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# Space Quantization And Vector Atomic Model

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***ABSTRACT***
*J. J. Thomson was the first to discover electron and observed that all the materials comprises a common constituent particle – the electron. He was the first who gives us structure of atom. After him many physicists come forwards with their new ideas and put light on the structure of atom among which Rutherford's atomic model, Neil Bohr's and Sommerfeld's atomic model was taken under consideration. Moreover, In this particular paper we will summarise the concepts of various physicists and to get a new atomic model called vector atomic model. We also put light on the space quantisation part of atomic model.*

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## I. INTRODUCTION

Starting from J. J. Thomson various physicist including Rutherford, Neil Bohr, Sommerfeld attempts to explain the structure of atom. J. J. Thomson was the first to discover electron and observed that all the materials comprises a common constituent particle – the electron. According to J. J. Thomson, an atom is a spherical shell of positively charged protons in which electrons are embedded like the seeds in watermelon. Rutherford gives the concept of nucleus - the central core in which whole mass of atom is concentrated and the electrons are revolving around it. This model fails to explain the hydrogen spectrum and the stability of atom. Bohr's model was quite satisfactory at that time and explains various properties of atom but fails to explain the structure of atomic species having more than one valance electron. Sommerfeld extend Bohr model and gave the concept of elliptical orbitals. His model was based on Einstein's theory of Relativity. It is also applicable for only single electron species – hydrogen and hydrogen like species. Even though Sommerfeld explains most of the phenomenon observed experimentally but it has some serious limitations too. It could not explain the distribution of electrons in an atom, variations in the intensities of spectral lines, anomalous Zeeman effect.

To explain the limitations of all the previously based atomic models, Vector atomic models was taken under consideration. Vector atomic models gains its popularity because of the fact that it is the only model that throw light on the distribution and arrangement of electrons in atoms. Not only that this model also explains the phenome like Anomalous Zeeman effect, Stark effect which was not explain by the rest of models given before. However Sommerfeld model was able to explain the phenomenon like fine structure of the spectral lines of simplest hydrogen atom to some extend but does not gave any information about the relative intensities of the lines. To explain the complex spectra of atoms and the correlation to atomic structure, vector model was introduced. In this particular paper we will summarise the concepts of various physicists and to get a new atomic model called vector atomic model. We also put light on the space quantisation part of atomic model.

### VECTOR ATOMIC MODEL

Two distinct features of vector atomic model for which it is know is:

1. The concept of spatial quantisation and
2. The concept of spinning electrons

**Space Quantisation:-** According to Bohr Sommerfeld theory, electron possesses two degree of freedom and the motion of electron in an elliptical orbit is two dimensional. hence two quantum numbers are sufficient to define the energy state and the orbit of electron in which it revolves. But in general it is found that the motion of electron in an atom is three dimensional and therefore possesses three degree of freedom. So an additional quantum number and the corresponding quantum condition is required to describe its actual state. On the other hand classical mechanics suggest that , when an electron moves in three dimensional orbits, the orbit may have all the orientations w.r.t. a fixed direction.

Based on quantum theory, the vector atomic model consider orbits to be quantised both in magnitude (i.e., size and form) as well as in the directions (i.e., orientation of orbits in space), making only certain discrete orientation of electron allowed. Hence, space quantisation forms the orbits vector quantities. The third quantum condition is that to quantise the orientation of elliptical orbit in three dimensions and does not alter the original Sommerfeld Orbits in regards to their size and shape. Certain preferred direction is needed for the purpose of space quantisation to which the orbits may receive their orientation and this is possibly obtained by the placing of an atom in a uniform magnetic field. In most of the cases we would like to take the magnetic field direction along the z -axis for our convenience. Moreover, to preserve the deformation of an orbit, tiny field is chosen. Theta is the angle taken between the field direction and the direction which is perpendicular to the plane of the orbit in such a way that it must be taken on certain discrete values i.e., the electron can only set itself in certain discrete positions with respect to the field direction. By using the idea of space quantisation we can explain the phenomenon like Zeeman Effect.

**Spinning Electron:** - By using high resolving power spectrometer, it is found that the yellow line of sodium comprising of two components viz. D1 and D2 (doublet) with a separation of 6A0, while for the spectral lines of magnesium, it reveals to be close triplets. Moreover the splitting goes to be more and more complex as we go to the higher elements i.e., the separation between component lines increases with increasing atomic number. In case of mercury, the spread of mercury triplet was observed to be over 1400 A0. Such splitting of components of various spectral lines in presence of a magnetic field is known as Zeeman Effect.

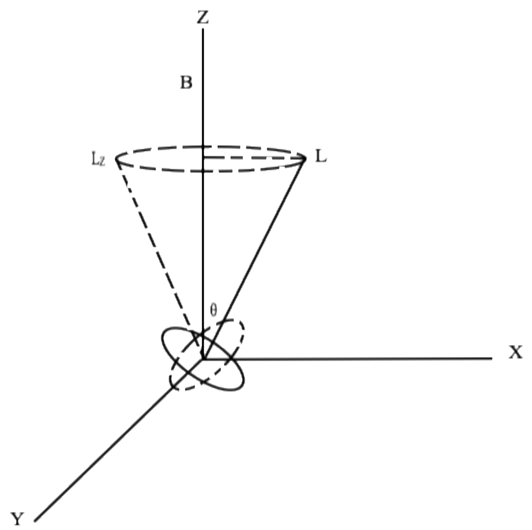
To account the effect of magnetic field on spectral lines and to explain the multiple characters of these lines, **Uhlenbeck** and **Goudsmit** in 1925 put forward the hypothesis of electron spin. They compare the motion of electrons around the nucleus to the motion of planets revolving around the sun in a solar system. According to this hypothesis, the electron in an orbit not only revolves around the nucleus but also rotates about its own axis. Thus two types of motions are required to define the behaviour of electron. One is orbital motion commonly known is orbital angular motion and the second one is spin motion also known as spin angular motion because of the fact that the angular momentum of atom is due to the spinning and orbital motion of electron. Going towards the quantum theory, the spinning and the orbital motion of electron should be quantised. Hence a new quantum number is introduced by the name of spin quantum number denoted by “s”. By using the concept of space quantisation, both the orbital motion and the spin motion of electron are considered to be quantised vectors as they are quantised in both magnitude as well as direction. Hence the atomic model based on these quantised vectors is called the Vector Atomic Model to which the vector laws apply.

As we are well aware about the motion of charged particle (electron) about an axis, it produces the mechanical angular momentum and the circular current thereby giving rise a magnetic moment, in-fact two magnetic moments - one due to the orbital motion and other due to spin motion of electron. Moreover, total magnetic moment of an atom should be the vector sum of orbital and the spin magnetic moments. Similarly, the total angular momentum of an atom should be the vector sum of orbital and spin angular momentum. By studying the interaction between these two angular momenta viz orbital and spin we came to know about the multiplicity of the spectral lines while the interaction between the magnetic moment of both spin and orbital magnetic moment accounts for the fine structure of spectral lines.

While studying the behaviour of atoms in non – homogenous magnetic fields, **Stern** and **Gerlach** experimentally verified these two fundamental concepts of vector atomic model.

**Space Quantisation of Orbital Angular Momentum**

Consider an electron revolving around a nucleus in an orbit placed in an external magnetic field B in such a way that electron orbit precesses about the field direction as axis having angular momentum vector L traces a cone around B shown in fig.



Let θ be the angle between the angular momentum vector and the direction of external magnetic field B taken along Z – axis.

Now resolve L in XZ plane as

$$L = L_x + L_z$$

If we consider the magnetic field along Z – axis, then only Z – component of L is taken under consideration.

$$\text{i.e., } L_z = L\cos\theta \qquad \text{….(1)}$$

By using the concept of quantum mechanics, the magnitude of angular momentum and its z – component are quantized according to the relations

$$L = \sqrt{l(l+1)}\ \hbar \qquad \text{….(2)}$$

$$\text{and } L_z = m_l\hbar \qquad \text{….(3)}$$

Here l and $m_l$ are orbital and magnetic orbital quantum numbers respectively.

Angle & between L and Z – axis is given by

$$\text{Cos}\theta = \frac{L_z}{L} \qquad \text{….(4)}$$

Using equation (1), (2), (3) in (4), we get

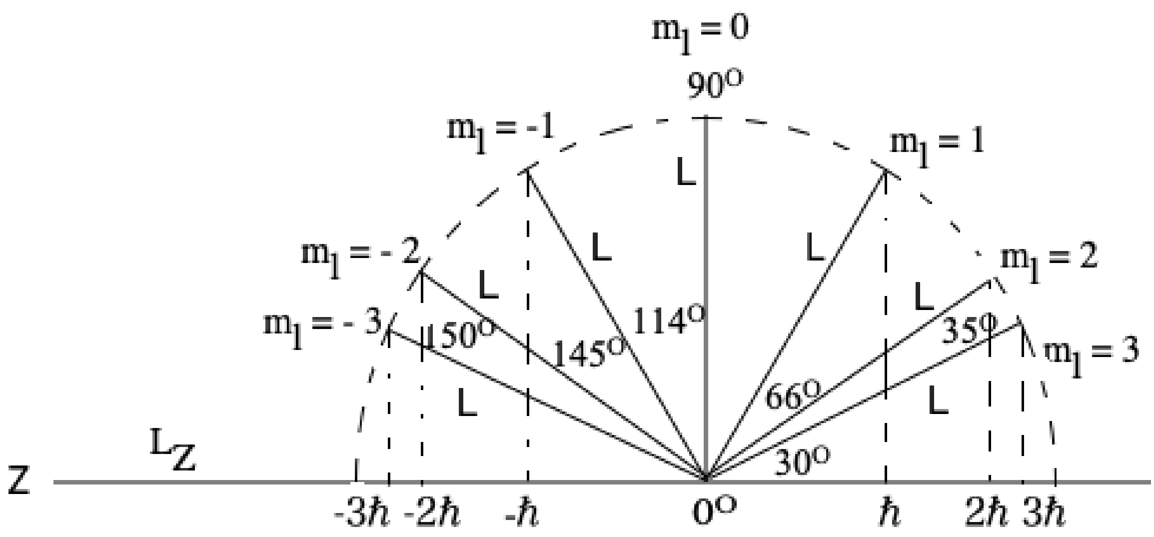
$$\text{Cos}\theta = \frac{m_l\hbar}{\sqrt{l(l+1)}\ \hbar}.$$

$$= \frac{m_l}{\sqrt{l(l+1)}}.$$

For given value of l, there are (2l +1) possible values of $m_l$.

The possible value of $m_l$ ranges from – l to + l. Thus angle θ can assume (2l+1) discrete values. In other words, angular momentum vector L can have (2l+1) discrete orientations in space. This quantisation of orientations in space is known as space quantisation.

The space quantisation of the orbital angular momentum vector corresponding to l = 3 is shown as



For l = 3 we have

$m_l$ = - 3, - 2, - 1, 0, 1, 2, 3

and $L_z = -3\hbar, -2\hbar, -\hbar, \hbar, 2\hbar, 3\hbar$

$\therefore \quad L = \sqrt{12}\,\hbar$

$= 2\sqrt{3}\,\hbar$

For $L_z = -3\hbar$

$$\text{Cos}\theta = \frac{-3\hbar}{2\sqrt{3}\,\hbar}$$

$$= \frac{-3}{2\sqrt{3}}$$

$$= \frac{-3 \times \sqrt{3}}{2\sqrt{3} \times \sqrt{3}} \qquad \{\text{Multiply and divide by } \sqrt{3}\}$$

$$= \frac{-\sqrt{3}}{2}$$

$$\therefore \quad \theta = \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) \qquad \{\cos^{-1}(-\theta) = \pi\text{-v}\cos^{-1}(\theta)\}$$

$$= \pi - \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$= \pi - \frac{\pi}{6}$$

$$= \frac{5\pi}{6}$$

Or $\boldsymbol{\theta \cong 150^{O}}$

Similarly, for $L_z = -2\hbar$

$$\text{Cos}\theta = \frac{-2}{\sqrt{6}}$$

$\therefore \quad \boldsymbol{\theta \cong 145^{O}}$

For $L_z = -\hbar$

$$\text{Cos}\theta = \frac{-1}{\sqrt{6}}$$

$\therefore \quad \boldsymbol{\theta \cong 114^{O}}$

For $L_z = 0$

$$\text{Cos}\theta = 0$$

$\therefore \quad \boldsymbol{\theta \cong 90^{O}}$

For $L_z = \hbar$

$$\text{Cos}\theta = \frac{1}{\sqrt{6}}$$

$$\therefore \quad \theta \cong 66^{O}$$

For $L_z = 2\hbar$

$$\cos\theta = \frac{2}{\sqrt{6}}$$

$\therefore \theta \cong 35^{O}$

For $L_z = 3\hbar$

$$\cos\theta = \frac{(\sqrt{3})}{2}$$

$$\therefore \quad \theta \cong 30^{O}$$

Thus the number of possible orientations = (2l+1) = 7.

**Electron Spin**: - While observing the spectral line through a high resolving spectrometer, each line is found to consist of two or more closely spaced lines. The groups of lines so obtained is called fine structure of a spectral line. To explain such phenomenon, two scientists namely **Samuel Goudsmit** and **George Uhlenback** in 1925 comes forward and proposed the concept of electron spin about its own axis which was similar to the rotation of earth about its axis. Thus in addition to orbital angular momentum L, an electron possesses an intrinsic angular momentum called spin angular momentum S, due to the motion of electron around the nucleus. The existence of electron spin was confirmed by Dirac in 1929 in his theory of relativistic quantum mechanics.

Magnitude of spin angular momentum is given by

$$S = \sqrt{s(s+1)}\,\hbar$$

Here s = spin quantum number. The only value that **s** can have is ½.

$$\therefore \quad S = \frac{\sqrt{3}}{2}\,\hbar$$

Spin angular momentum is also space quantized just like the orbital angular momentum.

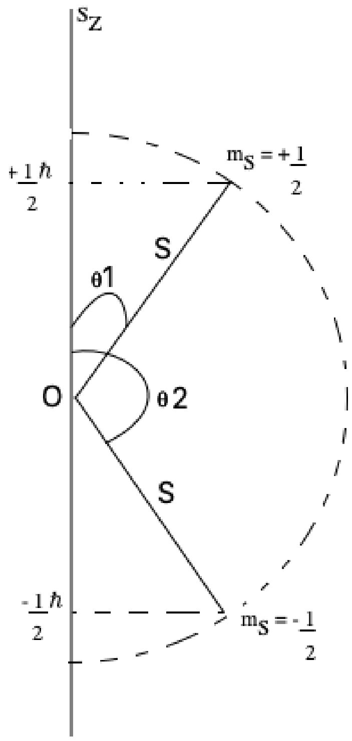
The Z – component of spin angular momentum is given by

$$S_z = m_s\hbar$$

$$= \pm \frac{1}{2}\,\hbar$$

Here the allowed values for the spin magnetic quantum number $m_s$ are $+\frac{1}{2}$ and $-\frac{1}{2}$ also known as up spin and down spin respectively.

The spin angular momentum S may have (2s + 1) = 2 orientations in space that shows space quantization of electron spin.




If we take θ as the angle between S and the Z – axis, then

$$\cos\theta = \frac{S_z}{S}$$

We know that

$$S_z = m_s\hbar$$

$$\text{and } S = \sqrt{s(s+1)}\,\hbar$$

$$\therefore \cos\theta = \frac{m_s\hbar}{\sqrt{s(s+1)}\,\hbar}$$

$$= \frac{m_s}{\sqrt{s(s+1)}}$$

For spin Up; $m_s = +\frac{1}{2}$

$$\cos\theta = \frac{1}{\sqrt{3}}$$

$$\therefore \theta = 54^{O}45'$$

For spin Down; $m_s = -\frac{1}{2}$

$$\cos\theta = -\frac{1}{\sqrt{3}}$$

$$\therefore \theta = 125^{O}\,15'$$

Magnetic moment of negatively charged electron is opposite to the angular momentum vector S. The concept of electron spin not only defines the fine structure and anomalous Zeeman effect but also proves a wide range of other atomic effects as well.

## II. Conclusion

The two distinct features of vector atomic model for which it is know is the concept of spatial quantisation and the concept of spinning electrons. By using the concept of orbital angular momentum we came to know about the orientation of electrons in an atom while the concept of electron spin not only defines the fine structure and anomalous Zeeman effect but also proves a wide range of other atomic effects as well.

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