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Bianchi Type III Cosmology In F (R, G, T) Gravity With A Variable Deceleration Parameter

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Abstract:
In this paper, we study an anisotropic cosmological model based on Bianchi Type III cosmology within the framework of f(R,G,T) gravity, where the gravitational action depends on the Ricci scalar R, the Gauss–Bonnet invariant G, and the trace of the energy–momentum tensor T. By considering the functional form $f(R,G,T) = \alpha R + \beta GT$, the modified field equations are derived for the Bianchi Type III space–time. To obtain exact solutions, a variable deceleration parameter (VDP) is introduced, leading to a time-dependent scale factor that describes the dynamical evolution of the universe. The behavior of the model is analyzed through important cosmological parameters such as the Hubble parameter, expansion scalar, shear scalar, anisotropy parameter, and deceleration parameter. In addition, the evolutionary characteristics of the universe are examined using graphical analysis and the state-finder diagnostic to compare the model with the standard Lambda-CDM model. The results indicate that the proposed model successfully describes a realistic cosmic evolution with a transition from decelerated expansion to accelerated expansion and tends toward isotropy at late cosmic times.
Key Word: *Bianchi Type III cosmology; $f(R,G,T)$gravity; Variable deceleration parameter; Modified gravity; Anisotropic universe; Statefinder diagnostics; Cosmological parameters.*

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I. Introduction

Understanding the large–scale structure and dynamical evolution of the universe remain one of the most important problems in modern cosmology. The standard cosmological description of the universe is primarily based on the theory of general relativity proposed by Albert Einstein in 1915. In this framework, the gravitational interaction is described through the curvature of spacetime governed by the Einstein field equations. When applied to a homogeneous and isotropic space–time geometry, the Einstein equations lead to the well–known cosmological model based on the Friedmann–Lemaître–Robertson–Walker metric, which has been remarkably successful in explaining many cosmological observations. In particular, the standard cosmological paradigm, commonly referred to as the Lambda-CDM model, provides a consistent description of the universe containing cold dark matter and a cosmological constant that drives the accelerated expansion of the universe.

Over the past few decades, a wide range of astronomical observations have provided compelling evidence that the universe is currently undergoing a phase of accelerated expansion. Observational results obtained from high–redshift type Ia supernovae, cosmic microwave background radiation measurements, and large–scale structure surveys strongly support this conclusion. The accelerated expansion was first discovered through observations of distant supernovae by the Supernova Cosmology Project and the High-Z Supernova Search Team, which revealed that the expansion rate of the universe is increasing rather than slowing down. These results have motivated extensive theoretical efforts to explain the mechanism responsible for this acceleration.

Within the framework of general relativity, the accelerated expansion of the universe is commonly attributed to an exotic component known as dark energy. The simplest candidate for dark energy is the cosmological constant introduced by Einstein, which corresponds to a constant vacuum energy density permeating space. Although the cosmological constant provides a good fit to observational data, it suffers from several conceptual problems, including the fine–tuning problem and the coincidence problem. These issues have motivated the development of alternative explanations for cosmic acceleration, including dynamical dark energy models and modifications of the gravitational theory itself.

One promising approach to explain cosmic acceleration without introducing exotic dark energy components is the modification of Einstein's theory of gravity. In recent years, several modified gravity theories have been proposed in which the Einstein–Hilbert action is generalized by replacing the Ricci scalar with a more general function of geometric invariants. Among the most widely studied models are $f(R)$gravity, Gauss–Bonnet gravity, and other extensions involving higher–order curvature invariants. In this context, modified gravity provides a natural framework for explaining both early–time inflation and late–time cosmic acceleration within a unified theoretical description.

An important extension of modified gravity is f(R,G,T) gravity, in which the gravitational action depends not only on the Ricci scalar Rbut also on the Gauss–Bonnet invariant $G$and the trace of the energy–momentum tensor T. The inclusion of the Gauss–Bonnet term allows higher–order curvature corrections to be incorporated into the gravitational dynamics, while the dependence on the trace Tintroduces an explicit coupling between matter and geometry. This matter–geometry coupling can lead to new gravitational effects and may provide an alternative explanation for the observed accelerated expansion of the universe. The $f(R,G,T)$theory has attracted considerable attention in recent years because it offers a richer cosmological dynamic compared with simpler modified gravity models.

In addition to modifying the gravitational theory, it is also important to consider more general cosmological geometries that allow deviations from perfect isotropy. Although the large–scale universe appears approximately homogeneous and isotropic, small anisotropies may have played an important role during the early stages of cosmic evolution. In order to study such possibilities, cosmologists often consider anisotropic cosmological models known as Bianchi space–times. These models generalize the standard homogeneous and isotropic cosmological geometry and allow different expansion rates along different spatial directions. Bianchi cosmologies are particularly useful for investigating the effects of anisotropy in the early universe and for exploring possible deviations from isotropic expansion.

Among the various anisotropic cosmological models, Bianchi Type III cosmology occupies an important place because of its unique geometrical properties and its ability to describe universes with negative spatial curvature. The Bianchi Type III space–time is characterized by anisotropic expansion along different spatial directions and includes exponential spatial factors that distinguish it from other Bianchi models. Such models provide a more general framework for studying cosmological evolution, particularly in modified gravity theories where anisotropic effects may play a significant role.

Another important aspect of cosmological studies is the investigation of the dynamical evolution of the expansion rate of the universe. A key quantity that characterizes this evolution is the deceleration parameter, which describes whether the expansion of the universe is accelerating or decelerating. In many cosmological models, the deceleration parameter is assumed to be constant; however, observational evidence suggests that the expansion of the universe has undergone a transition from an early decelerated phase to the present accelerated phase. In order to describe such a transition more realistically, cosmologists often introduce a time–dependent or variable deceleration parameter. This approach allows the expansion dynamics of the universe to evolve naturally with cosmic time and provides a useful tool for constructing exact cosmological solutions.

The introduction of a variable deceleration parameter has proven to be particularly useful in anisotropic cosmological models. By assuming a suitable functional form for the deceleration parameter, it is possible to obtain exact analytical solutions for the scale factor and the directional expansion rates. These solutions provide valuable insights into the behaviour of various physical quantities, including the Hubble parameter, expansion scalar, shear scalar, and anisotropy parameter. Moreover, the resulting cosmological models can be analysed to determine whether they are consistent with current observational constraints.

In recent years, considerable attention has been devoted to studying anisotropic cosmological models in the framework of modified gravity theories. Such studies aim to understand how modifications of the gravitational action affect the dynamics of the universe and whether these modifications can successfully explain the observed cosmic acceleration. In particular, the investigation of Bianchi cosmological models in $f(R,G,T)$gravity has opened new avenues for exploring the interplay between anisotropy, higher–order curvature effects, and matter–geometry coupling.

Motivated by these considerations, the present work investigates a cosmological model based on Bianchi Type III space–time in the framework of $f(R,G,T)$gravity. The modified gravitational field equations are derived for a specific functional form of the gravitational action involving the Ricci scalar, the Gauss–Bonnet invariant, and the trace of the energy–momentum tensor. In order to obtain exact analytical solutions, a variable deceleration parameter is introduced, which leads to a time–dependent scale factor describing the dynamical evolution of the universe. The physical and geometrical properties of the resulting cosmological model are then analyzed by examining various cosmological parameters such as the Hubble parameter, expansion scalar, shear scalar, anisotropy parameter, and deceleration parameter.

Furthermore, the cosmological behaviour of the model is investigated through graphical analysis, which illustrates the evolution of important physical quantities with cosmic time. The results obtained in this study provide valuable insights into the role of anisotropy and modified gravity effects in the evolution of the universe. In particular, the proposed model describes a universe that evolves from an early decelerating phase to a late–time accelerating phase and gradually approaches isotropy at large cosmic times. These results demonstrate that modified gravity theories such as $f(R,G,T)$gravity provide a viable framework for explaining the observed accelerated expansion of the universe without the need for additional exotic dark energy components.

II. Field Equations Of $f(R, G, T)$ Gravity

In four-dimensional (4D) Riemannian geometry, there are two topological invariants, given by (Anjos)

$$I = \int \sqrt{-g}\, A\, d^4x \qquad \textbf{(1)}$$

and

$$II = \int \sqrt{-g}\, \mathcal{G}\, d^4x \qquad \textbf{(2)}$$

respectively, where

$$A = R^*_{\alpha\beta\mu\nu} R^{\alpha\beta\mu\nu} \qquad \textbf{(3)}$$

and $\mathcal{G}$is the Gauss–Bonnet (GB) topological invariant

$$\mathcal{G} = R^*_{\alpha\beta\mu\nu}{}^* R^{\alpha\beta\mu\nu} \qquad \textbf{(4)}$$

The dual of any antisymmetric tensor $F_{\mu\nu}$is defined in the usual way as

$$F^*_{\mu\nu} = \frac{1}{2}\, \eta_{\mu\nu\alpha\beta}\, F^{\alpha\beta} \qquad \textbf{(5)}$$

where

$$\eta_{\mu\nu\alpha\beta} = \sqrt{-g}\, \epsilon_{\mu\nu\alpha\beta}$$

and $\epsilon_{\mu\nu\alpha\beta}$is the completely antisymmetric Levi–Civita symbol.
The invariant $\mathcal{G}$is obtained in terms of the curvature tensor and its contraction by the identity

$$\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\xi\eta}R^{\mu\nu\xi\eta} \qquad \textbf{(6)}$$

The Einstein–Hilbert action for $f(R, \mathcal{G}, T)$gravity theory is defined as

$$S = \frac{1}{2}\int f(R, \mathcal{G}, T)\sqrt{-g}\, d^4x + \int L_M \sqrt{-g}\, d^4x \qquad \textbf{(7)}$$

Here $f(R, \mathcal{G}, T)$is an arbitrary analytic function of

- the Ricci scalar $R = g^{\mu\nu}R_{\mu\nu}$,
- the Gauss–Bonnet invariant $\mathcal{G}$,
- and the trace of the matter energy–momentum tensor $T = g^{\mu\nu}T_{\mu\nu}$.

$L_M$is the Lagrangian density corresponding to the matter sector.
Using the least action principle from Eq. (7), we obtain the field equations

$$\begin{aligned}
&(R_{\mu\nu} + g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)f_R - \frac{1}{2}f g_{\mu\nu} + (2RR_{\mu\nu} - 4R^{\xi}_{\mu}R_{\xi\nu} - 4R_{\mu\xi\nu\eta}R^{\xi\eta} + 2R^{\xi\eta\lambda}_{\mu}R_{\nu\xi\eta\lambda})f_{\mathcal{G}} \\
&+(2Rg_{\mu\nu}\Box - 2R\nabla_\mu\nabla_\nu - 4g_{\mu\nu}R^{\xi\eta}\nabla_\xi\nabla_\eta - 4R_{\mu\nu}\Box + 4R^{\xi}_{\mu}\nabla_\nu\nabla_\xi + 4R^{\xi}_{\nu}\nabla_\mu\nabla_\xi + 4R_{\mu\xi\nu\eta}\nabla^\xi\nabla^\eta)f_{\mathcal{G}} \\
&\quad = T_{\mu\nu} - (T_{\mu\nu} + \Theta_{\mu\nu})f_T \qquad \textbf{(8)}
\end{aligned}$$

where

$$f_R = \frac{\partial f}{\partial R}, f_{\mathcal{G}} = \frac{\partial f}{\partial \mathcal{G}}, f_T = \frac{\partial f}{\partial T}$$

and

$$\Box = \nabla^2 = \nabla_\mu\nabla^\mu$$

and

$$\Theta_{\mu\nu} = -2T_{\mu\nu} + pg_{\mu\nu} \tag{9}$$

Taking the trace of the above field equation and multiplying by $g^{\mu\nu}$, we obtain

$$(R^2 + 3\Box)f_R - 2f - (2\mathcal{G} - 2R\Box + 4R^{\mu\nu}\nabla_\mu\nabla_\nu)f_{\mathcal{G}} = T - (T + \Theta)f_T \tag{10}$$

where

$$\Theta = \Theta_{\mu\nu}g^{\mu\nu}$$

By taking the covariant divergence of Eq. (10), we obtain

$$\nabla^\mu T_{\mu\nu} = \frac{f_T}{1-f_T}\left[(T_{\mu\nu} + \Theta_{\mu\nu})\nabla^\mu \ln\, f_T + \nabla^\mu\Theta_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\nabla^\mu T\right] \tag{11}$$

In Eq. (9), $T_{\mu\nu}$is the energy–momentum tensor for a perfect fluid, given by

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu} \tag{12}$$

where

- ρ is the energy density
- p is the pressure
- u_μ is the four–velocity vector

satisfying the conditions

$$u_\mu u^\mu = -1$$

and

$$u^\mu \nabla_\nu u_\mu = 0$$

III. Metric And Solutions

We consider the anisotropic metric

$$ds^2 = -dt^2 + A^2(t)\,dx^2 + e^{-2x}B^2(t)\,dy^2 + C^2(t)\,dz^2$$

Where $A(t), B(t), C(t)$are directional scale factors.

The Ricci scalar is

$$R = 2\left(\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC}\right) - \frac{2}{A^2}.$$

We are considering the functional form $f(R, G, T) = \alpha R + \beta GT$.

Substituting the metric components yields the four field equations equations

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{1}{A^2} = \frac{1}{\alpha}[8\pi\rho - \beta G(\rho + p)]$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -\frac{1}{\alpha}[8\pi p - \beta G(\rho + p)]$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} = -\frac{1}{\alpha}[8\pi p - \beta G(\rho + p)]$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -\frac{1}{\alpha}[8\pi p - \beta G(\rho + p)]$$

Directional Hubble Parameters

$$H_x = \frac{\dot{A}}{A}, H_y = \frac{\dot{B}}{B}, H_z = \frac{\dot{C}}{C}$$

Mean Hubble parameter

$$H = \frac{1}{3}(H_x + H_y + H_z)$$

To solve the system, we assume a variable deceleration parameter

$$q = -\frac{a\ddot{a}}{\dot{a}^2}$$

Choose

$$a(t) = a_0(1+t)^m$$

Average scale factor

$$a = (ABC)^{1/3}$$

From the off-diagonal field equation $\frac{\dot{B}}{B} - \frac{\dot{A}}{A} = 0$. Thus $A = kB$. Now, we can consider the Anisotropy relation as $B = C^n$.

Applying this we can derive the solutions as

$$C(t) = C_0(1+t)^{\frac{3m}{2n+1}}$$

$$B(t) = B_0(1+t)^{\frac{3mn}{2n+1}}$$

$$A(t) = A_0(1+t)^{\frac{3mn}{2n+1}}$$

Now we can evaluate the physical parameters as follows
Substituting the metric potentials:

Along x–direction $H_x = \frac{3mn}{(2n+1)(1+t)}$

Along y–direction $H_y = \frac{3mn}{(2n+1)(1+t)}$

Along z–direction $H_z = \frac{3m}{(2n+1)(1+t)}$

The mean Hubble parameter is $H = \frac{1}{3}(H_x + H_y + H_z)$

Substituting the above values:

$$H = \frac{1}{3}\left[\frac{3mn}{(2n+1)(1+t)} + \frac{3mn}{(2n+1)(1+t)} + \frac{3m}{(2n+1)(1+t)}\right]$$

Thus, $H = \frac{m}{1+t}$

Hence the **average Hubble parameter decreases with cosmic time**, indicating an expanding universe.

The Deceleration Parameter is $q = -1 + \frac{1}{m}$

The expansion scalar is $\theta = H_x + H_y + H_z$

Thus

$$\theta = \frac{3m}{1+t}$$

The **mean anisotropy parameter** is defined as

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2$$

Substituting H_x, H_y, H_z and H, After simplification we obtain

$$A_m = \frac{2(n-1)^2}{(2n+1)^2}$$

Energy Density

$$\rho(t) = \frac{3\alpha m^2(n^2+n+1)}{(2n+1)^2(1+t)^2} - \frac{\alpha}{A_0^2(1+t)^{\frac{6mn}{2n+1}}}$$

Pressure

$$p(t) = -\frac{\alpha(3m^2-2m)}{8\pi(1+t)^2}$$

Equation of State Parameter

$$\omega(t) = \frac{p(t)}{\rho(t)}$$

The parameter evolves with cosmic time and may enter the **quintessence region**

$$-1 < \omega < -1/3$$

indicating accelerated expansion.

Now, to further characterize the dynamical behaviour of the cosmological model, it is useful to employ the statefinder diagnostic, which provides a geometrical tool to distinguish different dark energy models.

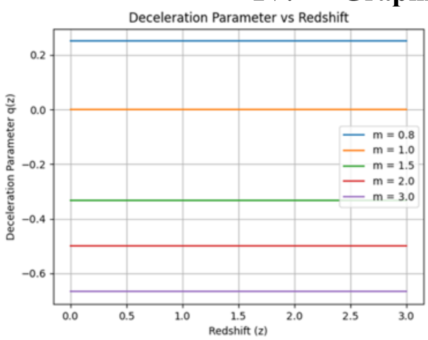
The statefinder parameters (r, s) are defined in terms of the scale factor and its higher-order time derivatives as follows:

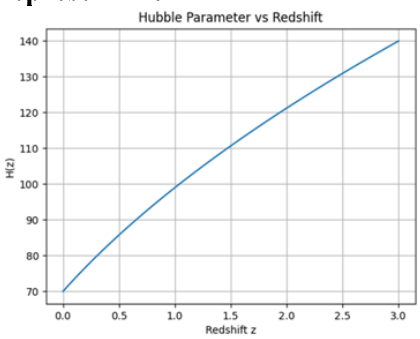
$$r = \frac{\dddot{a}}{aH^3}$$

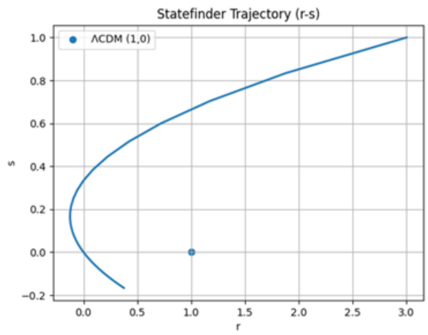
$$s = \frac{r-1}{3\left(q-\frac{1}{2}\right)}$$

where $a(t)$ is the average scale factor, $H = \frac{\dot{a}}{a}$ is the Hubble parameter, and $q = -\frac{a\ddot{a}}{\dot{a}^2}$ is the deceleration parameter. The parameter r is often referred to as the **jerk parameter**, since it contains the third derivative of the scale factor and measures the rate of change of cosmic acceleration. The parameter s is a dimensionless combination of r and q, which provides a convenient way to classify different dark energy models.

IV. Graphical Representation







V. Discussion

1. The present study investigated an anisotropic cosmological model within the modified gravity framework $f(R,G,T) = \alpha R + \beta GT$. The Bianchi Type III geometry allows the study of anisotropic expansion in the early universe. By assuming a variable deceleration parameter, the field equations were solved analytically and explicit expressions for the directional scale factors were obtained.
2. The Hubble Parameter decreases with cosmic time, which is typical in power-law cosmological models.
3. For $m > 1$, the Bianchi Type III model predicts a late-time accelerating universe, which is consistent with observational cosmology.
4. The physical parameters such as energy density and pressure show that the energy density decreases with time while the pressure becomes negative at late times. This behaviour is consistent with the presence of dark energy. The equation of state parameter evolves dynamically and can enter the quintessence region depending on the model parameters.
5. Furthermore, the statefinder diagnostic was employed to analyze the geometrical behaviour of the model. The obtained expressions for the parameters $r$and $s$indicate that the model can approach the ΛCDM limit for certain values of the constant m.

VI. Conclusion

By assuming a variable deceleration parameter, exact analytical solutions of the modified field equations were obtained, leading to explicit expressions for the directional scale factors $A(t)$, $B(t)$, and $C(t)$, which describe the anisotropic expansion of the universe. The derived physical parameters show that the energy density $\rho(t)$decreases with cosmic time while the pressure $p(t)$becomes negative at late times, indicating the presence of a dark-energy–dominated accelerating phase. The equation of state parameter $\omega(t)$evolves dynamically and can enter the quintessence region $(-1 < \omega < -1/3)$, which is consistent with the observed accelerated expansion of the universe. Moreover, the statefinder diagnostic parameters $r$and $s$suggest that the obtained model can approach the standard ΛCDM cosmological scenario for suitable values of the model parameters. Overall, the results indicate that the considered Bianchi Type III model in the modified gravity framework provides a consistent description of the transition from an early anisotropic universe to a late-time accelerating phase, thereby offering a viable alternative explanation for cosmic acceleration without introducing an explicit cosmological constant.

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