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# Anti De Sitter Space And Big Crunch

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**Abstract**
A biggest challenge for today's astrophysicist is to solve the mystery of primordial evolution of the universe. This is also known as belongs to Planck's era. In this stage space-time singularity and inflationary phase all occur when universe was just of the size $10^{-5}$m. All this evolution occurs in a tiny fraction of a second. This description is made by joining the cosmological principle of isotropy and homogeneity, the Hubble law, and the Einstein's field equations of General Relativity in the Big Bang Theory or Standard Cosmological Model (SCM). The SCM is failed to describe first moment in which universe emerges out in presence of singularity. Due to failure of above model quantum theory of gravitation is needed to understand the evolution of early universe. Quantum gravity (QG) is a field of theoretical physics that need to describe gravity by the principles of quantum mechanics, and where quantum effects cannot be ignored, such as in the vicinity of black holes or similar compact astrophysical objects where the effects of gravity are strong(such as neutron stars). Though the theory is able to unfold many mystery but the observational results to prove it are still not exist. The Quantum Cosmology provides some simulating models to understand the primordial evolution by adding some hypothesis. The SCM well explain the observational data with consistency but have some limitations. In present paper Anti de Sitter and Conformal Field Theory are combined together to study condensed matter made up of extra dimensional particles like Dilaton and Strings which are going to decide ultimate fate; the Big Crunch of our Universe. The AdS/CFT correspondence is used to understand the transition from expanding universe to non-relativistic transformation to obtain Big Crunch. In this study a in-depth analysis of de Sitter and Anti de Sitter is given to explain how AdS is related to understand Big Crunch.

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## I. Introduction

A biggest challenge for today's astrophysicist is to solve the mystery of primordial evolution of the universe. This is also known as belongs to Planck's era. In this stage space-time singularity and inflationary phase all occur when universe was just of the size $10^{-5}$m. All this evolution occurs in a tiny fraction of a second. This description is made by joining the cosmological principle of isotropy and homogeneity, the Hubble law, and the Einstein's field equations of General Relativity in the Big Bang Theory or Standard Cosmological Model (SCM). The SCM is failed to describe first moment in which universe emerges out in presence of singularity. Due to failure of above model quantum theory of gravitation is needed to understand the evolution of early universe. Quantum gravity (QG) is a field of theoretical physics that need to describe gravity by the principles of quantum mechanics, and where quantum effects cannot be ignored,[1] such as in the vicinity of black holes or similar compact astrophysical objects where the effects of gravity are strong(such as neutron stars). Though the theory is able to unfold many mystery but the observational results to prove it are still not exist. The Quantum Cosmology provides some simulating models to understand the primordial evolution by adding some hypothesis [2-6]. The SCM well explain the observational data with consistency but have some limitations. In present investigation Anti de Sitter and Conformal Field Theory are combined together to study condensed matter made up of extra dimensional particles like Dilaton and Strings which are going to decide ultimate fate; the Big Crunch of our Universe. The dilaton appear in theory when we study volume of compact dimensions which vary in extra dimension. The cosmic singularity is a point where function takes infinite value for example it exist at centre of black hole. The Conformal Field Theory is a type of quantum field theory which remains invariant and follows Liouville's theorem. The present work is based on 2-dimensional dilaton gravity-scalar theories. The 2-dimensional dilaton gravity helps to understand Big Crunch cosmological singularities. The two dimensional model of gravity convert higher dimensional difficult calculation into simplest way to solve the problems. The gravitational theories are defined in a higher dimensional space-time containing at least the d dimension of particle theory plus one extra dimension of infinite extent. They often include number of finite dimensions in the form of sphere for example depending on the context, the correspondence is known as a Gauge/Gravity duality, Gauge/String duality or AdS/CFT (Anti de Sitter/ Conformal Field Theory) correspondence[7]. The AdS/CFT correspondence is used to understand the transition from expanding universe to non relativistic transformation to obtain Big Crunch. AdS is the theory of quantum gravity while CFT is the theory of quantum field theory.

**De Sitter Space (dS symbolically)**

De Sitter (dS) spacetime is defined as having no matter (ordinary or dark), but having a positive cosmological constant (or in other terms, dark energy. It is an empty 4D universe, but one that, due to its cosmological constant, has positive curvature. The positive curvature is responsible for everlasting expansion of the universe. It is to be noted that as the matter of universe get diluted with the time its effect on radius of curvature also get reduced even it becomes zero at $t\rightarrow\infty$. Although it is to early to come on this type of conclusion where one starts to say that our universe is asymptotically de Sitter. The reason behind this is lying in de Sitter itself as de Sitter space has no mass or energy but our universe has. And second most acceptable reason is the fact that this is not possible that all mass and energy will become zero at the final destination of our universe.

In all debates on destination of the universe one thing is common that all models agreed on present inflation of the universe. In de Sitter the idea of inflation is given by following equation of state:

$$p_V = -\rho_V c^2$$

The cosmic expansion would be exponential hence

$\alpha \; \alpha \; e^{H_V t}$

Where

$$H_V = \frac{8\pi}{3} G\rho_V$$

The pressure is negative due to the consequence of physics of the vacuum at very high temperature in early universe. The exponential expansion phase is referred as de Sitter phase.

**Anti de Sitter space ( symbolically AdS)**

Like dS, anti-de Sitter (AdS) spacetime has no matter in it. However, in other regards, anti-de Sitter (AdS) spacetime is the opposite of de Sitter spacetime. It has a negative cosmological constant, which results in negative curvature of spacetime. This would cause an expanding AdS universe to slow down, i.e., decelerate.

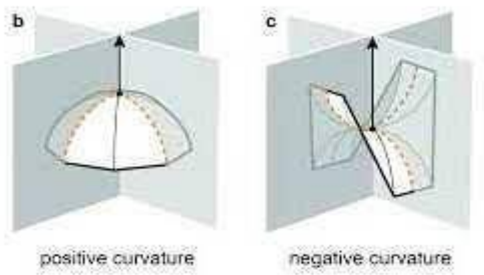
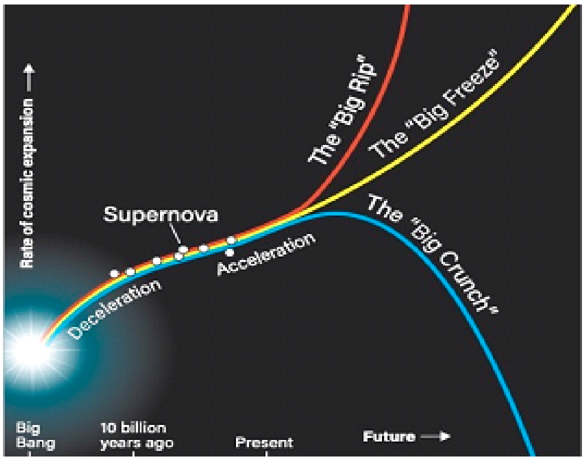



**Fig1: Positive and negative curvature in example, pic courtesy Bob Klauber in student friendly QFT.**

AdS spacetime, like dS spacetime, has constant curvature everywhere, though that curvature is negative. It is difficult to visualize a 4D saddle shape, of the same shape at every point in spacetime. So AdS is also symmetric in all space as de Sitter. AdS spacetime is not the vacuum of our particular universe, but in the multi-verse scenario, there are probably many universes that do have AdS vacuums.

**Big Crunch In Brief**

The expansion of universe started some 15 billion year ago with Big Bang. The studies of red shift in the spectrum of galaxies in near vicinity suggest the expansion of our universe. The experimental studies of Cosmic Microwave Background (CMB) also suggest that Universe is originated due to Big Bang. Early of the twentieth century some astrophysicist started to propound that how long this expansion will last and what is going to be happened with this universe. There are three different fate of our universe suggested in popular literature which are (i) Big Rip, (ii) Big Freez and (iii) Big Crunch[8]. Here we are discussing the Big Crunch as other two are not part of our study.



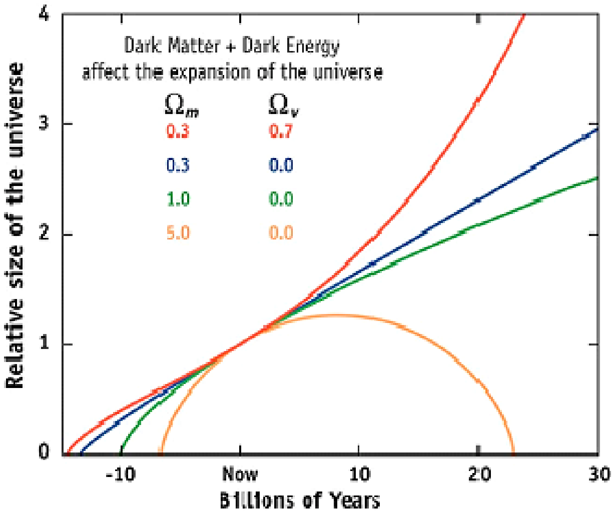




**Fig2: The Beginning to the end of the Universe The Big Crunch vs**
**Fig 3 : Fate of Universe in the view of DM and DE Courtesy Big Freeze by Eric Betz "Astronomy" 31 Jan 2021 NASA WMAP Collaboration [12]**

The Fig 2 above is described by [8] and above Fig 3 on right is by WMAP (Wilkinson Microwave Anisotropy Probe experiment) which shows that the fate of Universe is ultimately depend on density of dark matter and dark energy. In the next section we discuss the Anti de Sitter and Conformal Field theory and its aspects to explain the Big Crunch.

**Anti de Sitter/ Conformal Field Theory (AdS/CFT) & Big Crunch**

The idea of AdS/CFT duality arises from superstring theory. As we know that superstring theory is a major tool of Grand Unification Theory (GUT). The GUT unify four fundamental forces in nature which are gravity, electromagnetic force, strong force and weak force. The AdS/CFT duality claims following equivalence between two theories:

Since AdS/CFT claims relation between four dimensional physics and five dimensional physics therefore it is also called a holographic theory. We know that an optical hologram encodes a three dimensional image on a two dimensional object. Similarly the holographic theory encodes 5-dimensional theory by a 4-dimansional theory. The gauge theory well explain the three forces namely electromagnetic (by U(1)), Strong force (SU(3)) also known as QCD (Quantum Chromo Dynamics) and weak force but gravity is not explained by gauge theory.

| |
|---|
| Strongly-coupled 4-dimensional gauge theory<br>= Gravitational theory in 5-dimensional AdS spacetime |

The strong force is really strong that we are not able to understand it for strong coupling by gauge theory. For the same we use AdS/CFT duality which explain strong coupling using curved space time namely AdS spacetime.

De Sitter solved the Einstein's equation by considering a space as sphere with constant positive curvature (For expansion). On the other hand Anti de Sitter (AdS) space time have negative curvature (For contraction!). The gauge theory live on 4-dimansional boundary of space time [9]. The AdS/CFT is first adopted for explaining singularity in black hole. Using a black hole one can get glimpse of holography. The AdS/CFT duality may also give details of singularity in big crunch.

AdS/CFT enables one to analyze a strongly-coupled gauge theory using the AdS spacetime. However, there are a number of significant differences between the realistic SU(3) gauge theory and the gauge theory studied in AdS/CFT.

First, AdS/CFT typically considers a SU($N_c$) gauge theory. In such a theory, $N_c$ plays a role of a parameter, and the "strong coupling" is the so-called large-$N_c$ limit, where one tunes $N_c$ in an appropriate way.

**Fig4: The above is the famous cows figure used to explain effect of negative curvature. All the cows are having same shape (invariance) and proper size but due to negative curvature they appear smaller near boundary. This happened because of projection of 3 dimensional space on 2 dimensional paper. [7]**

Second, AdS/CFT typically considers a supersymmetric gauge theory which has supersymmetry. In particular, the $\mathcal{N}$ = 4 Super-Yang-Mills theory (SYM) provides the simplest example of AdS/CFT. Here, $\mathcal{N}$ = 4 denotes the number of supersymmetry the theory has. The theory also has the scale invariance since the theory has no dimensionful parameter. Furthermore, the theory has a larger symmetry known as the conformal invariance which contains the Poincar´e invariance and the scale invariance. Such a theory is in general known as a conformal field theory, or CFT. This is the reason why the duality is called AdS/CFT.

**Poincaré Coordinate System of AdS Spacetime**

In this section we discuss the Poincaré coordinates (t,r) however Static coordinate and Conformal coordinate can also be used for consideration but that is beyond the scope of this work. The Poincaré coordinate system is defined as

$$Z = \frac{Lr}{2}\left(-t^2 + \frac{1}{r^2} + 1\right)$$
$$X = Lrt$$
$$Y = \frac{Lr}{2}\left(-t^2 + \frac{1}{r^2} - 1\right)$$

When $(r > 0, t{:} -\infty \to \infty)$ . The metric become

$$\frac{ds^2}{L^2} = -r^2dt^2 + \frac{dr^2}{r^2}$$

Above is most often used coordinate system in AdS/CFT duality. $AdS_{p+2}$ spacetime has SO(2,p+1) invariance which are defined by

$$ds_{p+3}^2 = -dX_0^2 + dX_1^2 + \cdots + dX_{p+2}^2$$
$$-X_0^2 + X_1^2 + \cdots + X_{p+2}^2 = +L^2$$

For $p = 0$, we set $X_0 = Z, X_1 = X, and\ X_2 = Y$ just like $dS_2$ the $dS_{p+2}$ spacetime becomes $S^{p+2}$ by switching X to its complex variable.

**AdS Spacetime Summary** [9]**:**

- The AdS spacetime is the spacetime of constant negative curvature.
- The$AdS_5$ spacetime has the (3+1)-dimensional timelike boundary known as the AdS boundary.
- The $AdS_5$ spacetime has the SO(2,4) invariance. This symmetry is the same as the (3+1)-dimensional conformal invariance of the $\mathcal{N}$ = 4 SYM.
- The $AdS_5$ spacetime in Poincaré coordinates coincides with the near-horizon limit of the D3-brane.
- In AdS/CFT, the gauge theory time is the coordinate time not the proper time.
- Given a proper energy of an excitation, the gauge theory energy is larger if the excitation is nearer the AdS boundary.

Quantum Extremal Surfaces and Singularity in Big Crunch: The Big Bang Singularity is a very striking and cosmological solvable phenomenon introduced by Einstein. In the year 1959, the question regarding Cosmological Singularity (CS) and its relation to Einstein's equation was first formulated by L. Landau. In the rear 1969, this question was answered by V. Belinski, I. Khalatnikov, E. Lifshitz (BKL). According to BKL, singularity is a property of a generic cosmological solution of the classical gravitational equations and it behaves in chaotic manner. The chaotic behavior can be explained in terms of “Cosmological Billiard” system, in which cosmological evolution is described at each spatial point as the relativistic motion of the fictious billiard ball in the Lorentzian space of the logarithmic scale factors. In 1965, Roger Penrose stated that under some conditions incomplete geodesics in space-time can’t be avoided. Both BKL conjecture and Penrose represent two sides of phenomenon but the links between these two can’t be established. The general solution with singularity can be applied both to the Big Crunch and Big Bang although both are physically different. The higher dimensional background were studied as time dependent deformation of AdS/CFT via gauge/gravity duality into singularity of Big Bang or Big Crunch[10]. When bulk spacetime develops a cosmological big crunch singularity and breakdown, then in $AdS_5$ case the AdS/CFT duality living on the space that crunch itself have strong time-dependent gauge coupling. The scaling assumptions considered for singularity of big crunch are

$$X^2 = t^k r^m, e^f = \frac{1}{r^2} = t^a r^b, e^\emptyset = t^\alpha r^\beta$$

Then

$$ds^2 = \frac{e^f}{X^{2(d_i-1)/d_i}}(-dt^2 + dr^2) + X^{4/d_i} dx_i^2$$

Above equation is for simulation of Big Crunch (or Big Bang) [9-11].

## II. Conclusion

Anti de Sitter and Conformal Field Theory are combined together to study condensed matter made up of extra dimensional particles like Dilaton and Strings which are going to decide ultimate fate; the Big Crunch of our Universe. The AdS/CFT correspondence is used to understand the transition from expanding universe to non-relativistic transformation to obtain Big Crunch. The detailed study in this field can improve our understanding about origin and final destination of our universe.

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