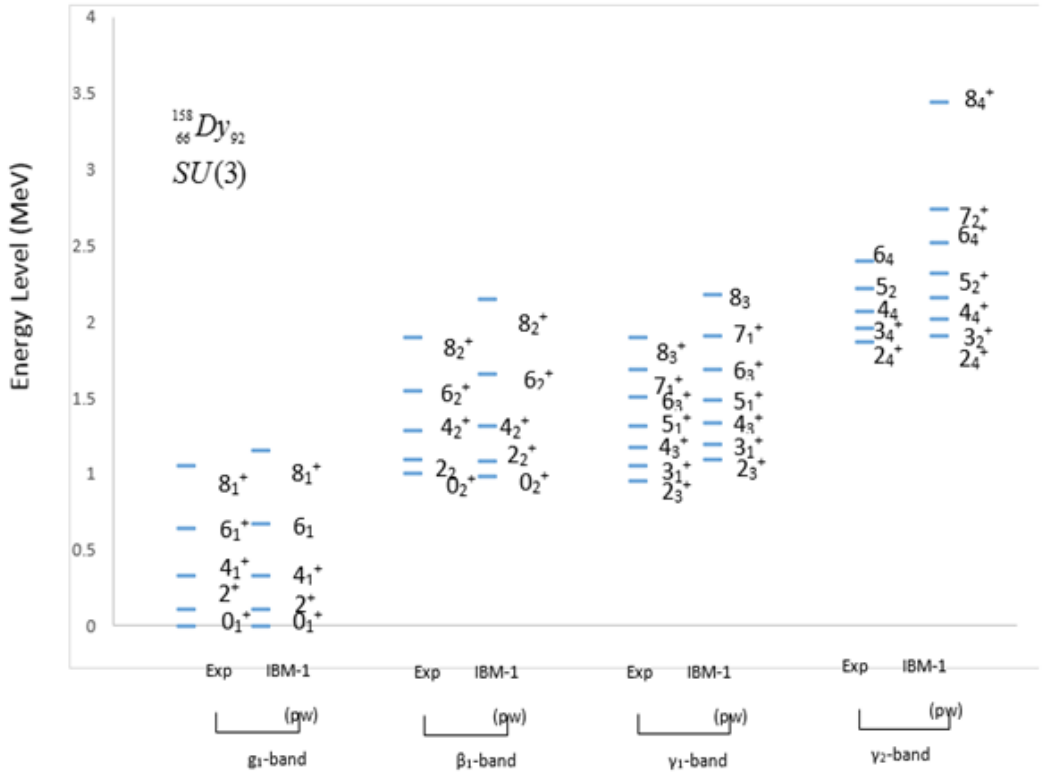
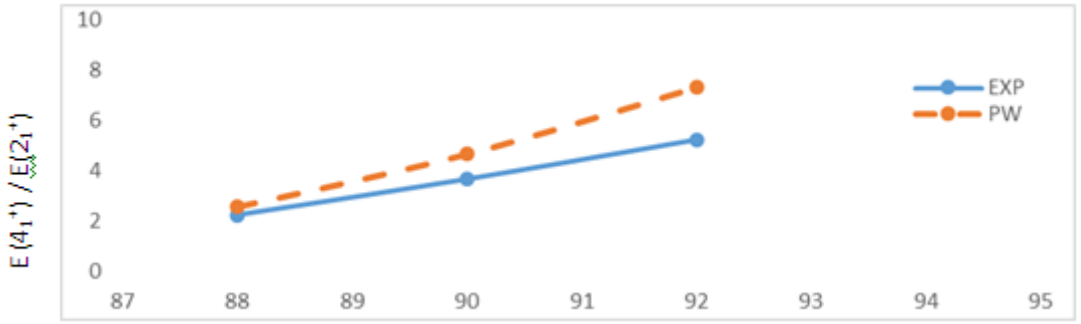
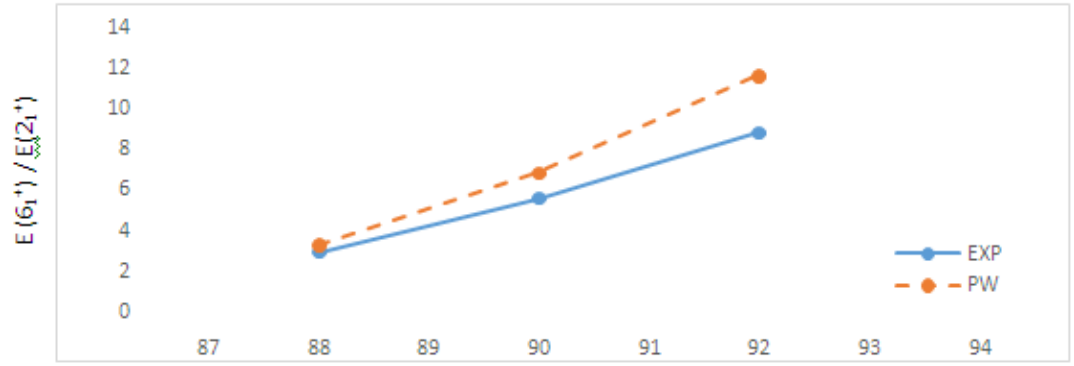


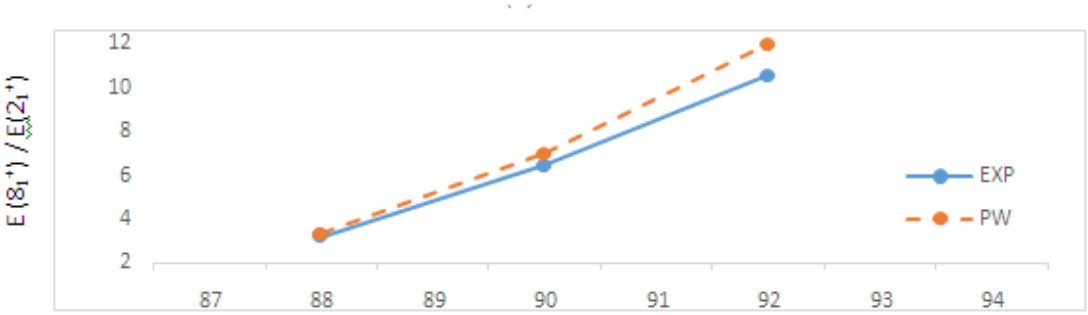
Fig 3 Comparison between calculated IBM (pw) and experimental energy bands states(g, β, γ) in isotope $^{158}_{66}Dy_{94}$ of the dynamical symmetry SU(3) – O(6).



Number of Neutron N(A)



Number of Neutron N (B)



Number of Neutron N.(C).

The relation between the energy ratios as a function of number of neutron N for the even-even Dy (A=154-158) isotopes.

References

[1]. A. Arima and F. Iachello, Advances in Nuclear Physics, **13**, 139 (Plenum Press, New York) (1984).
[2]. Arima A. and Iachello F. Interacting Boson Model of Collective States IV: The O(6) limit. Annals of Physics,, v.102, 123–468 1979,.
[3]. O. Scholten, Intercating Boson in Nuclear Physics, edited by F. Iachello (Plenum Press, New York), 17 (1979).
[4]. O.S. Van Roosmalen, A.E.L.Dieperink, F.Iachello, Chem. Phys. Lett., 85, 2 (1982).
[5]. Arima A. and Iachello F., Phys.Lett.B. Vol.53, 309(1974).
[6]. Casten R.F and Warner D.D.,"The Interacting Boson Approximation"vol.60, 391 (1988).
[7]. R. Casten and D. Warner, Rev. Mod. Phys. 60, 389(1988).
[8]. Zhang J. Y., Casten R. F. and N V Zamfir, Phys. Rev. C, 48 (1993) R10.
[9]. Iachello F. and Arima., (Interacting Boson Models) pub-Cambridge University press, Cambridge, England P.132 (1987).
[10]. Feshbach H., Theoretical nuclear Phys., Ed. Wiley J., New York, Pub. Canada, P.670-671(1974).
[11]. Casten R.F., and Warner D.D., Rev. Mod. Phys. Vol. 60, 391 (1988).
[12]. Casten R.F., and Warner D.D., phys. Rev. C, Vol. 28, (1983).
[13]. Pefeifer W., (Introduction to Interacting boson model of atomic nucleic) PI, p,5 (1998).
[14]. Mitsuo Sakal, Atomic Data and Nuclear Data Tables, 31,399-432 (1984).