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Studying The Properties Of A Superpotential Using Algebraic Equations

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Abstract:
In attempts to understand the fundamentals of string theory, it has become clear that we need a better understanding of conformal theories as these are the building blocks of string vacuum. This approach is most powerful when applied to superconformal models with N = 2 worldsheet supersymmetry. They are related to the superpotential with the corresponding central charge c, dimension of chiral field, and the ring of the corresponding minimal model. This means that we can study the properties of Calabi-Yau manifold as the tensor product of the minimal discrete models from the point of view of LG theory.
***Key Word**: Calabi-Yau; Superpotential; Supersymmetry.*

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I. Introduction

The connection and evolution of string and field theories is complex and associated with dimensional reduction. Bosonic string theory is a consistent relativistic quantum theory of one-dimensional objects which includes:

- Gravity with a massless spin-2 state;
- Grand unification;
- Extra dimensions;
- Supersymmetry;
- No free parameters.

It is determined in a spacetime of 26 dimensions to prevent breaking of Weyl invariance. The five consistente superstring theories include the heterotic string theories differing in their ten-dimensional gauge groups: the heterotic E8×E8 string and the heterotic SO (32) string, which are anomaly-free chiral theories in ten dimensions with Calabi–Yau compactification. The main difficulty of these theories relates to the enormous number of potential vacuums (ground state) and problems with understanding the dynamics to choose the right vacuum. So, as the vacuum plays an important role in electroweak interaction, in strong interaction, in string theory, the study of its properties is an important component of high energy physics. Superstring theory is tachyon-free string theory that accounts for both fermions and bosons in ten-dimensional space-time, that is invariant under super conformal transformations. New theories were needed that would make it possible to fix string dynamics in a certain way. Super conformal theories relate to conformal methods used to study string compactification and dynamics. N=4 super Yang–Mills theory derived from 10-dimensional theory is a mathematical and physical model created to study particles in system with conformal symmetry. Since conformal transformation is a combination of a coordinate transformation and a Weyl transformation, conformal field theories (CFT) are important for studying the general properties of quantum field theories. The connection of the presented theories is shown in figure 1. Our study of the properties of CFT in two dimensions is motivated by their importance in constructing string vacuum.

Let's consider extension of the Landau-Ginzburg (LG) theory to N = 2 theories with n chiral superfields Φ_i in the action

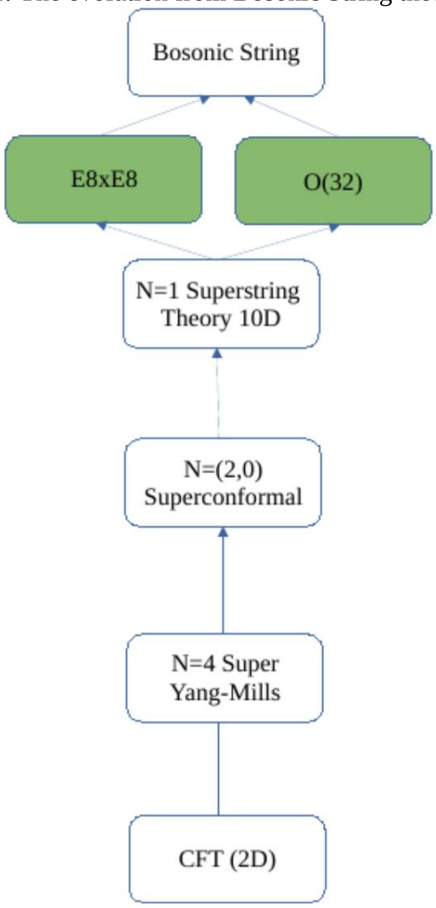
$$\int d^2zd^4\theta K(\Phi_i\Phi_i) + \left(\int d^2zd^2\theta W(\Phi_i) + c.c.\right)$$

with term K called the D-term and the second F-term with W - holomorphic function of the fields, called superpotential, which corresponds exactly to the conformal theory for each superpotential[1]. At the fixed points of the renormalization group flow the theory must be scale invariant. Rescaling of the two-dimensional metric $g \rightarrow \lambda^2 g$ by λ leads to rescaling of the superpotential, connected with rescaling of the superfields Φ_i

$$W(\lambda^{\omega_i}\Phi_i) = \lambda W(\Phi_i) \quad (1)$$

Functions with this property are called quasi-homogeneous with weights ω_i . So, to classify N = 2 LG superconformal models, it is necessary to make a classification of quasi-homogeneous holomorphic functions (1) of n variables. The classification of stable equivalence classes of functions in CFT is one of the main aims of singularity theory with isolated points. As the superpotentials as quasihomogeneous functions of superfields are in correspondence with CFT, then the physically interesting singularities are the singularities of quasi-homogeneous functions. To review singularity theory, it is necessary to compute the central charge $c = \sum_i 6(1/2 - \omega_i)$ of the fixed point of the RG flow, which is read off from the partition function after rescaling[2]. The using of the quasihomogeneity of $Z \rightarrow \lambda^{\sum_i (1/2-\omega_i)} Z$

Figure no1: The evolution from Bosonic String theory to CFT.



Classification of the quasi-homogeneous singularities relates to the A-D-E series and corresponds to models with central charge, c. For example, the superpotential $W(\Phi) = \Phi^{P+2}$ corresponds to the A-series modular invariant N = 2 minimal theory, therefore, for a tensor product of minimal models ($P_1 \dots P_r$), superpotential

$$W(\Phi_1, \dots, \Phi_r) = \Phi_1^{P_1+2} + \cdots + \Phi_r^{P_r+2}.$$

From the other hand we know that the corresponding Calabi-Yau is determined by the zero-locus of weighted projective space

$$P^4_{\omega_1,\dots,\omega_5} = P^4 / Z_{\omega_1} \times \cdots \times Z_{\omega_5}$$

satisfying the condition $W(p) = 0$. The Calabi-Yau manifold $Y_{4,5}$ defined by $z_1^5 + z_2^5 + z_3^5 + z_4^5 + z_5^5 = 0$ in CP^4 is identified with the theory of tensor product of five copies of the N = 2 discrete series[3]. For K3 surface, the polynomials define the singular geometries[4]

$$\begin{aligned} F_{A_{N-1}} &= x_1^N + x_2^2 + x_3^2, \qquad (N \geq 2) \\ F_{D_{N/2+2}} &= x_1^{N/2} + x_1 x_2^2 + x_3^2, \quad (N{:}even \geq 6) \\ F_{E_6} &= x_1^4 + x_2^3 + x_3^2, \\ F_{E_7} &= x_1^3 x_2 + x_2^3 + x_3^2, \\ F_{E_8} &= x_1^5 + x_2^3 + x_3^2, \end{aligned} \tag{2}$$

For singular Calabi-Yau 3,4-folds, we add $x_4^2, x_4^2 + x_5^2$ to above polynomials. The quadric terms do not change the type of singularity.

II. Calculations

In this paper, we'll consider different central charges to study their influence at the geometry of Calabi-Yau models and at the roots value of the Calabi-Yau polynomials. To obtain an N-1 (complex) dimensional theory, we can choose N+1 minimal models at level $P_i(\omega_i)$with central charge[3]

$$c_i = 3P_i/(P_i + 2)$$

such that

$$\sum_{i=1}^{N+1} \frac{P_i}{P_i+2} = N - 1 \quad \text{and } c = 3(N-1).$$

To relate these minimal models to Calabi-Yau compactifications, we'll consider different examples.

a) Input (p=3, N (complex space dimension) =4), c=9, N=4

$\frac{p}{p+2} + \frac{p}{p+2} + \frac{p}{p+2} + \frac{p}{p+2} + \frac{p}{p+2} = 5p/(p+2) = 3$ - dimensional complex theory, p=3, c=9, i=5

$W(\Phi) = \Phi^{p+2} = \Phi^5$

$W(\Phi_1, \Phi_2, \Phi_3, \Phi_4, \Phi_5) = \Phi_1^5 + \Phi_2^5 + \Phi_3^5 + \Phi_4^5 + \Phi_5^5$

$x^5 + y^5 + z^5 + u^5 + v^5 = 0$

Solutions:

$z = \sqrt[5]{-u^5 - v^5 - x^5 - y^5}$

$z = -\sqrt[5]{-1}\sqrt[5]{-u^5 - v^5 - x^5 - y^5}$

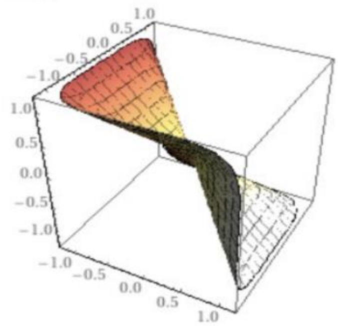
$z = (-1)^{2/5}\sqrt[5]{-u^5 - v^5 - x^5 - y^5}$

$z = -(-1)^{3/5}\sqrt[5]{-u^5 - v^5 - x^5 - y^5}$

$z = (-1)^{4/5}\sqrt[5]{-u^5 - v^5 - x^5 - y^5}$

u = indeterminate

Figure no2: A particular example of such polynomials, the corresponding geometric surface $x^5 + y^5 + z^5 = 0$.



b) Input (p=2, N (complex space dimension) =3), c=6, $z^4 + x^4 + y^4 + u^4 + v^4 = 0$

N=3, $\frac{p}{p+2} + \frac{p}{p+2} + \frac{p}{p+2} + \frac{p}{p+2} = 4p/(p+2) = 2$ - dimensional complex theory, p=2, c=6

$W(\Phi) = \Phi^{p+2} = \Phi^4$

$$W(\Phi_1, \Phi_2, \Phi_3, \Phi_4) = \Phi_1^4 + \Phi_2^4 + \Phi_3^4 + \Phi_4^4$$

Solutions:

$$z = -\sqrt[4]{-u^4 - v^4 - x^4 - y^4}$$

$z = -i\sqrt[4]{-u^4 - v^4 - x^4 - y^4}$

$z = i\sqrt[4]{-u^4 - v^4 - x^4 - y^4}$

$z = \sqrt[4]{-u^4 - v^4 - x^4 - y^4}$

c) Input (p=1, N (complex space dimension) =2), c=3, $z^3 + x^3 + y^3 + u^3 + v^3 = 0$

$\frac{p}{p+2} + \frac{p}{p+2} + \frac{p}{p+2} = 3p/(p+2) = 1$ - dimensional complex theory, p=1, c=3

$W(\Phi) = \Phi^{p+2} = \Phi^3$

$$W(\Phi_1, \Phi_2, \Phi_3) = \Phi_1^3 + \Phi_2^3 + \Phi_3^3$$

Solutions:
$z = \sqrt[3]{-u^3 - v^3 - x^3 - y^3}$
$z = -\sqrt[3]{-1}\sqrt[3]{-u^3 - v^3 - x^3 - y^3}$
$z = (-1)^{2/3}\sqrt[3]{-u^3 - v^3 - x^3 - y^3}$
Solutions for the variable u
$u = \sqrt[3]{-v^3 - x^3 - y^3 - z^3}$

$$u = -(-1)^{\frac{1}{3}}\sqrt[3]{-v^3-x^3-y^3-z^3}$$

$u = (-1)^{2/3}\sqrt[3]{-v^3 - x^3 - y^3 - z^3}$
For the polynomials of K3 surface we concentrate on the isolated rational ADE singularity of the types presented in Figure 3

Figure no3: Roots in the complex plane for (from top left to right)
1) E6 type $x^4 + x^3 + x^2 = 0$, 2) A3 type $5x^4 + x^2 + 3x^2 = 0$,
3) E8 type $x^5 + x^3 + x^2 = 0$, 4) A4 type $x^5 + x^2 + x^2 = 0$.

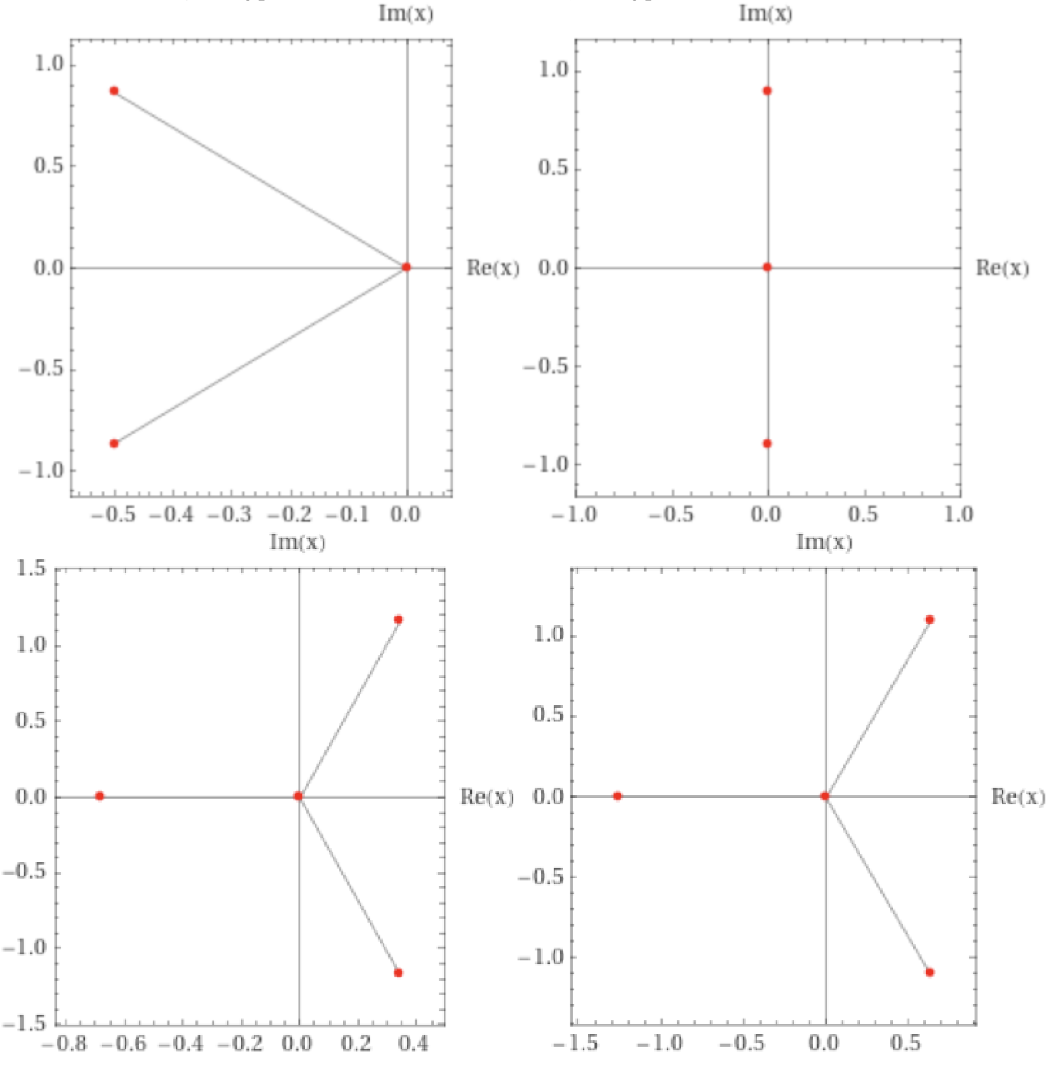
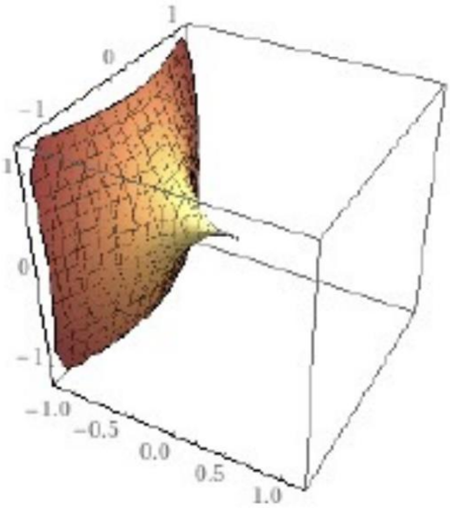


Figure no4: The surface for polynomials of this kind has the form $5x^5 + 6y^2 + 3z^2 = 0$.



III. Conclusion

The real world at the energies below the Planck scale is well described by effective field theory. These effective field theories arise as low-energy descriptions of some "vacuums" of string theory, which could be considered as solutions of the equations of motion for a compactified space. In attempts to understand the fundamentals of string theory, it has become clear that we need a better understanding of conformal theories as these are the building blocks of string vacuum. Conformal theories are in general very complicated but using the RG theory with the identification of fixed points of RG flow with conformal theories leads to the possibility of characterization of the conformal theory by the corresponding data. This approach is most powerful when applied to superconformal models with N = 2 worldsheet supersymmetry. The special property of superconformal theory is displayed in the quasi-homogeneity of the superpotential. This property of superpotential is realized by checking the correspondence between the central charge c, the dimension of chiral fields, and the ring of the corresponding minimal model. This means that we can obtain Calabi-Yau manifold with the tensor product of the minimal discrete models from the point of view of LG theory. We considered different N = 2, 3, 4 models, calculated corresponding central charges, c = 3, 6, 9 and investigated the forms and roots of such manifolds for singular 2-fold, or K3 surface, defined by the corresponding polynomials.

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