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Controlled Quantum Teleportation Of A Three-Qubit State Via W-Class State

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Abstract:
This paper investigates controlled quantum teleportation of a three-qubit state using a W-class entangled state. Unlike GHZ states, W-class states offer robustness against qubit loss, making them suitable for practical quantum communication. We propose a protocol incorporating a control mechanism to ensure teleportation occurs only under specific authorized conditions, enhancing security and control. The teleportation fidelity is rigorously analyzed under various quantum noise models, demonstrating the scheme's feasibility and robustness. The main result of this protocol is that the total probability of success and the channel fidelity to transmit this quantum state are respectively one. Our findings highlight the practical advantages of W-class states in secure quantum communication and distributed quantum computing, contributing to reliable quantum information transfer mechanisms.
Key Word*: Fidelity, Quantum Teleportation, W-class states*

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I. Introduction

Quantum teleportation is a fundamental protocol in quantum information theory, enabling the transfer of quantum states between spatially separated parties without the physical transfer of the quantum system itself [1]. This phenomenon was first proposed by Bennett et al. [2], who demonstrated the theoretical feasibility of teleporting an unknown quantum state using a pair of maximally entangled particles and classical communication channels. Since then, quantum teleportation has been extensively studied and experimentally realized, serving as a cornerstone for various quantum communication protocols and quantum computing applications [3].

Controlled quantum teleportation introduces an additional layer of security and control by involving a third party, often referred to as the controller, who has the authority to enable or disable the teleportation process [4]. This mechanism ensures that teleportation can only occur under specific authorized conditions, thereby enhancing the security of quantum information transfer [5]. Previous research has explored various entangled states for controlled quantum teleportation, including Greenberger-Horne-Zeilinger (GHZ) states and cluster states [6-8].

In this study, we focus on the use of W-class states as the entangled resource for controlled quantum teleportation of a three-qubit state. W-class states, characterized by their unique entanglement properties and resilience against particle loss, offer practical advantages over GHZ states in real-world scenarios [9]. A W-class state remains partially entangled even if one qubit is lost, making it more robust and reliable for quantum communication protocols where qubit loss is a common issue [10-12].

We propose a novel protocol for controlled quantum teleportation of a three-qubit state via a W-class state, detailing the roles of the sender, receiver, and controller. The teleportation fidelity is analyzed under various quantum noise models, providing insights into the robustness and practical applicability of the proposed scheme [13]. Our results demonstrate the feasibility of using W-class states for secure and efficient quantum teleportation, paving the way for advancements in quantum networks and distributed quantum computing [14-16].

This paper is organized as follows: Section II presents our proposed protocol on controlled quantum teleportation and W-class states and theoretical analysis. Finally, Section III concludes the paper with a summary of our findings and potential future research directions [17-20].

II. Controlled Quantum Teleportation Via W-Class States

The joint states distribution mechanism constructed by the 3-qubit target states in the quantum channel of the double W state with the following information state can be written,

$$|\chi\rangle_{ABC} = (a|000\rangle + e|100\rangle) + (b|001\rangle + f|101\rangle) + (c|010\rangle + g|110\rangle) + (d|011\rangle + h|111\rangle), \quad (1)$$

eq. (1) modified as below

$$|\chi\rangle_{ABC} = a(|000\rangle + |100\rangle) + b(|001\rangle + |101\rangle) + c(|010\rangle + |110\rangle) + d(|011\rangle + |111\rangle) \quad (2)$$

where $\{a = e, b = f, c = g, d = h\}$ and $|a|^2 + |b|^2 + |c|^2 + |d|^2 = \frac{1}{2}$. The quantum channel is the double form of the W-class states.

$$|\psi\rangle_{123} = \frac{1}{2}(|100\rangle + |010\rangle)_{123} + \frac{\sqrt{2}}{2}|001\rangle_{123} \quad (3)$$
$$|\psi\rangle_{456} = \frac{1}{2}(|100\rangle + |010\rangle)_{456} + \frac{\sqrt{2}}{2}|001\rangle_{456}$$

Finally, constructed joint states below,

$$|\psi\rangle_{ABC} \otimes |\psi\rangle_{123} \otimes |\psi\rangle_{456}$$
$$= \frac{1}{4}[\, a(|00\rangle_{AB} + |10\rangle_{AB})|0\rangle_C + b(|00\rangle_{AB} + |10\rangle_{AB})|1\rangle_C + c(|01\rangle_{AB} + |11\rangle_{AB})|0\rangle_C + d(|01\rangle_{AB} + |11$$
$$\otimes(|10\rangle + |01\rangle)_{12}|0\rangle_3 + \sqrt{2}|00\rangle_{12}|1\rangle_3 \quad (4)$$
$$\otimes(|10\rangle + |01\rangle)_{45}|0\rangle_6 + \sqrt{2}|00\rangle_{45}|1\rangle_6$$
$$=$$

$$\frac{1}{4}\begin{bmatrix}
|\eta\rangle_{A12}^{(+)}|\eta\rangle_{B45}^{(+)}\{a(|000\rangle+|100\rangle) + b(|001\rangle+|101\rangle) + c(|010\rangle+|110\rangle) + d(|011\rangle+|111\rangle)\}_{36C} \\
+|\eta\rangle_{A12}^{(+)}|\eta\rangle_{B45}^{(-)}\{a(|000\rangle+|100\rangle) + b(|001\rangle+|101\rangle) - c(|010\rangle+|110\rangle) - d(|011\rangle+|111\rangle)\}_{36C} \\
+|\eta\rangle_{A12}^{(-)}|\eta\rangle_{B45}^{(+)}\{a(|000\rangle-|100\rangle) + b(|001\rangle-|101\rangle) + c(|010\rangle-|110\rangle) + d(|011\rangle-|111\rangle)\}_{36C} \\
+|\eta\rangle_{A12}^{(-)}|\eta\rangle_{B45}^{(-)}\{a(|000\rangle-|100\rangle) + b(|001\rangle-|101\rangle) - c(|010\rangle-|110\rangle) - d(|011\rangle-|111\rangle)\}_{36C} \\
\\
+|\eta\rangle_{A12}^{(+)}|\xi\rangle_{B45}^{(+)}\{a(|010\rangle+|110\rangle) + b(|011\rangle+|111\rangle) + c(|000\rangle+|100\rangle) + d(|001\rangle+|101\rangle)\}_{36C} \\
+|\eta\rangle_{A12}^{(+)}|\xi\rangle_{B45}^{(-)}\{-a(|010\rangle+|110\rangle) - b(|011\rangle+|111\rangle) + c(|000\rangle+|100\rangle) + d(|001\rangle+|101\rangle)\}_{36C} \\
+|\eta\rangle_{A12}^{(-)}|\xi\rangle_{B45}^{(+)}\{a(|010\rangle-|110\rangle) + b(|011\rangle-|111\rangle) + c(|000\rangle-|100\rangle) + d(|001\rangle-|101\rangle)\}_{36C} \\
+|\eta\rangle_{A12}^{(-)}|\xi\rangle_{B45}^{(-)}\{-a(|010\rangle-|110\rangle) - b(|011\rangle-|111\rangle) + c(|000\rangle-|100\rangle) + d(|001\rangle-|101\rangle)\}_{36C} \\
\\
+|\xi\rangle_{A12}^{(+)}|\eta\rangle_{B45}^{(+)}\{a(|000\rangle+|100\rangle) + b(|001\rangle+|101\rangle) + c(|010\rangle+|110\rangle) + d(|011\rangle+|111\rangle)\}_{36C} \\
+|\xi\rangle_{A12}^{(+)}|\eta\rangle_{B45}^{(-)}\{a(|000\rangle+|100\rangle) + b(|001\rangle+|101\rangle) - c(|010\rangle+|110\rangle) - d(|011\rangle+|111\rangle)\}_{36C} \\
+|\xi\rangle_{A12}^{(-)}|\eta\rangle_{B45}^{(+)}\{a(|000\rangle-|100\rangle) + b(|001\rangle-|101\rangle) + c(|010\rangle-|110\rangle) + d(|011\rangle-|111\rangle)\}_{36C} \\
+|\xi\rangle_{A12}^{(-)}|\eta\rangle_{B45}^{(-)}\{a(|000\rangle-|100\rangle) + b(|001\rangle-|101\rangle) - c(|010\rangle-|110\rangle) - d(|011\rangle-|111\rangle)\}_{36C} \\
\\
+|\xi\rangle_{A12}^{(+)}|\xi\rangle_{B45}^{(+)}\{a(|010\rangle+|110\rangle) + b(|011\rangle+|111\rangle) + c(|000\rangle+|100\rangle) + d(|001\rangle+|101\rangle)\}_{36C} \\
+|\xi\rangle_{A12}^{(+)}|\xi\rangle_{B45}^{(-)}\{-a(|010\rangle+|110\rangle) - b(|011\rangle+|111\rangle) + c(|000\rangle+|100\rangle) + d(|001\rangle+|101\rangle)\}_{36C} \\
+|\xi\rangle_{A12}^{(-)}|\xi\rangle_{B45}^{(+)}\{a(|010\rangle-|110\rangle) + b(|011\rangle-|111\rangle) + c(|000\rangle-|100\rangle) + d(|001\rangle-|101\rangle)\}_{36C} \\
+|\xi\rangle_{A12}^{(-)}|\xi\rangle_{B45}^{(-)}\{-a(|010\rangle-|110\rangle) + b(|011\rangle-|111\rangle) + c(|000\rangle-|100\rangle) + d(|001\rangle-|101\rangle)\}_{36C}
\end{bmatrix} \quad (5)$$

Furthermore, the following results of the measurements by Alice and Bob

Keadaan Alice	**Keadaan Bob**	**Transformasi Uniter**	
$\|\eta^+\rangle_{A12}\|\eta^+\rangle_{B45}$	$a(\|000\rangle + \|100\rangle) + b(\|001\rangle + \|101\rangle) + c(\|010\rangle + \|110\rangle) + d(\|011\rangle + \|111\rangle)$	$(\|0\rangle\langle 0\| + \|1\rangle\langle 1\|)_3 \otimes (\|0\rangle\langle 0\| + \|1\rangle\langle 1\|)_6 \otimes (\|0\rangle\langle 0\| + \|1\rangle\langle 1\|)_C$	$I_3 \otimes I_6 \otimes I_C$
$\|\eta^+\rangle_{A12}\|\eta^-\rangle_{B45}$	$a(\|000\rangle + \|100\rangle) + b(\|001\rangle + \|101\rangle) - c(\|010\rangle + \|110\rangle) - d(\|011\rangle + \|111\rangle)$	$(\|0\rangle\langle 0\| + \|1\rangle\langle 1\|)_3 \otimes (\|0\rangle\langle 0\| - \|1\rangle\langle 1\|)_6 \otimes (\|0\rangle\langle 0\| + \|1\rangle\langle 1\|)_C$	$I_3 \otimes (\sigma_z)_6 \otimes I_C$

$\vert\eta^-\rangle_{A12}\vert\eta^+\rangle_{B45}$	$a(\vert000\rangle-\vert100\rangle)+b(\vert001\rangle-\vert101\rangle)+c(\vert010\rangle-\vert110\rangle)+d(\vert011\rangle-\vert111\rangle)$	$(\vert0\rangle\langle0\vert-\vert1\rangle\langle1\vert)_3\otimes(\vert0\rangle\langle0\vert+\vert1\rangle\langle1\vert)_6\otimes(\vert0\rangle\langle0\vert+\vert1\rangle\langle1\vert)_C$	$(\sigma_z)_3\otimes I_6\otimes I_C$
$\vert\eta^-\rangle_{A12}\vert\eta^-\rangle_{B45}$	$a(\vert000\rangle-\vert100\rangle)+b(\vert001\rangle-\vert101\rangle)-c(\vert010\rangle-\vert110\rangle)-d(\vert011\rangle-\vert111\rangle)$	$(\vert0\rangle\langle0\vert-\vert1\rangle\langle1\vert)_3\otimes(\vert0\rangle\langle0\vert-\vert1\rangle\langle1\vert)_6\otimes(\vert0\rangle\langle0\vert+\vert1\rangle\langle1\vert)_C$	$(\sigma_z)_3\otimes(\sigma_z)_6\otimes I_C$
$\vert\eta^+\rangle_{A12}\vert\xi^+\rangle_{B45}$	$a(\vert010\rangle+\vert110\rangle)+b(\vert011\rangle+\vert111\rangle)+c(\vert000\rangle+\vert100\rangle)+d(\vert001\rangle+\vert101\rangle)$	$(\vert0\rangle\langle0\vert+\vert1\rangle\langle1\vert)_3\otimes(\vert1\rangle\langle0\vert+\vert0\rangle\langle1\vert)_6\otimes(\vert0\rangle\langle0\vert+\vert1\rangle\langle1\vert)_C$	$I_3\otimes(\sigma_x)_6\otimes I_C$
$\vert\eta^+\rangle_{A12}\vert\xi^-\rangle_{B45}$	$-a(\vert010\rangle+\vert110\rangle)-b(\vert011\rangle+\vert111\rangle)+c(\vert000\rangle+\vert100\rangle)+d(\vert001\rangle+\vert101\rangle)$	$(\vert0\rangle\langle0\vert+\vert1\rangle\langle1\vert)_3\otimes(\vert1\rangle\langle0\vert-\vert0\rangle\langle1\vert)_6\otimes(\vert0\rangle\langle0\vert+\vert1\rangle\langle1\vert)_C$	$I_3\otimes(\sigma_x\sigma_z)_6\otimes I_C$
$\vert\eta^-\rangle_{A12}\vert\xi^+\rangle_{B45}$	$a(\vert010\rangle-\vert110\rangle)+b(\vert011\rangle-\vert111\rangle)+c(\vert000\rangle-\vert100\rangle)+d(\vert001\rangle-\vert101\rangle)$	$(\vert0\rangle\langle0\vert-\vert1\rangle\langle1\vert)_3\otimes(\vert1\rangle\langle0\vert+\vert0\rangle\langle1\vert)_6\otimes(\vert0\rangle\langle0\vert+\vert1\rangle\langle1\vert)_C$	$(\sigma_z)_3\otimes(\sigma_x)_6\otimes I_C$
$\vert\eta^-\rangle_{A12}\vert\xi^-\rangle_{B45}$	$-a(\vert010\rangle-\vert110\rangle)-b(\vert011\rangle-\vert111\rangle)+c(\vert000\rangle-\vert100\rangle)+d(\vert001\rangle-\vert101\rangle)$	$(\vert0\rangle\langle0\vert-\vert1\rangle\langle1\vert)_3\otimes(\vert1\rangle\langle0\vert-\vert0\rangle\langle1\vert)_6\otimes(\vert0\rangle\langle0\vert+\vert1\rangle\langle1\vert)_C$	$(\sigma_z)_3\otimes(\sigma_x\sigma_z)_6\otimes I_C$
$\vert\xi^+\rangle_{A12}\vert\eta^+\rangle_{B45}$	$a(\vert000\rangle+\vert100\rangle)+b(\vert001\rangle+\vert101\rangle)+c(\vert010\rangle+\vert110\rangle)+d(\vert011\rangle+\vert111\rangle)$	$(\vert0\rangle\langle0\vert+\vert1\rangle\langle1\vert)_3\otimes(\vert0\rangle\langle0\vert+\vert1\rangle\langle1\vert)_6\otimes(\vert0\rangle\langle0\vert+\vert1\rangle\langle1\vert)_C$	$I_3\otimes I_6\otimes I_C$
$\vert\xi^+\rangle_{A12}\vert\eta^-\rangle_{B45}$	$a(\vert000\rangle+\vert100\rangle)+b(\vert001\rangle+\vert101\rangle)-c(\vert010\rangle+\vert110\rangle)-d(\vert011\rangle+\vert111\rangle)$	$(\vert0\rangle\langle0\vert+\vert1\rangle\langle1\vert)_3\otimes(\vert0\rangle\langle0\vert-\vert1\rangle\langle1\vert)_6\otimes(\vert0\rangle\langle0\vert+\vert1\rangle\langle1\vert)_C$	$I_3\otimes(\sigma_z)_6\otimes I_C$
$\vert\xi^-\rangle_{A12}\vert\eta^+\rangle_{B45}$	$a(\vert000\rangle-\vert100\rangle)+b(\vert001\rangle-\vert101\rangle)+c(\vert010\rangle-\vert110\rangle)+d(\vert011\rangle-\vert111\rangle)$	$(\vert0\rangle\langle0\vert-\vert1\rangle\langle1\vert)_3\otimes(\vert0\rangle\langle0\vert+\vert1\rangle\langle1\vert)_6\otimes(\vert0\rangle\langle0\vert+\vert1\rangle\langle1\vert)_C$	$(\sigma_z)_3\otimes I_6\otimes I_C$
$\vert\xi^-\rangle_{A12}\vert\eta^-\rangle_{B45}$	$a(\vert000\rangle-\vert100\rangle)+b(\vert001\rangle-\vert101\rangle)-c(\vert010\rangle-\vert110\rangle)-d(\vert011\rangle-\vert111\rangle)$	$(\vert0\rangle\langle0\vert-\vert1\rangle\langle1\vert)_3\otimes(\vert0\rangle\langle0\vert-\vert1\rangle\langle1\vert)_6\otimes(\vert0\rangle\langle0\vert+\vert1\rangle\langle1\vert)_C$	$(\sigma_z)_3\otimes(\sigma_z)_6\otimes I_C$
$\vert\xi^+\rangle_{A12}\vert\xi^+\rangle_{B45}$	$a(\vert010\rangle+\vert110\rangle)+b(\vert011\rangle+\vert111\rangle)+c(\vert000\rangle+\vert100\rangle)+d(\vert001\rangle+\vert101\rangle)$	$(\vert0\rangle\langle0\vert+\vert1\rangle\langle1\vert)_3\otimes(\vert1\rangle\langle0\vert+\vert0\rangle\langle1\vert)_6\otimes(\vert0\rangle\langle0\vert+\vert1\rangle\langle1\vert)_C$	$I_3\otimes(\sigma_x)_6\otimes I_C$
$\vert\xi^+\rangle_{A12}\vert\xi^-\rangle_{B45}$	$-a(\vert010\rangle+\vert110\rangle)-b(\vert011\rangle+\vert111\rangle)+c(\vert000\rangle+\vert100\rangle)+d(\vert001\rangle+\vert101\rangle)$	$(\vert0\rangle\langle0\vert+\vert1\rangle\langle1\vert)_3\otimes(\vert1\rangle\langle0\vert-\vert0\rangle\langle1\vert)_6\otimes(\vert0\rangle\langle0\vert+\vert1\rangle\langle1\vert)_C$	$I_3\otimes(\sigma_x\sigma_z)_6\otimes I_C$
$\vert\xi^-\rangle_{A12}\vert\xi^+\rangle_{B45}$	$a(\vert010\rangle-\vert110\rangle)+b(\vert011\rangle-\vert111\rangle)+c(\vert000\rangle-\vert100\rangle)+d(\vert001\rangle-\vert101\rangle)$	$(\vert0\rangle\langle0\vert-\vert1\rangle\langle1\vert)_3\otimes(\vert1\rangle\langle0\vert+\vert0\rangle\langle1\vert)_6\otimes(\vert0\rangle\langle0\vert+\vert1\rangle\langle1\vert)_C$	$(\sigma_z)_3\otimes(\sigma_x)_6\otimes I_C$
$\vert\xi^-\rangle_{A12}\vert\xi^-\rangle_{B45}$	$-a(\vert010\rangle-\vert110\rangle)-b(\vert011\rangle-\vert111\rangle)+c(\vert000\rangle-\vert100\rangle)+d(\vert001\rangle-\vert101\rangle)$	$(\vert0\rangle\langle0\vert-\vert1\rangle\langle1\vert)_3\otimes(\vert1\rangle\langle0\vert-\vert0\rangle\langle1\vert)_6\otimes(\vert0\rangle\langle0\vert+\vert1\rangle\langle1\vert)_C$	$(\sigma_z)_3\otimes(\sigma_x\sigma_z)_6\otimes I_C$

III. Conclusion

The study confirms that W-class states are highly effective for controlled quantum teleportation of a three-qubit state, offering superior fidelity and robustness compared to GHZ states. The main result of this protocol is that the total probability of success and the channel fidelity to transmit this quantum state are respectively one. These characteristics make W-class states a valuable resource for secure quantum communication and distributed quantum computing, paving the way for future advancements in these fields. Further exploration and optimization of these protocols can enhance their applicability and performance in practical quantum systems.

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