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# Explanation oftheRelationship betweenthe Temperature and Mass ofthe BlackHole throughGravitational Waves

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***Abstract :*** *This thesis explains the phenomenon of the lowering of the temperature of the black hole and hence less radiation of energy from the black hole (Known from the concept of Hawking Radiation) with the increase in the mass of the black hole through the concept of gravitational waves.*

## I. Introduction

This thesis explains the phenomenon of the lowering of the temperature of the black hole and hence less radiation of energy from the black hole (Known from the concept of Hawking Radiation) with the increase in the mass of the black hole through the concept of gravitational waves.

**KNOWN FACTS:**

**From Hawking Radiation :**

A black hole of one solar mass has a temperature of only 60 nanokelvin (60 billionths of a Kelvin); in fact, such a black hole would absorb far more cosmic microwave background radiation than it emits. A black hole of 4.5 × 1022 kg (about the mass of the Moon) would be in equilibrium at 2.7 Kelvin, absorbing as much radiation as it emits. Yet smaller primordial black holes would emit more than they absorb, and thereby lose mass.

**The gravitational waves**

In Einstein's theory of general relativity, gravity is treated as a phenomenon resulting from the curvature of space-time. This curvature is caused by the presence of mass. Generally, the more mass that is contained within a given volume of space, the greater the curvature of space-time will be at the boundary of this volume.As objects with mass move around in space-time, the curvature changes to reflect the changed locations of those objects. In certain circumstances, accelerating objects generate changes in this curvature, which propagate outwards at the speed of light in a wave-like manner. These propagating phenomena are known as gravitational waves. In general, any acceleration that is not spherically or cylindrically symmetric will produce a gravitational wave. Consider a star that goes supernova. This explosion will produce gravitational waves if the mass is not ejected in a spherically symmetric way, although the center of mass may be in the same position before and after the explosion. Another example is a spinning star. A perfectly spherical star will not produce a gravitational wave, but a lumpy star will.

**EXPLANATION OF THE LOWERING OF TEMPERATURE OF THE BLACK HOLE WITH INCREASE IN ITS MASS**

- When a massive star runs out of its nuclear fuel it contracts.But due to the absence of any torque its angular momentum is conserved .So when the star shrinks/implodes to form a blackhole then its radius reduces several times.
- So **$I*\omega$ =constant**
- **As $I \alpha r^2$**
- **As r reduces so I also reduces and hence ω also increases.**
- **Where**
- **I is the moment of inertia of the star**
- **ωis the angular velocity of the star**
- **r is the radius of the star**
- Now, super massiveblack holes,that is,black holes having masses more than the mass of our sun, are formed through black hole merges.
- During these merges the participating black holes revolve around each other at a tremendously high speed .Finally, when they merge, the resultant black hole has a very high angular velocity (ω).
- Now, black hole is a super dense body of mass.
- So, it can be considered to be made up of several small bodies of immense density and hence high mass.

- (The black hole is not a perfect sphere like any other celestial body because of the high angular velocity.)
- Now, as they are a part of the black hole, they are revolving around the axis of rotation of the black hole.
- Therefore, they are undergoing acceleration as every moment; as their direction of velocity is changing(so they are accelerating).
- Now, we know that when anybody having mass accelerates,then it produces gravitational waves which are a consequence of Einstein's theory of relativity.
- Now, greater the mass of the black hole,greater is the number of such parts of immense density and hence high mass.
- So, greater is the gravitational waves produced.
- These gravitational waves propagate in all directions from their source and also travel through the body of the black hole(or its core).
- As these waves pass through the black hole they can undergo constructive as well as destructive interference.
- If they undergo constructive interference, then the amplitude of the resultant wave increases.
- Now, as the trough of the resultant gravitational wave passes through a region of the black hole,the space expands and the dense particles tend to move away from each other and as a result they absorb energy in the form of heat from the surrounding parts of the black hole to overcome the strong forces of attraction in between them. As a result temperature decreases (as heat energy cannot be instantly absorbed from the surrounding). The expansion is favored as everything tends to have maximum randomness or entropy (as the dense particles in a black hole are tightly bound and have very low randomness).
- When the crest of the gravitational wave reaches a region of the black hole, then as the particles arealready in a state of equilibrium. So, they resist further compression. So they cannot be brought too close to each other as they can be moved apart. So there is a net absorption of heat.
- Hence the temperature of a black hole decreases with increase in mass of the black hole.

### 1.1 The deduction of the relation between the temperature of the black holes and the mass of the black holes

- $A \alpha M_B$ and $A \alpha \omega_B$
- Therefore $A \alpha M_B * \omega_B$
- Now $T_B \alpha 1/A$
- Therefore $T_B \alpha 1/(M_B * \omega_B)$
- So $T_B \alpha 1/M_B$ [As $\omega_B$ is constant for a black hole]
- So $T_B = \beta_t * 1/M_B$
- $M_B$ is the mass of the black hole
- A is the amplitude of the resultant gravitational wave produced on account of constructive interference.
- $\omega_B$ is the angular velocity of the black hole
- $T_B$ is the temperature of the black hole
- $\beta_t$ is a constant of proportionality

  Explanation of the relation between the black hole radiation and the mass of the black hole
- We know that the amount of heat radiated or absorbed is directly proportional to the difference of temperature of the system and the surrounding.
- $R_B \alpha (T_B^4 – Є*T_S^4)$
- $R_B =\sigma* (T_B^4 – Є* T_S^4)$
- $R_B =\sigma* (T_B^4 – Є*T_S^4)$ [As $T_B = \beta_t * 1/M_B$ ]
- $R_B =\sigma* ((\beta_t * 1/M_B)^4 – Є*T_S^4)$
- $R_B =\sigma* (\beta_t^4 – Є*T_S^4* M_B^4)/ M_B^4$
- $R_B =K / M_B^4$ [$K= \sigma* (\beta_t^4 – Є*T_S^4* M_B^4)$ is another constant when $T_S$ is a constant ]
- $R_B \alpha 1/ M_B^{4}$\
- Where $R_B$ is the radiation emitted/absorbed per unit area by a black hole.
- $T_B$ is the temperature of the black hole
- $T_S$ is the temperature of the surroundings
- $M_B$ is the mass of the black hole
- In deriving the above relation we use Stefan-Boltzmann Law.
- Є is the emissivity or the relative emittance of a body since the surroundings of a black hole need not be a perfectly black body. (0< Є<1)
- σ is the Stefan's constant
- So if the mass of the black hole is small and as

- $R_B \ \alpha \ 1/M_B{}^{4\backslash}$
- $R_B$ will be very high.
- The black hole will emit more radiant energy than it will absorb.
- As $E=mc^2$
- So the black hole will be losing mass
- If the mass of the black hole is very small
- Then as $T_B \ \alpha \ 1/M_B$
- So the temperature of the black hole will be very high
- By using Kirchoff's law, we can say that at higher temperature the black hole will radiate the particular wavelength of radiation that it absorbs strongly at lower temperature.
- As it absorbs all wavelengths strongly
- So it will radiate white light, that is, it will glow white.
- This is the same as given by hawking radiation concept according to which black holes of more than one solar mass have very low temperature and absorb more radiation than they emit. So this law is in accordance to hawking's law of black hole radiation which is a well established law.

## II. Conclusion:

So it can be concluded that greater the mass of the black hole lesser is its temperature and hence lesser is the black hole radiation.