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First Order Coherence of an Electromagnetic Wave: From Classical Theory to Quantum States

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Abstract:
In this article, the physical significance of the 1st order coherence in respect of the classical and quantum world is discussed with due example of Young's double slit experiment. Mathematical equations governing these phenomenon are established. The corresponding correlation functions in the classical and quantum analogue are discussed.
Key Word*: Quantum state;* superposition; Young's double slit experiment; Coherence

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I. Introduction

Coherence is a basic requirement for interference to take place, be it classical world or the quantum world. If the two interfering field-amplitudes (in classical physics) or the two interfering wave-functions are able to yield a visible interference pattern, these field-amplitudes/wave-functions ought to be coherent, in other words, correlated. Although classical meaning of interference is based on the division of the wavefront/amplitude or the energy of the interfering waves. Whereas, in quantum mechanics, the interference implies division of wave function, and not the division of energy. It has been proven that a single photon can interfere with itself to produce an interference pattern.

II. Correlation and Coherence

In the context of optics, coherence is generally associated with the monochromaticity of the optical electromagnetic fields (Luo and Sun 2017; Schlosshauer 2007; Schlosshauer 2019). This implies that the phase of the optical fields is maintained constant for all times and at any point in space. This kind of optical coherence is an example of first order coherence. A classical world example to study 1st order coherence is Young's double slit experiment (Born and Wolf 2013; Born and Wolf 1999; Ghatak et al. 2013).

Mathematically:

The interference pattern obtained in Young's double slit experiment consists of maxima and minima regions. The contrast between maxima and minima depends upon the amount of correlation that exists between the interfering entities. For example, in classical interference pattern of two electric fields $\vec{\boldsymbol{E}}_1$ and $\vec{\boldsymbol{E}}_2$, the resultant field can be written as (Glauber 1963):

$$\vec{\boldsymbol{E}} = \vec{\boldsymbol{E}}_1 + \vec{\boldsymbol{E}}_2 \quad (1)$$

$$\text{Where } \vec{\boldsymbol{E}}_{1,2} = \vec{\boldsymbol{E}}_{o1,o2} \exp j(\vec{\boldsymbol{k}}_{1,2}.\vec{\boldsymbol{r}} - \omega_{1,2}t) \quad (2)$$

The corresponding intensity of the interfering waves is given by:

$$I = \left|\vec{\boldsymbol{E}}\right|^2 \quad (3)$$

$$I = K\left|\vec{\boldsymbol{E}}\right|^2 = K\left|\vec{\boldsymbol{E}}_1 + \vec{\boldsymbol{E}}_2\right|^2 = K(\vec{\boldsymbol{E}}_1 + \vec{\boldsymbol{E}}_2)(\vec{\boldsymbol{E}}_1 + \vec{\boldsymbol{E}}_2)^* \quad (4)$$

(Where K is the proportionality constant and * denotes the complex conjugate.)

$$I = K(\vec{\boldsymbol{E}}_1.\vec{\boldsymbol{E}}_1^* + \vec{\boldsymbol{E}}_2.\vec{\boldsymbol{E}}_2^* + \vec{\boldsymbol{E}}_2.\vec{\boldsymbol{E}}_1^* + \vec{\boldsymbol{E}}_1.\vec{\boldsymbol{E}}_2^*) \quad (5)$$

$$I = K(\boldsymbol{E}_{o1}^2 + \boldsymbol{E}_{o2}^2 + 2\boldsymbol{E}_{o1}\boldsymbol{E}_{o2}\cos[(\vec{\boldsymbol{k}}_1 - \vec{\boldsymbol{k}}_2).\vec{\boldsymbol{r}} - (\omega_1 - \omega_2)t] \quad (6)$$

In Eqn. (6), we can write $(\vec{\boldsymbol{k}}_1 - \vec{\boldsymbol{k}}_2).\vec{\boldsymbol{r}} = l_1\boldsymbol{\phi_s}$; and $(\omega_1 - \omega_2)t = l_2\boldsymbol{\phi_t}$.

$$I = K(\boldsymbol{E}_{o1}^2 + \boldsymbol{E}_{o2}^2 + 2\boldsymbol{E}_{o1}\boldsymbol{E}_{o2}\cos\ (l_1\boldsymbol{\phi_s} - l_2\boldsymbol{\phi_t}) \quad (7)$$

Eqn. (7) gives the correlation between $\vec{E}_1$ and $\vec{E}_2$. Here $\boldsymbol{\phi_s}$ is the difference in the spatial phases of the electric fields at position $\vec{r}$ in the interfering space. Since the electric field generated from a real physical source is inherently chaotic (resulting in a random value of the phase $\boldsymbol{\phi}$), due to the participation of infinite number of independently emitting emitters present in the real physical source. Hence the interference is observed to an extent till the interfering fields are mutually coherent at two different space-time points. Also $\boldsymbol{\phi_t}$ depends upon the monochromaticity of the physical source. Young's double slit interference experiment is an example of 1sr order of coherence which depends upon the randomness of the electric fields generated (the spatial phase coherence) and the monochromaticity of the physical sources (the temporal phase coherence). l_1 and l_2 are the integers ranging from $-\infty$ to $+\infty$. The classical correlation function can be defined as:

$$W(\boldsymbol{\phi_s}, \boldsymbol{\phi_t}) = \sum_{l_1=-\infty}^{\infty} \sum_{l_2=-\infty}^{\infty} \langle\, (E_{o1})_{l_1}\, (E_{o2})^*_{l_2}\, \rangle_e \exp -j(l_1\boldsymbol{\phi_s} - l_2\boldsymbol{\phi_t}) \qquad (7a)$$

The quantum analogue of a 1st order coherent system can be explained as follows:

A pure Q state (abbreviation of quantum state) $|\psi\rangle$ can be written in terms of its state vectors (basis vectors) $|l\rangle$ as:

$$|\psi\rangle = c_l|l\rangle \qquad (8)$$

where c_i is a complex number from where one can evaluate the probability of the quantum state to be in the basis vector $|i\rangle$. For a mixed state, Eqn. (8) gets modified to

$$|\psi\rangle = \sum_{l=-\infty}^{\infty} c_l|l\rangle \qquad (9)$$

In order to find the correlation between the mixed Q states (which define the quantum randomness), we evaluate density matrix ρ which is defined as the outer product of the quantum state with itself:

$$\rho = |\psi\rangle\langle\psi| \qquad (10)$$

From the above equation, the ensemble average is obtained as:

$$\rho = \langle\, |\psi\rangle\langle\psi|\, \rangle_{en} \qquad (11)$$

Substituting Eqn. (9) into Eqn. (11), we get

$$\rho = \sum_{l_1=-\infty}^{\infty} \sum_{l_2=-\infty}^{\infty} \langle\, c_{l_1} c_{l_2}{}^*\, \rangle_e\, |l_1\rangle\, \langle l_2| \qquad (12)$$

Taking the matrix elements of ρ over $\boldsymbol{\phi_s}$ and $\boldsymbol{\phi_t}$ (the elements responsible for 1st order coherence):

$$\langle\phi_s|\rho|\phi_t\rangle = \sum_{l_1=-\infty}^{\infty} \sum_{l_2=-\infty}^{\infty} \langle\, c_{l_1} c_{l_2}{}^*\, \rangle_e\, \langle\phi_s|l_1\rangle\, \langle l_2|\phi_t\rangle \qquad (13)$$

Eqns. (7a) and (13) are classical and quantum counterpart definitions of the 1st order coherence. Where $(E_{o1})_{l_1}$ and $(E_{o2})_{l_2}$ are analogous to c_{l_1} and c_{l_2} respectively.

Now in quantum world, we do not study the intensity variation of the interference pattern (Because division of energy as studied in classical interference pattern would imply division of photon, which is not a linear phenomenon.) as done in Eqns. (5-7), but instead we study the absorption probability of a photon, i.e., the detection probability of a single photon. Because in quantum physics, a single photon can result in interference with itself by the virtue of it being in a superposition state. Hence, the photon absorption probability in quantum state $|i\rangle$ is given by:

$$P(\vec{r}, t) = \langle\langle i|\hat{E}^{(-)}(\vec{r}, t)\, \hat{E}^{(+)}(\vec{r}, t)|i\rangle\rangle_{en} = Tr(\rho\hat{E}^{(-)}(\vec{r}, t)\, \hat{E}^{(+)}(\vec{r}, t)) \qquad (14)$$

$\hat{E}^{(-)}(\vec{r}, t)$ and $\hat{E}^{(+)}(\vec{r}, t)$ are the fields corresponding to negative and positive frequency-components (spatial and temporal) in the Fourier expansions of $\vec{E}$ at any space-time point $(\vec{r}, t)$.

The expression for 1st order correlation between the fields at space-time points $(\vec{r}_1, t_1)$ and $(\vec{r}_2, t_2)$ in Q state $|i\rangle$ becomes:

$$G^{(1)}(\vec{r}_1, t_1; \vec{r}_2, t_2) \equiv \langle\langle i|\hat{E}^{(-)}(\vec{r}_1, t)\, \hat{E}^{(+)}(\vec{r}_2, t)|i\rangle\rangle_{en} = Tr(\rho\hat{E}^{(-)}(\vec{r}_1, t)\, \hat{E}^{(+)}(\vec{r}_2, t)) \qquad (15)$$

This expression gives the 2-point correlation function of finding a single photon at space-time coordinates $(\vec{r}_1, t_1)$ and $(\vec{r}_2, t_2)$.

III. Discussion

The authors have compared the 1st order coherence for classical interference and the quantum interference giving detailed mathematical analysis. The corresponding correlation functions are also evaluated.

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