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Modeling of the Collect and Discharge Tank for Rain Driven Hydro Power Plant

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Abstract: *This paper presents a mathematical model of the Collect and Discharge Tank unit of the rain driven hydro power generator currently being designed using the University of Port Harcourt's faculty of science building at Ofrima, Abuja Campus. The model revealed that over 2.0KW of power can be generated using rain water collected from the building, and that a minimum of 3,751 gallons storage capacity tank is needed to make sure that the unused water accumulated from a 0.6 inches per hour of rainfall is effectively stored if rainfall lasts for 33.33 minutes. This means that for the tank to effectively serve its purpose when rainfall at the rate of 0.6 inches per hour lasts for over 33.33 minutes, a larger capacity size of tank is needed, or the hydro power generating system is redesigned to generate power greater than 2.0KW.*
Keywords: *collect and discharge tank, hydro power generator, mathematical model, rainfall, uniport*

I. Introduction

The collect and discharge tank to be modeled here, is a unit of the rain driven hydro power generator currently being designed using the University of Port Harcourt Faculty of Science building at Ofrima. The tank is to collect water from the roof of the building and then discharge it to an installed Francis turbine on the ground floor in the course of a rainfall and until the unused accumulated water in the tank is exhausted.

Rain is liquid water in the form of droplets that have condensed from atmospheric water vapour and then precipitated (that is, become heavy enough to fall under gravity). It is a major component of the water cycle and is responsible for depositing most of the fresh water on the Earth. The major cause of rainfall is moisture moving along three-dimensional zones of temperature and moisture contrasts known as weather fronts. If enough moisture and upward motion is present, precipitation falls from convective clouds (those with strong upward vertical motion) such as cumulonimbus (thunder clouds) which can organize into narrow rain bands [1].
Rainfall intensity is classified according to the rate of precipitation:

- Light rain — when the precipitation rate is < 2.5 mm (0.098 in) per hour,
- Moderate rain — when the precipitation rate is between 2.5 mm (0.098 in) - 7.6 mm (0.30 in) or 10 mm (0.39 in) per hour,
- Heavy rain — when the precipitation rate is > 7.6 mm (0.30 in) per hour, or between 10 mm (0.39 in) and 50 mm (2.0 in) per hour, and
- Violent rain — when the precipitation rate is > 50 mm (2.0 in) per hour [2] [3].

For hydro turbine, the power generated from a stream of water is given by

$$P = \eta\rho g h \dot{q} \qquad (1.1)$$

Where: η = turbine efficiency, ρ = density of water (kg/m^3), g = acceleration due to gravity (9.81m/s^2), h = net head (m), $\dot{q}$ = flow rate (m^3/s), and the power, P is in J/s or watts [4].

The net head, $h = h_g - h_f - h_m \qquad (1.2)$

Where: h_g = gross head (total head available, that is, difference of water level between head race and tail race), h_f = head loss due to friction in the pipe, h_m = other minor losses.

A model is a miniature representation of something; a pattern of something to be made; an example for imitation or emulation; a description or analogy used to help visualize something (e.g., an atom) that cannot be directly observed; a system of postulates, data and inferences presented as a mathematical description of an entity or state of affairs [5]. This definition suggests that modeling is an activity, a cognitive activity in which we think about and make models to describe how devices or objects of interest behave. There are many ways in which devices and behaviors can be described. We can use words, drawings or sketches, physical models, computer programs, or mathematical formulas. In other words, the modeling activity can be done in several languages, often simultaneously. If the language used is mathematics, the model is now called mathematical model. A mathematical model can therefore be defined as a representation in mathematical terms of the behavior of real devices and objects.

II. Purpose Of The Tank

The purpose of this tank is to make the system work despite the variation in the rate of rain fall. Rainfall rate can be light (< 0.10inches of rain per hour), moderate (0.10 to 0.30inches of rain per hour), heavy (> 0.30 inches of rain per hour, or between 0.39 in and 2.0 in of rain per hour), and violent (> 2.0 inches of rain per hour).heavy. The rate of flow to the turbine should be approximately constant, therefore a tank is needed to temporarily store the water collected from the roof and then discharge the required amount of water to drive the turbine on the ground floor.

III. Modeling Questions And Answers

This unit will be modeled using the question and answer method as follows:

3.1 How Much Water Can Be Gathered From The Roof Of The Building?

The top view of the roof of the building is of the shape and dimension given in figure 3.1.

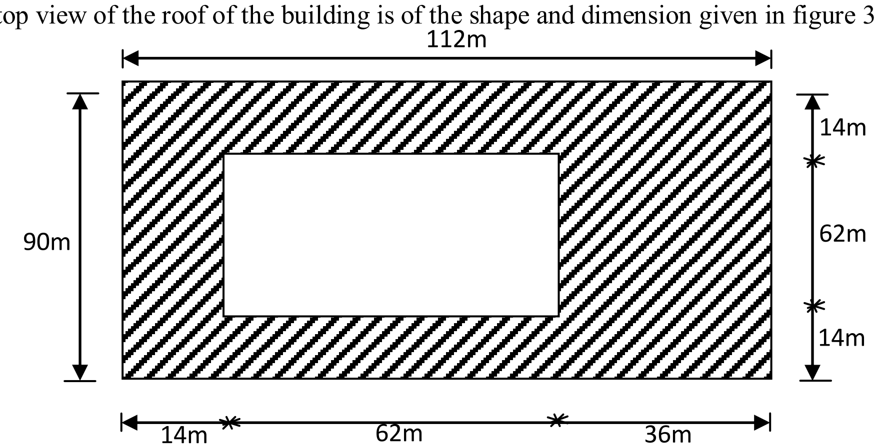

Fig 3.1 Top view of the roof of the UNIPORT faculty of science building, Ofrima.

The area of the roof cover can be determined thus:

Roof cover area = Total area – Internal open space area

$= (112\text{m} \times 90\text{m}) - (62\text{m} \times 62)$

$= 10{,}080\text{m}^2 - 3{,}844\text{m}^2$

$= 6{,}236\text{m}^2$

One inch of rainfall equals 4.7 gallons of water per square yard. This is converted to liter per square meter thus: 1 yard = 0.9144m and 1 gallon = 3.7854L

Therefore, 4.7 gallons/yd^2 = $\frac{4.7\times 3.7854\,\text{L}}{(0.9144)^2\text{m}^2}$

$= 21.2783\text{Lm}^{-2} \approx 21.28\text{Lm}^{-2}$

For moderate rain of 0.10 inches per hour, the amount of water that can be accumulated from the roof is determined thus:

0.10 inches of rain per hour = $0.10/60^2$ inches of rain per second

$= 2.78 \times 10^{-5}$ inches of rain per second

$= (2.78 \times 10^{-5})(21.28)\text{Lm}^{-2}\text{s}^{-1}$

$= 5.92 \times 10^{-4}\text{Lm}^{-2}\text{s}^{-1}$

This means that one square meter of the roof top will accumulate 5.92×10^{-4} liter of water in one second for moderate rain of 0.10 inches.

Therefore, the roof top of $6{,}236\text{m}^2$ will accumulate ($6{,}236 \times 5.92\text{x}10^{-4}$) Liters of water in one second.

The amount of water that can be accumulated from the roof of the building for a moderate rain of 0.10 inch is 3.69Ls-^{1}

For moderate rain of 0.30 inches per hour, we have

0.30 inch of rain per hour = $0.30/60^2$ inches of rain per second

$= 8.33 \times 10^{-5}$ inch of rain per second

$= (8.33 \times 10^{-5})(21.28)\ \text{Lm}^{-2}\text{s}^{-1}$

$= 1.77\text{x}10^{-3}\text{Lm}^{-2}\text{s}^{-1}$

One square meter of the roof will accumulate $1.77\text{x}10^{-3}$ liter of water in one second. Therefore, the roof of $6{,}236\text{m}^2$ will accumulate 11.04 liters of water per second for a moderate rain of 0.3inch per hour.

For a heavy rainfall of 0.6 inches per hour, we have

0.6 inch of rain per hour = $0.6/60^2$ inches of rain per second

$= 1.67 \times 10^{-4}$ inch of rain per second
$= (1.67 \times 10^{-4})(21.28)\ Lm^{-2}s^{-1}$
$= 3.55 \times 10^{-3} Lm^{-2}s^{-1}$

One square meter of the roof will accumulate 3.55×10^{-3} litre of water in one second. Therefore, the roof of 6,236m^2 will accumulate 22.14 liters of water per second from a heavy rainfall of 0.6 inches per hour. The answer to the number one modeling question is: the amount of water that can be accumulated from the roof of the building is 3.69Ls^{-1} for moderate rain of 0.1 inch per hour, 11.04Ls^{-1} for moderate rain of 0.3 inch per hour, 22.14Ls^{-1} for heavy rain of 0.6 inch per hour, etc.

3.2 How Much Power Is To Be Generated?

The flow rate is related to the power by equation 1.1 given in section 1.0, which is

$P = \eta\rho g h\dot{q}$

The tank is to be placed on the last floor which has height of 15m, and therefore ignoring head losses, h = 15m, the density of water ρ = 1000kg/m^3. Assuming an efficiency of 90% (as Francis turbine can have efficiency of over 90%), we have:

For moderate rain of 0.1 inch per hour and flow rate of 3.69Ls^{-1} (= 0.0037m^{3}s^{-1}) the power that can be generated is

$$P = \eta\rho g h\dot{q} = A\dot{q} \qquad (3.1)$$

Where:

$A = \eta\rho g h$
$= (0.9)(1000kg/m^3)(9.81ms^{-2})(15m)$
$= 13.24 \times 10^4 J/m^3$
$\therefore P = A\dot{q}$
$= (13.24 \times 10^4 Jm^{-3})(0.0037m^3s^{-1})$
$= 490W$

For moderate rain of 0.3inch per hour and flow rate of 11.04Ls^{-1} (= 0.011m^{2}s^{-1}), the power that can be generated is

$P = \eta\rho g h\dot{q} = A\dot{q}$
$= 13.24 \times 10^4 J/m^3 \times 0.011m^3s^{-1}$
$= 1{,}456W \approx 1.5KW$

For heavy rain of 0.6 inch per hour and flow rate of 22.14Ls^{-1} (=0.0221m^{3}s^{-1}), the power that can be generated is

$P = \eta\rho g h\dot{q} = A\dot{q}$
$= 13.24 \times 10^4 J/m^3 \times 0.0221m^3s^{-1}$
$= 2{,}926W \approx 3.0KW$

This means that for moderate rainfall, 490 to 1,456W of power can be generated and for heavy rainfall over 1,456W of power can be generated. Therefore, the choice of the power output of this system should be over 1.5KW so as to fully harness the energy from heavy rain falls and reduce water wastage.

The answer to the second modeling question is a choice of 2.0KW.

3.3 What Flow Rate Out Of The Tank Will Produce This Power?

The flow rate is given by

$\dot{q} = \frac{P}{\eta\rho g h} = \frac{P}{A}$

For $P = 2.0KW$

$\dot{q} = \frac{2000W}{13.24 \times 10^4 J/m^3} = 0.0151m^3s^{-1}$

The flow rate that will generate 2.0KW of power is $0.0151m^3s^{-1}$

3.4 How Does The Volume Of Water In The Tank Vary?

For rates below $0.0151m^3s^{-1}$ of water entering the tank, the turbine will either generate power below its capacity of 2.0KW or will not generate power at all if the flow rate cannot drive it, and water leaves the tank at the same rate in which it enters. In this case, the volume of water in the tank is approximately equal to zero. For rates equal to $0.0151m^3s^{-1}$ of water entering the tank, the turbine generates power at the designed capacity, and water also leaves the tank at the same rate it enters. The volume of water in the tank is approximately zero. For rates above $0.015m^3s^{-1}$ of water entering the tank, the turbine generates power at the designed capacity, and the volume of water in the tank increases with time. Let Fig 3.2 represent the tank model.

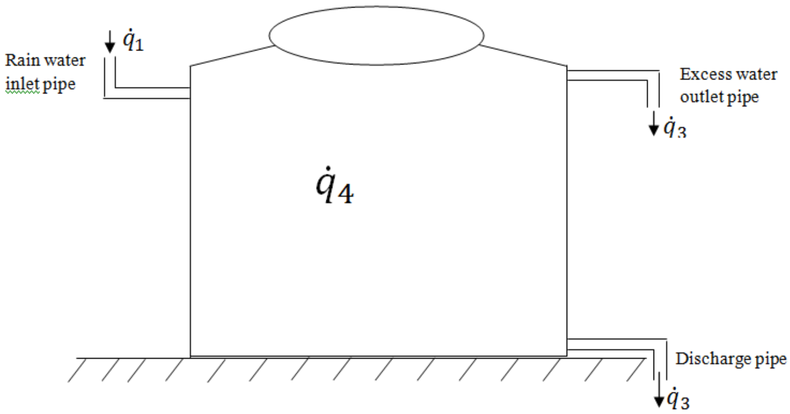

Fig 3.2: The tank model

$\dot{q}_1$= collected rain water in-flow rate
$\dot{q}_2$ = water discharge flow rate to the turbine
$\dot{q}_3$ = excess water out flow rate
$\dot{q}_4$ = rate at which volume of water in the tank increases
At any point in time,

$$\dot{q}_1 - \dot{q}_2 - \dot{q}_3 - \dot{q}_4 = 0 \qquad (3.2)$$

For heavy rainfall of say 0.6 inch per hour $\dot{q}_1$ will be $0.221m^3s^{-1}$ (as earlier calculated)
$\dot{q}_2 = 0.015m^3s^{-1}$ (the normal discharge rate required)
$\dot{q}_3$ = 0 at the initial time and remains zero until the tank fills up.
Therefore, as long as the thank is not filled up,

$$q_4 = \frac{dq_4}{dt} = \frac{dV}{dt} = \dot{q}_1 - \dot{q}_2 \qquad (3.3)$$

Where V = q_4 is the volume of water in the tank

$$\Delta V = (\dot{q}_1 - \dot{q}_2)\Delta t \qquad (3.4)$$

ΔV is the incremental variation of the volume of water in the tank within the time interval Δt.
Therefore, for 0.6 inch per hour of rainfall for one second

$$\Delta V = (0.0221\text{m}^3\text{s}^{-1} - 0.015\text{m}^3\text{s}^{-1}) (1\text{s})$$
$$= 7.10 \text{ x } 10^3\text{m}^{-3}$$

Using equation 3.5 , the volume of water in the tank, ΔV for continuous rainfall of rate 0.6 inch/hour for10s to 4hours is calculated as given in Table 3.1

Table 3.1 Volume variation of water in the tank

Time (s)	ΔV x 10^{-2} (m^3)	Time (m)	ΔV (Gal)
10	7.10	0.17	18.76
100	71.00	1.67	18.56
1000	710.00	16.67	1875.62
2000	1420.00	33.33	3751.24
3000	2130.00	50.00	5626.87
4000	2840.00	66.67	7,502.49
5000	3550.00	83.33	9,378.10
6000	4260.00	100.00	11,253.73
7000	4970.00	116.67	13,129.35
8000	5680.00	133.33	15,004.97
9000	6390.00	150.00	16,880.59
10,000	7100.00	166.67	18,756.20
11,000	7810.00	183.33	20,631.84
12,000	8520.00	200.00	22,507.46
13,000	9230.00	216.67	24,383.08
14,000	9940.00	233.33	26,258.70

IV. Discussion

Table 3.1 gives the volume of water stored in the tank at different time intervals if rain falls continuously at rate of 0.6 inches per hour. From this table, it can be deduced that if the storage capacity of the tank is 3,751 gallons, water will start wasting after 33.33 minutes. To avoid this, the tank's storage capacity should be increased or the turbine power generating capacity should be increased. Increasing the power generating capacity of the hydro turbine will lead to increase in the flow rate to the turbine ($\dot{q}_2$).

V. Conclusion

Over 2.0KW of power can be generated from rain water accumulated from the University of Port Harcourt Faculty of Science building at Ofrima. To ensure that unused accumulated rain water does not fill up the collect and discharge tank and starts wasting if rain falls at the rate of 0.6 inches per hour for 33.33 minutes, a storage tank of 3,751 gallons storage capacity is needed.

Since rain can fall in Port Harcourt at the rate of 0.6 inches per hour for over 33.33 minutes, a bigger sized tank is needed or a redesign of the system to increase the power generating capacity is necessary.

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