

Optimization of Aberrated Coherent Optical Systems

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Abstract: The images of a straight edge in coherent illumination, produced by an optical system with circular aperture and apodised with amplitude filters have been studied. The image quality assessment parameters such as edge-ringing, edge-gradient and edge-shift of the edge fringes have been studied as a function of apodisation parameter for various degrees of defocus, Coma and primary spherical aberrations. It is found that, at certain combinations of aberrations the quality of the image of straight edge objects can be improved.

Keywords: Aberrations, Apodisation, edge-ringing, edge-gradient, edge-shift and Optical system.

I. Introduction

A common feature of all the optical systems is the presence of optical aberrations. Even in the most highly corrected systems there are some residual aberrations and most of the systems are not well corrected. Aberrations results in phase errors in the wave front as it traverses the optical systems. Aberrations can introduce undesirable results and degradation of performance of optical systems. Edge ringing and edge shifting phenomena are more highly enhanced in the aberrated optical system than in the aberration-free one. BARAKAT (1969) and ROWE (1969) studied in detail about apodised images of coherently illuminated edges in the presence of aberrations. Marechal has worked on the optimum balancing conditions for a coherently illuminated line in the presence of spherical aberration and defocusing (BORN and WOLF, 1984).

Apodisation is the deliberate modification of the transmittance of the optical system, which in turn affects the imaging characteristics of the optical system. It will be shown that the use of apodisation is also useful in controlling the effects of aberrations in coherent systems. Defocus is the simplest type of the aberration, in that; the real wave front differs from the spherical reference wave front only in its radius of curvature. Spherical aberration is a radially symmetric aberration. It describes a wave front having the largest deviation from the spherical reference wave front of any of the aberration considered. Coma aberration occurs either due to the incident wavefront being tilted, or decentered with respect to the optical surface. Hence, it is either an aberration affecting off-axis image points, or the result of axial misalignment of optical surfaces, respectively. In the coherent imagery of edge objects the three major effects encountered which degrade the image are edge ringing, edge gradient and edge shifting. In this paper, a systematic study was carried out and the results from the investigations on the effects of defocusing, coma and primary spherical aberrations on the performance of the apodized coherent optical imaging systems in the formation of straight edge images are presented.

Fourier analytical methods are applicable to the optical systems with space invariance, and unit magnification, where the conditions of linearity are satisfied. The image of a one dimensional edge object illuminated coherently is obtained at the image plane following an image formation scheme employing Fourier techniques. Coherent illumination may be regarded as a linear image forming system only at the extreme condition where the system is linear in object and image amplitudes with the pupil function relating them. The final image intensity distribution is the resultant of the addition of the complex amplitudes to produce the complex amplitude distributions, with its squared modulus. The Edge response, being the basic mathematical model, has been obtained in the case of coherent illumination by applying the coherent image formation scheme using the Fourier analytical methods. The response of the optical system with generalized shaded aperture for various values of the apodisation parameters has been studied.

II. Theory

An opaque straight edge is one which is bright on one side of a line and dark on the other.

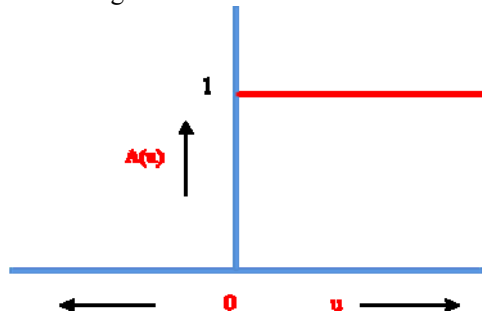


Fig.1 Edge response

The mathematical representation of amplitude transmission of an opaque straight edge is given by

$$\begin{aligned} A(u, v) &= 1 \quad \text{for } u \geq 0 \\ A(u, v) &= 0 \quad \text{for } u < 0 \end{aligned} \tag{1}$$

This indicates that the transmission function is discontinuous at $u = 0$. The Fourier transform for this equation gives the amplitude spectrum of the object and is given by [18]

$$a(x, y) = \frac{1}{2} \cdot \delta(y) \left[\delta(x) + \frac{1}{i\pi x} \right] \tag{2}$$

where $\delta(x)$ is the Dirac-delta function. The modified object amplitude spectrum at the exit pupil of the optical system is given by

$$a'(x, y) = a(x, y) \cdot f(x, y) \tag{3}$$

where $f(x, y)$ is the pupil function of the optical system. For the given optical system the complex amplitude distribution in the image plane is given by the inverse Fourier transform of expression (3). Thus

$$A(u', v') = \iint_{pupil} a(x, y) \cdot f(x, y) \{ \exp 2\pi i (u'x + v'y) \} dx dy \tag{4}$$

The present work constitutes one-dimensional edge condition and hence, the general form of amplitude distribution is given by

$$A'(u', v') = \frac{1}{2} + \frac{1}{\pi} \int_0^1 f(x, 0) \frac{\sin(Zx)}{x} dx \tag{5}$$

where $Z=2\pi u'$ and $f(x, 0)$ is the coherent transfer function of the system. The coherent transfer function $f(x, 0)$ in the current study is rotationally symmetric and satisfies the condition

$$f(x, 0) = f(-x, 0) \tag{6}$$

Pupil function $f(r)$ for the shaded aperture amplitude filter is given by

$$f(x, y) = 1 - \beta r^2 \tag{7}$$

where r is the normalized distance of an arbitrary point on the pupil from its centre and β is the apodisation parameter. The term β controls the degree of non-uniformity of transmission over the pupil. A value of $\beta=0$, corresponds to diffraction limited Airy system having uniform transmission of unity over the entire aperture.

On introducing wave aberrations such as defocus, Coma and Primary spherical aberration expression (5) takes the form

$$A'(u', v') = \frac{1}{2} + \frac{1}{\pi} \int_0^1 (1 - \beta r^2) \exp \left[-i \left(\phi_d \frac{x^2}{2} + \phi_c \frac{x^3}{3} \cos(\theta) + \phi_s \frac{x^4}{4} \right) \right] \frac{\sin(Zx)}{x} dx \tag{8}$$

Now the intensity distribution of an edge image formed by an apodised optical system is given by the squared modulus of expression (8).

Thus

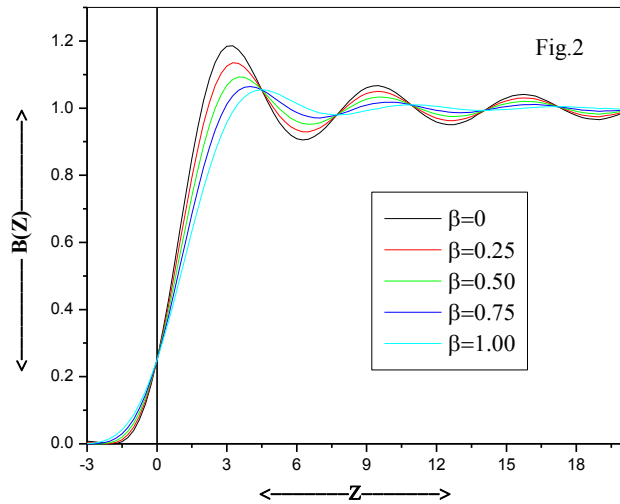
$$B(u') = B(Z) = |A'(u')|^2$$

$$B(Z) = |A'(Z)|^2 = \left| \frac{1}{2} + \frac{1}{\pi} \int_0^1 (1 - \beta r^2) \exp \left[-i \left(\phi_d \frac{x^2}{2} + \phi_c \frac{x^3}{3} \cos(\theta) + \phi_s \frac{x^4}{4} \right) \right] \frac{\sin(Zx)}{x} dx \right|^2 \tag{9}$$

III. RESULTS AND DISCUSSIONS

The investigations on the effects of aberrations on the images of straight edge objects formed by coherent optical systems apodised by the Shaded aperture in the case of circular aperture have been evaluated using the expression (9) by employing Matlab 7.8. The intensity distribution in the images of straight edge objects has been obtained for different values of dimensionless diffraction variable Z varying from -3 to 20 . The image quality assessment parameters such as edge-ringing, edge-gradient and edge-shift of the edge fringes have been studied as a function of apodisation parameter for various degrees of defocus, Coma and primary spherical aberration parameters.

Fig.2 shows the intensity distribution curves in the case of circular aperture for aberration-free optical system. It is clear that the edge-ringing is the maximum in the case of clear aperture ($\beta=0$), i.e., the magnitude of edge-ringing is the highest, in the case of unapodised optical system (Airy case). The edge-ringing is reducing along with edge-gradient at the cost of increasing in edge shift as the apodisation parameter β is increasing from 0 to 1. Figures from 3 to 6 depict the intensity distribution curves in the case of circular aperture in the presence of aberrations such as defocus, coma and primary spherical aberration, where, defocus and coma kept fixed at $\phi_d = \phi_c = \pi$ with $\theta = 0$ for different



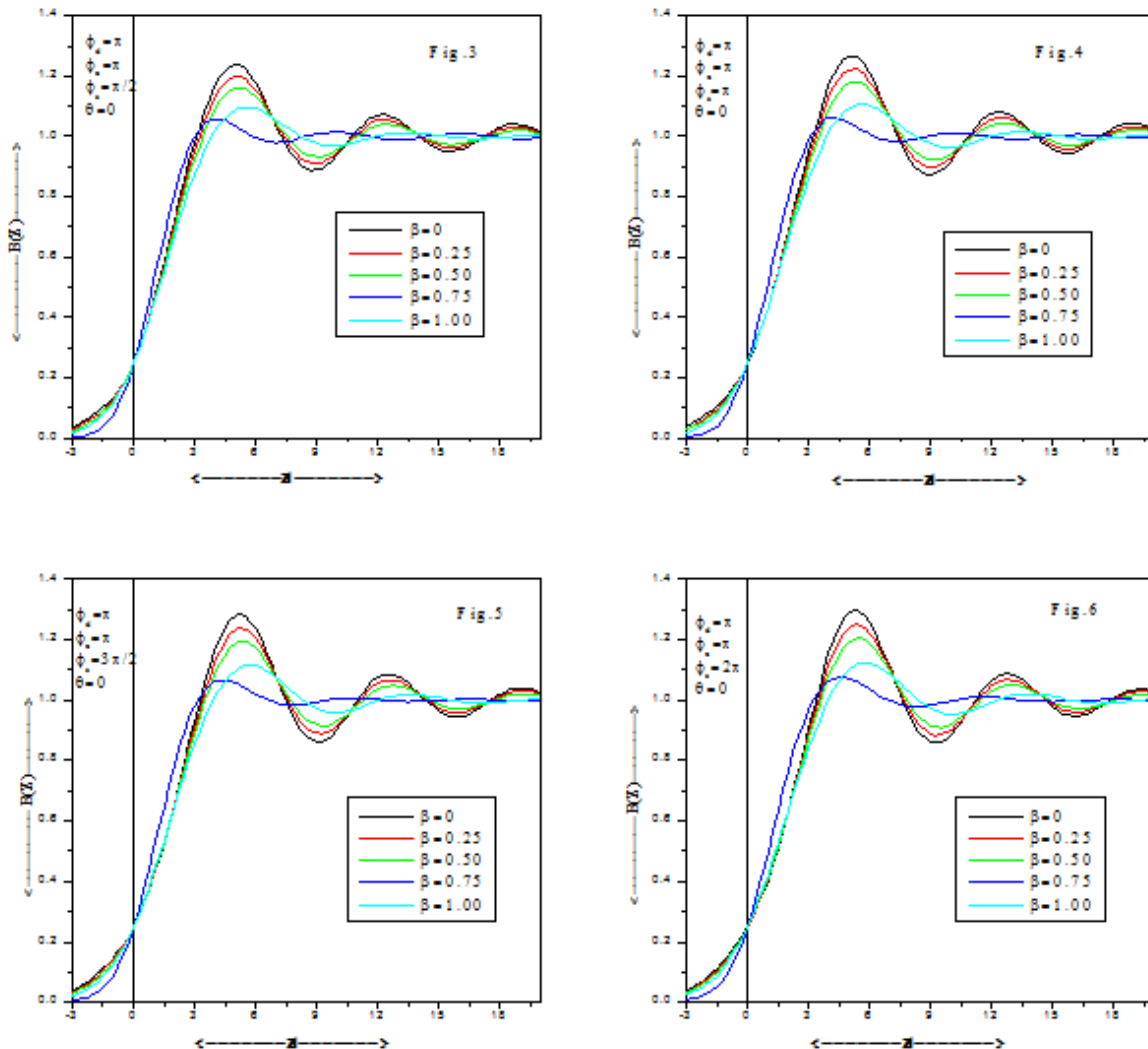
magnitudes of primary spherical aberration when β is varied from 0 to 1 in steps of 0.25. It is observed that for $\beta = 0.75$ there is a maximum suppression in edge-ringing for the given combination of $\phi_d = \phi_c = \pi$ and for all values of $\phi_s = \pi/2, \pi, 3\pi/2$ and 2π at $\theta = 0$ while the edge-gradient and edge shift attains the maximum and the minimum values respectively.

Fig.7 shows that the edge ringing is decreasing with apodisation parameter β for all values of ϕ_s at $\phi_d = \phi_c = \pi$. And it is also observed that for a given β value, the ringing is increasing with ϕ_s . The edge ringing is the maximum at $\phi_s = 2\pi$ for $\beta=0$ i.e., 0.29665 and it is the minimum at $\phi_s = 0$ for $\beta=1$ i.e., 0.08724. It means the edge ringing can be minimized by the operation of apodization even in the presence of aberrations. The edge ringing attains the minimum values at $\beta=1$ for all values of $\phi_s = 0, \pi/2, \pi, 3\pi/2$ and 2π respectively.

Fig.8 shows the variation of edge gradient with apodisation parameter β for given value of $\phi_d = \phi_c = \pi$. For $\phi_s = 0$, the edge gradient is gradually decreasing as β varies from 0 to 1. For $\phi_s = \pi/2, \pi, 3\pi/2$ and 2π edge gradient shows an increasing trend from $\beta=0$ to 0.75, however it is decreasing for $\beta > 0.75$. It is the minimum at $\beta=0$ for $\phi_s = 2\pi$ i.e., 0.12584 and is the maximum at $\beta=0.75$ for $\phi_s = \pi/2$ i.e., 0.22788.

Fig.9 shows the variation of edge shift with apodisation parameter β for given value of $\phi_d = \phi_c = \pi$. For $\phi_s = 0$, the edge shift is increasing as β varies from 0 to 1. For $\phi_s = \pi/2, \pi, 3\pi/2$ and 2π edge shift shows oscillatory changes, however it attains the minimum values at $\beta=0.75$. The shift is the maximum at $\beta=0$ for $\phi_s = 2\pi$ i.e., 1.4950 and is the minimum at $\beta=0.75$ for $\phi_s = \pi/2$ i.e., 0.8850. The product of edge-gradient and edge-shift is also computed and found that it is almost constant for the given combination. It can be seen more clearly from the fig.10.

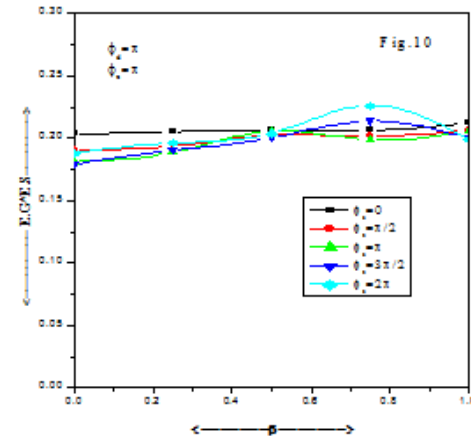
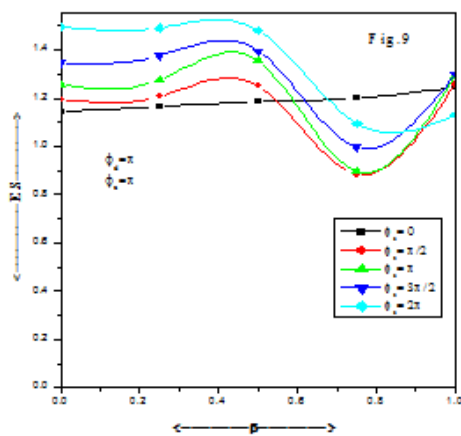
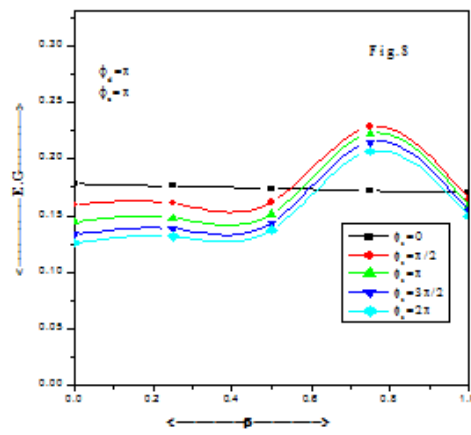
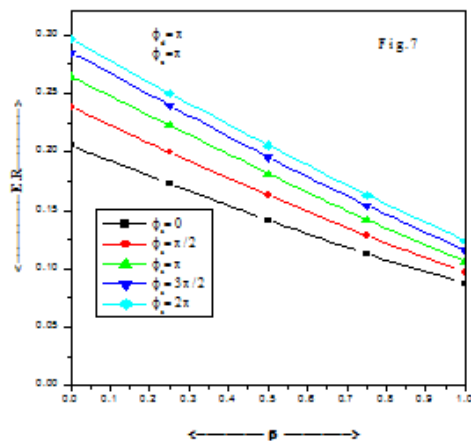
IMAGE INTENSITY DISTRIBUTIONS



IV. Conclusions

The important conclusions from the above investigations made on the aberrated, apodised optical systems are summarized as follows.

- i. The disturbing effects of aberrations on the image formation in the optical systems are well known.
- ii. Apodisation of the given optical system by the shaded optical filter mitigates the deleterious effect of the edge ringing in the absence of aberrations. The image intensity distribution at the straight edge rises monotonically from the dark region to the bright region.
- iii. The edge-ringing is increasing with coma as it is increasing from $\phi_c=0$ to 2π , for $\beta = 0.25$. But the pronounced ringing due the presence of coma has been mitigated gradually at certain defocused planes such as $\phi_d = \pi/2, \pi, 3\pi/2$ and 2π , when $\theta = \pi$.
- iv. The edge ringing is almost absolutely eliminated when the optical system is apodised with $\beta = 0.75$.
- v. The elimination of ringing is achieved with minimum edge shift and there is a considerable improvement in edge gradient for certain combinations of aberrations at $\beta=0.75$.
- vi. The reduction in the ringing pattern is more effective at suitable combination of aberrations than the aberration free cases and attains the minimum value for $\phi_c = 2\pi$ where it is completely balanced with ϕ_d .
- vii. Thus the degraded edge imaging characteristics due to one type of aberration has been recompensed by introducing some other type of aberrations.



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