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Stellar Measurements with the New Intensity Formula

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Abstract: *In this paper a linear relationship in stellar optical spectra has been found by using a spectroscopical method used on optical light sources where it is possible to organize atomic and ionic data. This method is based on a new intensity formula in optical emission spectroscopy (OES). Like the HR-diagram, it seems to be possible to organize the luminosity of stars from different spectral classes. From that organization it is possible to determine the temperature , density and mass of stars by using the new intensity formula. These temperature, density and mass values agree well with literature values. It is also possible to determine the mean electron temperature of the optical layers (photospheres) of the stars as it is for atoms in the for laboratory plasmas. The mean value of the ionization energies of the different elements of the stars has shown to be very significant for each star. This paper also shows that the hydrogen Balmer absorption lines in the stars follow the new intensity formula.*
Keywords: *Astrophysics, Emission Spectroscopy Linear, Relationships*

I. Introduction

The author and the collegue Dr. Sten Yngström have earlier presented a new formula for the intensity of spectral lines in optical emission spectroscopy (OES) in many previous papers and conferences.
According to a new theory in Ref 1 the intensity I(hν) is given by equation 1

$$I (h\nu) = C \ \lambda^{-2} \exp (- J/ kT) / ((\exp (h\nu/kT) – 1) \qquad (1)$$

where ν is the frequency of the λ is the wavelength of atomic spectral line, J the ionization energy of the atom, and C is a product of factors about sample properties (number densities of atoms and electrons) and the transition probability of the atom.
In earlier papers by us about this formula in Ref 2 and Ref 3, we studied absolute intensities. The intensities came from arc measurements and are tabled in Ref (4), which we have used in our studies. In these studies the new intensity formula was used in the development of this method of analysis. In this method ln (I λ^2) was plotted versus
hν (1+θ/hν ln(1-exp(-hν/θ))) eV for 17 elements.
Each intensity value is the mean value of many individual values. By forming the maximum between the difference between ln I λ^2 and ln λ^2 the following formula will be the basic equation in this method of analysis.

$$\ln (I_{max} \ \lambda^2_{max}) = \text{const.} - 1.6 \ J / h\nu_{max} \qquad (2)$$

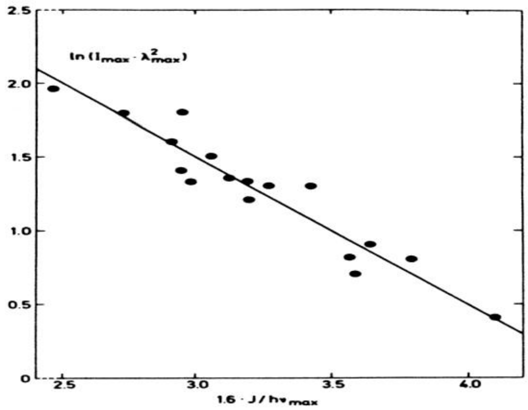

Fig 1 ln ($I_{max} \lambda^2_{max}$) plotted versus (1.6 J) / hν_{max}

for seventeen elements from the NBS tables in Ref 4.

This graph can be seen in Fig 1, where ln (I_{max} λ^2_{max}) has been plotted versus 1.6 J / hν_{max} = J /θ for 17 elements, where θ = k T_e (electron temperature). J denotes table value of ionization energy. This graph forms a good linear relationship, where hν_{max} = 1.6 θ. This means that this graph is a strong support of the new intensity formula, based on the new theory. It is also possible to measure the internal electron temperature for different elements. It has now shown to be possible to obtain similar linear relationships when using intensity data of stellar optical spectra. In Table 1 the electron temperature- and ionization energy values from

17 elements are shown with this method. The mean value of these electron temperature values are around 2 eV, which fit well with literature values Ref 5.
A very strong support of this new intensity formula has recently been published in two open access summary papers Ref 6 and Ref 7, where different methods from the literature have been used, which support the new intensity formula.

Table 1
Determination of the electron temperature for 17 elements of different ionization energies

Element	θ (eV)	J (eV)
Cs	1.6	3.89
Na	1.9	5.14
Ba	1.8	5.20
Li	1.8	5.39
Ca	2.1	6.11
Yb	2.1	6.25
Sc	2.1	6.70
Cr	2.3	6.76
Ti	2.1	6.83
Sn	2.1	7.33
Mo	2.3	7.38
Mn	2.3	7.43
Ag	2.1	7.57
Ni	2.1	7.63
Fe	2.1	7.86
Co	2.2	7.88
Pt	2.1	9.0

II. Ionic spectra

The intensity formula for ions has a similar appearance as equation 1 and is shown in equation 3. This formula include ionization energies for the first (J_1) and second (J_2) ionization energy, which has been proposed earlier in the detection limit method Ref 10 and in two open access ionic papers of Ref 8 and Ref 9, which have recently been published. C is a factor given by transition probabilities, number densities and sample properties. λ and ν are here the wavelength and frequency of the ionic spectral line. The ionic intensity formula has the following appearance :

$$I = C\, \lambda^{-2} (\exp (- (J_1+J_2)/ kT)) / (\exp (h\nu /kT) - 1) \qquad (3)$$

To show the validity of equation 3 with this method $\ln (I \lambda^2)$ was plotted versus $h\nu (1+\theta/h\nu \ln(1-\exp(-h\nu/\theta)))$ eV for 11 elements; each intensity value comes from many individual values from the NBS table of Ref 4. By forming the maximum between the difference between $\ln I\lambda^2$ and $\ln \lambda^2$, $\ln (I_{max}\ \lambda^2_{max})$ was plotted versus $1.6 (J_1+J_2) /\ h\nu_{max} =(J_1+J_2)/\theta$ in the same way as for atoms which is seen above. The points will follow an expression in equation 4 for ions which is similar to equation 2 for atoms.

$$\ln (I_{max}\ \lambda^2_{max}) = \text{const.} - 1.6 (J_1+J_2) / h\nu_{max} \qquad (4)$$

Equation 4 includes ionization energies for the first and second ionization energy. 11 different elements were plotted in this way and (J_1+J_2) and θ are tabled in Table 2 for 11 ionic elements. These values fit well with secondary electron temperature values from the literature in Ref 5.
A similar plot to Fig 1 for atoms has also been done for ions of 11 elements, which is seen in Fig 2. The mean value of electron temperatures are about 4 eV for ions, which fit well with the literature values Ref 5.

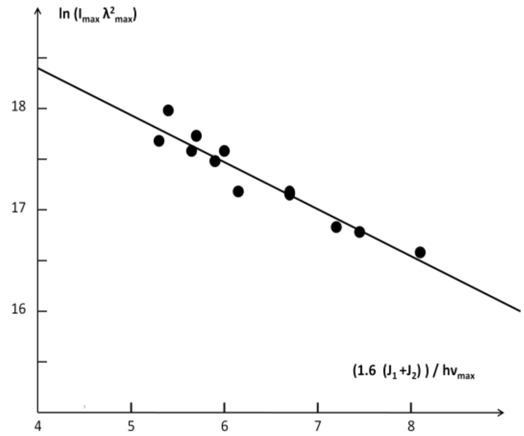

Fig 2 ln (I_{max} λ^2_{max}) plotted versus (1.6 (J_1 + J_2))/ h ν_{max} for eleven ionic elements from the NBS tables.

Table 2
Determination of the electron temperature for 11 ionic elements of different ionization energies

Element	(J_1 + J_2) (eV)	θ (eV)
Yb	18.36	3.3
Y	19.00	3.3
Sc	19.60	3.6
Ti	20.44	3.4
Mn	23.07	4.4
Cr	23.46	3.5
Fe	24.10	4.2
C	35.65	4.9
K	36.15	4.5
Cs	36.35	5.1
Cu	28.00	5.0

According to Refs 8 and 9 a general recursion formula for two adjacent ionic states(r and r+1) could be written in the following way :

$$I_{r+1} = C_r \lambda^{-2}(e x p (- (J_r+J_{r+1})/ k T)) / (e x p (h \nu /k T) – 1) \qquad (5)$$

III. Stellar Spectra

These stellar optical spectra extend over the spectral classes O – M and the photometrically well-calibrated luminosity measurements from star to star, and come from Ref 11 . Good temperature and luminosity coverage have been achieved. The data were digitalized from the main sequence classed O5 – F0 and F6 – K5 displayed in term of relative flux as a function of wavelength. The parameters that have been measured in this investigation are maximum luminosity L_{max}(Rel.fluxmax) of the Planck curve. In this maximum the wavelength λ_{max} and the maximum frequency ν_{max} were also measured.

Then ln (L_{max} λ_{max}^2) values were plotted versus (1.6 $J_{meanvalue}$ / hν_{max}) where $J_{meanvalue}$ is the mean value of the ionization energies of the elements of the stars measured. To obtain a similar linear relationship for the stellar data as in Fig 1 from the spectroscopical method from Refs 2 and 3, the following luminosity data from Ref 11 and data from Table 3 were used and plotted according to equation 6

$$\ln (L_{max} \lambda_{max}^2) = const – (1.6 J_{meanvalue}/ h\nu_{max}) \qquad (6)$$

which is similar to equation 2 for atoms and equation 4 for ions.

To obtain the values of Table 3 it is necessary to use a two step procedure. In the first step it is necessary to define the graph by calculating the $J_{meanvalue}$ of the G2-star. The $J_{meanvalue}$ can be
expressed in the following way :

$$J_{meanvalue} = \sum c_n J_n \qquad (7)$$

where c_n is the normalized content of an element of a star. It is plausible to consider the content values of G2-stars rather to be similar to the content values of the sun. Therefore
the c_n-values of the sun have been used here. J_n is here the ionization of a star. This $J_{meanvalue}$ has been calculated for the sun (G2 star), which gave $J_{meanvalue}$ = 16.2 eV according to the linear graph in Fig 3. This value

is 16.2 eV, too, for the sun when using equation 6 together with established chemical composition values of the sun. This means that we now have one point determined in Fig 3.
A more profound description of this method of creating Fig 3 and Table 3 is described in Ref 12.

Table 3
Determination of the electron temperature of the stars from different spectral classes

Spectral class	θ (eV)	$J_{meanvalue}$ (eV)
K5V	1.44	15.5
K4V	1.47	15.6
G9-K0	1.50	15.8
G6-G8	1.53	16.0
G1-G2	1.56	16.2
F8-F9V	1.63	16.7
F6-F7V	1.63	16.5
A9-F0V	1.72	16.9
A8	1.75	17.1
A5-A7	1.81	17.5
A1-A3	1.84	17.6
B6V	1.88	17.8
B3-B4V	1.94	18.0
O7-B0V	1.97	18.1
O5V	2.00	18.2

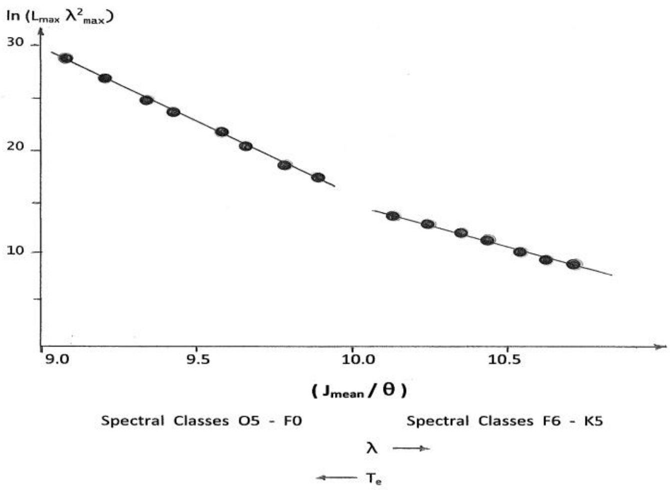

Fig 3
ln (L_{max} λ^2_{max}) plotted versus (1.6 $J_{meanvalue}$) / $h\nu_{max}$
for different stars from spectral classes O – M (from Ref 12)

The data in Fig 3 constitute a straight line in the classes O5 – F0 and F6 – K5.
In equation 6 $h\nu_{max} = 1.6\ \theta$, where θ = internal electron temperature in eV. This means that the classes O5 – F0 have higher temperature than the classes F6 – K5, which is also in accordance with the usual HR-diagram. For example a G2 star (the sun) has θ = 1.56 eV (T_e = 18110 K).

IV. The Use Of The Balmer Lines

It is shown in the in the paper by Ref 11 that the appearance of the continuous-and discrete spectra of stars seem to be the same, where the hydrogen Balmer absorption lines of different stars have been studied. These are the well known Planck curves with steep low wavelength side and a slow high wavelength side. The wavelength of the intensity maximum of continuous-and discrete spectrum seems to be the same. This in agreement with equation 1 and the new theory, where the Planck factor is a part of the new intensity formula. This isclearly seen in Fig 4 from the spectrum of two A-stars. The normalized flux is here propor-tional to the emissions from the continuous-and discrete spectrum. These curves show very good examples of Planck curves, where continuous-and discrete emissions seem to have the same wavelength maximum. The wavelengths of the Balmer lines are shown in Table 4 (Ref 13). By using equation 6 and Table 3intensity ratios have been determined theoretically(from intensity formula) and experimentally by using the data of Ref 11 , from different spectral classes of stars. At the use of these intensity ratios J_H = 13.595 eV for hydrogen was used. The electron temperatures for different spectral classes have earlier been determined in Table 1 in Ref (12). A summary of the values from the spectral classes of this paper is shown in Table 5 Nice correlation (r=0.98) has been achieved between theoretical-and experimental intensity ratios.This is shown in Fig 5 and is, together with Fig 4, a strong evidence of the fact that stars follow the new intensity formula, as atoms and ions do. Fig 5 shows very nice correlation between experiment and theory.

Table 4
Balmer lines used here

H_α	6562.80 Å
H_β	4861.32 Å
H_γ	4340.46 Å
H_δ	4101.73 Å
H_ε	3970.07 Å

Table 5
Spectral classes and mean electron temperature

class	**θ (eV)**
A8	1.75
A5-A7	1.81
A1-A3	1.84
B6V	1.88
B3-B4V	1.94

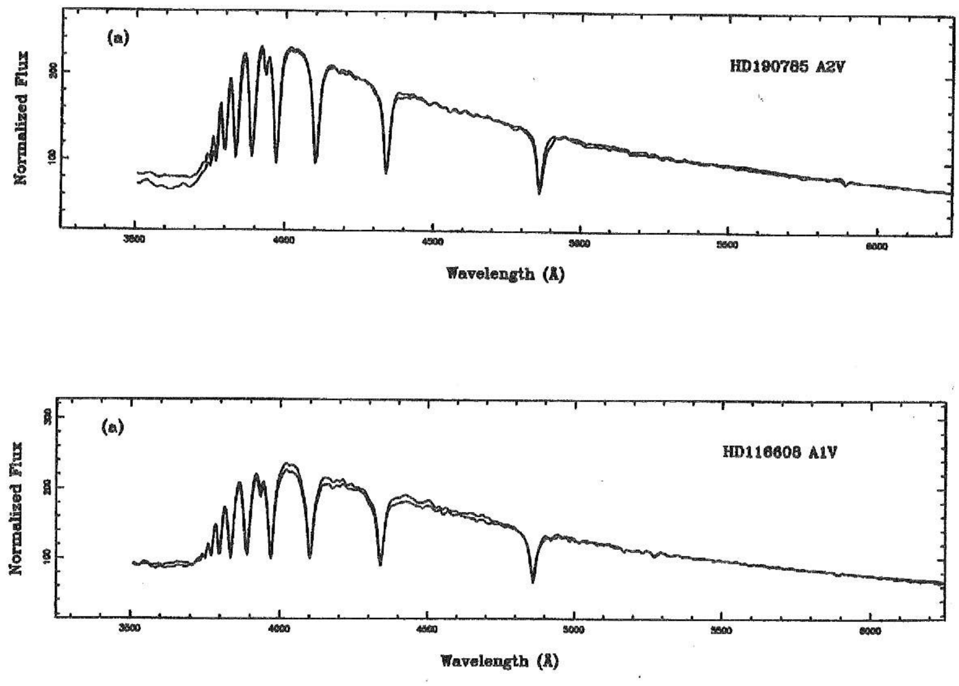

Fig 4
Plot of normalized flux versus the wavelength(Planck curve) for two different A-stars.
The absorption hydrogen Balmer lines are clearly observed. The wavelength of the intensity maxima for both continuous and discrete emissions seems to be the same.
(From Ref 11)

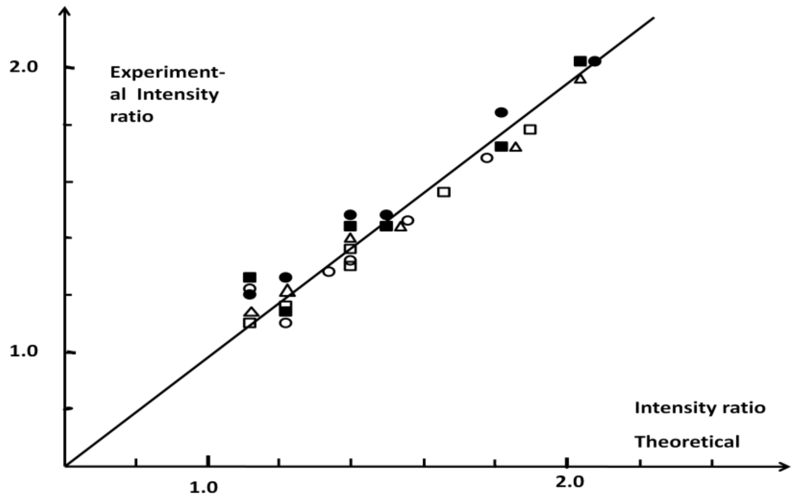

Fig 5 Spectral intensity ratios (experimental and theoretical) give very good correlation(r=0.98) using the Balmer lines from different spectral classes of stars using the new intensity formula.
Spectral classes used: A8=unfilled circles , A5-A7=unfilled squares, A1-A3=unfilled triangles, B6V= filled circles, B3-B4=filled squares.

V. Determination Of The Effective Temperature Of Stars

Table (66) p.564 in Ref (13) were then used, where the effective temperatures were tabled from many main sequence stars from different spectral classes (A-K).These effective temperature values were then plotted versus the electron temperature values from corresponding spectral class from Table 3 in this paper. In this way effective temperature values have been obtained for 12 main sequence stars and are tabled in Table 6.
Good correlation (+- 85 K) is here achieved between the values from this investigation and the literature values based on the Stefan-Boltzmann temperature law and can be seen in Table 6 and Fig 6, which show good correlation (r=0.99).

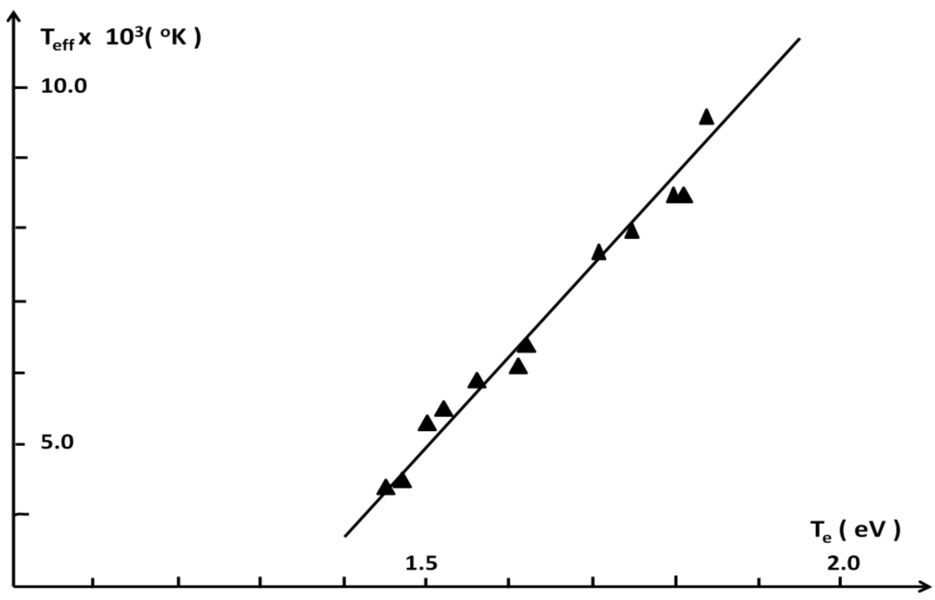

Fig 6 Effective temperature plotted versus electron temperature for a number of main sequence stars.(correlation r= 0.99)

Table 6
Determination of $T_{effective}$ of stars

	From the graph	From the literature	Spectral group
Vega	9300° K	9300° K	A0
Altair	8100	8000	A7
Procoyon A	7500	7500	F5
Sun	5700	5740	G2
Sirius A	9500	9700	A1
Aldebaran	4200	4100	K5
Pollux	4700	4500	K0
Capella B	5200	4940	G5-G0
Regulus	9700	10300	B7
Canopus	7250	7350	F0
Fomalhaut	8700	8500	A3
Sirius B	8400	8200	A5

VI. Determination Of The Density Of A Star By Using Balmer Lines.

According to equation 1 the C-factor is a product of factors of number densities of atoms and electrons. By using the approximate formula of equation of equation 1 we obtain :

$$\mathbf{I = C \lambda^{-2} \exp (- (h\nu + J)/ kT_e)} \qquad (8)$$

By expressing C as a function of the other parameters in equation 8 and by taking the ratio between the density of a star compared to the sun, we obtain the following expression

$$\mathbf{C_{star} / 1 = (I\gamma_{star} / I\gamma_{sun})(\lambda_{max\,star} / \lambda_{max\,sun})^2 \exp ((h\nu_{max\,star} + J_H)/ \theta_{star} - (h\nu_{max\,sun} + J_H)/ \theta_{sun})} \qquad (9)$$

where C = 1 is the sun value and $\theta = kT_e$. The intensity ratio ($I\gamma / I\gamma_{sun}$) here is the ratio between the γ-Balmer line from the star and the sun from the data of Ref (11). λ_{max} and the $h\nu_{max}$ have also been taken from Ref (11) and the electron temperature values have been taken from Ref (12) for different spectral classes. J_H is the ionization energy of hydrogen.

The results of 12 stars here, are shown in Fig 7 and Table 7 where ρ / ρ_0 –values been calculated for 12 different main sequence stars. In Fig 7 a straight line is achieved following
in the near of the Schwarzschild line Ref 13 (p.555).

Table 7
Determination of density ratio of 12 stars relative to the sun

Star	ρ / ρ_{sun}(new method)	spectral class
Aldebaran	1.26	K5
Pollux	0.95	K0
Capella B	0.91	G5
Sun	1.0	G2
Procyon A	0.66	F5
Canopus	0.55	F0
Altair	0.45	A7
Sirius B	0.37	A5
Formalhaut	0.42	A3
Sirius A	0.42	A1
Vega	0.39	A0
Regulus	0.28	B7
η Ori	0.14	B1

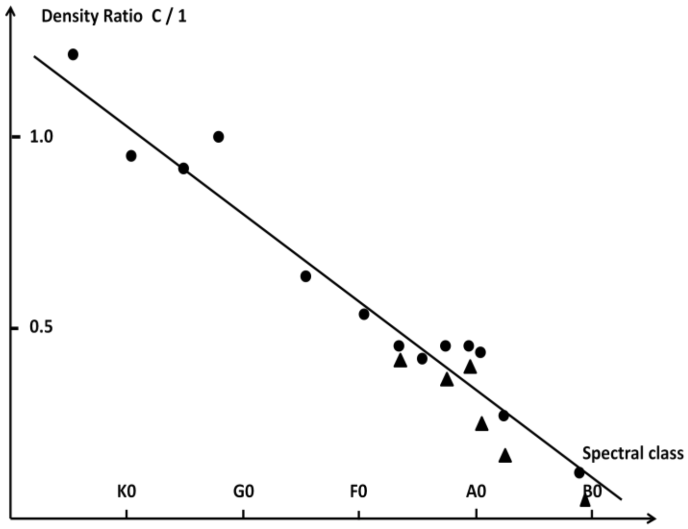

Fig 7 Density determination of stars relative to the sun at different spectral classes.Filled circles = new method , Filled triangles = Schwarzschild limit

VII. Determination Of The Mass Of The Stars

According to the usual Mass-Luminosity relation in astronomy, there is a linear relationship between luminosity and mass of a star. In a similar way there is a possibility to use the equation 1 in a similar way by the fact that M α kT_e .By using the approximate formula of equation 1 we obtain :

$$\mathbf{I = C\,\lambda^{-2}\exp(-(h\nu + J)/kT_e)} \qquad \mathbf{(10)}$$

By expressing kT_e as a function of the other parameters in equation 10 and by taking the ratio between the mass of a star compared to the sun (M_0), we obtain the following expression :

$$\mathbf{M_{star} / M_{0\,sun} = \ln(I\,\lambda^2/C_0)_{max\,sun}(J_{mean\,star} + h\nu_{max\,star}) / \ln(I\,\lambda^2/C)_{maxstar}(J_{mean\,sun} + h\nu_{max\,sun})} \qquad \mathbf{(11)}$$

The ln $I\lambda^2$, C , J_{med} and $h\nu_{max}$ - values in equation 11 can be determined from Fig 3 and Table 3 in this paper. The l n ($I\lambda^2$) –values can be shown directly from the graph in Fig 3 for a certain star and the C-values are shown as the prolongation of the two lines in Fig 3 for a certain star placed on one the lines. The M / M_0 – values have been tabled in Table 8 for 10 different stars, which show good agreement with the literature values. This good agreement is also shown in Fig 8 between (M / M_0) – values from this new method and literature values and show a nice linear relationship (r=0.97)

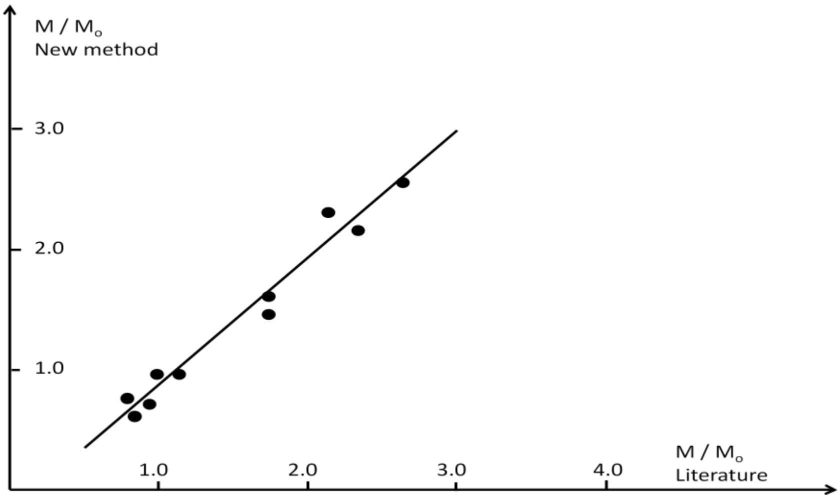

Fig 8. Determination of the mass of a number of stars with the new method together with literature values. (Correlation r= 0.97) .

Table 8
Determination of the mass ratio relative to the sun

Star	$M / M_{o\ (new\ method)}$	$M / M_{o(Literature)}$	Spectral class
Vega	2.55	2.50	A0
Formalhaut	2.15	2.30	A3
Sirius A	2.35	2.10	A1
Altair	1.62	1.70	A7
Dubhe	1.40	1.70	F0
α Centauri A	1.00	1.10	G2
Sun	1.00	1.00	G2
Capella B	0.76	0.80	G5
α Centauri B	0.72	0.90	K1
Eksilon	0.62	0.83	K2V

VIII. Discussion

This method of analysis has shown to be a simple method of verifying the new intensity formula by using atomic, ionic and stellar data. By using this method together with the new intensity formula it has been possible to determine the mean electron temperature in different laboratory plasmas and in the optical layers of a star without knowing so much about the chemical composition of the star. These mean electron temperature values fit well with other methods from the literature. The method also gives an organizing method for stars similar to the established HR-diagram. The $J_{meanvalue}$ has shown to be a kind of "signum" for every star. Fig 3 has shown to be a valuable and simple method of organizing and classifying the stars without knowing so many other details about the stars. The Balmer spectral absorption lines seem to follow the new intensity formula too, which is clearly seen in Figs 4 and 5.This is clearly seen by the correlation coefficient (0.98) . This means that discrete emissions in the star do follow the new intensity formula but are heavily absorbed in the star. Therefore, the light coming from the star is mostly continuous radiation following Planck radiation law.
It has also been possible to determine the effective temperature of a number of stars from different spectral classes on the main sequence. The results gave good agreement with the established temperature method by Stefan-Boltzmann.

It has also been possible to determine the density of a number of stars compared to the sun from different spectral classes on the main sequence. These values are in accordance with the Schwarzschild limit. The graph in Fig 7 shows a nice linear relationship.
It has also been possible to determine the mass of a number of stars compared to the sun from different spectral classes on the main sequence. These values are in accordance with the literature values. The graph in Fig 8 shows a nice linear relationship.

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