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The role of topological invariants in the study of the early evolution of the Universe

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Abstract:
The study of the stages of the early evolution of the Universe is of interest from the point of view of physical and mathematical interpretation. One of the most promising and relevant is the apparatus of D-brane representation of early states in Universe, which can be used to represent phase transitions in the Universe and potential energy representations of different states. The transition to the geometric representation of soliton objects of the D-brane type allows us to represent the phase transition as a nesting of polyhedra. Using the apparatus of vector bundles and the fact of isomorphism of Universal bundles to vector bundles we have done the calculations of homotopic groups. Physical interpretation of the obtained result is connected with the observation of a transition from one solitonic state to another one with the corresponding equidistant set of energy levels.

Key Word*: evolution of the Universe; D-brane; nesting of polyhedra; homotopic groups.*

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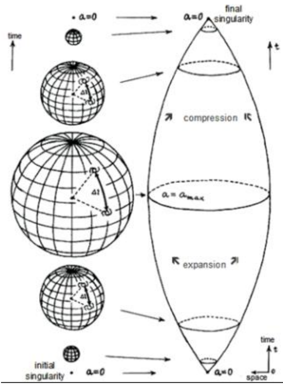
I. Introduction

Questions of the Universe evolution, the nature of forces and physical processes at an early stage of its origin and development are connected with an explosion of space itself - the Big Bang. Phase transitions in time and space could be presented by the following evolution picture as one of the possible options for the development process from point to point through a set of spheres of different dimensions, Fig.1.

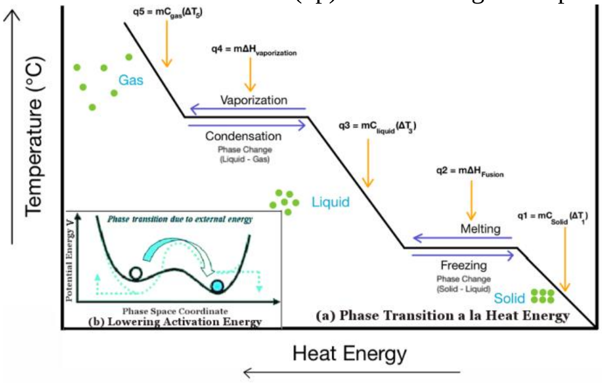
Figure 1: Evolution of the Universe from point to another point presented through a set of spheres of different dimensions.



The most promising and complex version of the theory describing processes at high matter densities and high energies is the theory of superstrings and *D*-branes. It is described in terms of vector bundles, where the fiber is complex *n*-dimensional vector space such as the Hilbert space H^n considered to be a bundle over the projective Hilbert space PH^n. Moreover, the (2*n* - 1)-sphere S^{2n-1} formed by the normalized vectors of the Hilbert space is a bundle with circular fibers (S^1 -bundle) on PH^n. The phase transitions of one sphere into another one are connected with the changes in vacuum energy when a change in potential energy is associated with a transition to a new state, or to a larger sphere.

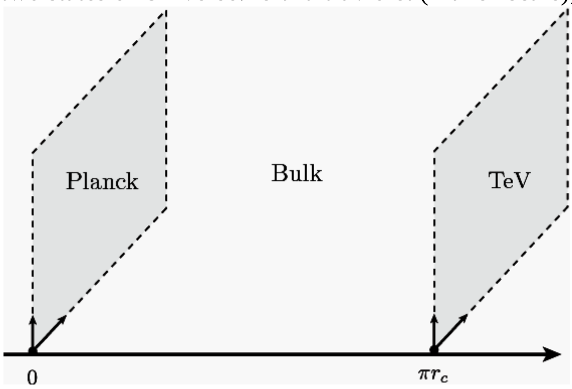
The discovery of the Higgs boson led to the problem of a metastable vacuum in the mechanism of electroweak symmetry breaking, Fig.2

Figure 2: Phase transitions in the hot Universe (up) and lowering in the potential energy (down).



Such a transition in D-brane language can be represented as follows, Fig.3

Figure 3: Two D-branes as two states of Universe: left- ultraviolet (Planck scale), right- infrared (TeV scale).



Such D-branes (Planck brane is the left minimum of potential and TeV brane is the right minimum of potential in Fig.2) could be presented in terms of vector bundles characterized by topological invariants, [1]. Our purpose will be to calculate these topological invariants of the corresponding vector bundles and to predict the possibility of phase transition between different states or vector bundles.

II. Mathematical apparatus of homotopic groups and bundle isomorphisms

For the realization of the purpose we considered two variants of inclusions as nesting of

$$S^1 \subset S^2 \subset \ldots \subset S^n \quad (1)$$

$$\mathbb{C}P^1 \subset \mathbb{C}P^2 \subset \ldots \subset \mathbb{C}P^n \quad (2)$$

spheres (1) and complex projective spaces (2). It is known that

$$V_1(\mathbb{R}^n) = S^{n-1} = O(n)/O(n-1) = SO(n)/SO(n-1)$$

where $V_1(\mathbb{R}^n)$ is stiefel space. Therefore, we can consider the inclusion of stiefel spaces

$$V_k(F^k) \subset V_k(F^{k+1}) \subset \ldots \subset V_k(F^n) \subset \ldots \subset V_k(F^\infty) = \bigcup_{n\geqslant 1} V_k(F^n)$$

instead of

$$S^1 \subset S^2 \subset \ldots S^n \subset \ldots \subset S^\infty = \bigcup_{n\geqslant 1} S^n$$

for $F = \mathbb{R}$ or

$$\mathbb{C}P^1 \subset \mathbb{C}P^2 \subset \ldots \subset \mathbb{C}P^\infty = \sum_{n\geq 1} \mathbb{C}P^n$$

for $F = \mathbb{C}$.

There is the isomorphism between homotopy groups [2] for

$$i \leqslant c(n+1) - 3$$

$$j_* : \pi_i(V_k(F^n)) \to \pi_i(V_{k+1}(F^{n+1}))$$

It is known that $\pi_{n-k}(\mathbb{R}^n)$ isomorphic to $\mathbb{Z}$, $k = 1$.

We can calculate homotopy groups for $k = 1$, $c = 1$, $n = 4,5$

$$\pi_2\left(V_1\left(\mathbb{R}^4\right)\right)=\pi_2\left(V_2\left(\mathbb{R}^5\right)\right)=\mathbb{Z}$$
$$\pi_3\left(V_1\left(\mathbb{R}^5\right)\right)=\pi_3\left(V_2\left(\mathbb{R}^6\right)\right)=\mathbb{Z}$$

and for k = 1, c = 2, n = 2,3

$$\pi_3\left(V_1\left(\mathbb{C}^2\right)\right)=\pi_3\left(V_2\left(\mathbb{C}^3\right)\right)=\pi_3\left(S^3\right)=\mathbb{Z}$$
$$\pi_5\left(V_1\left(\mathbb{C}^3\right)\right)=\pi_5\left(V_2\left(\mathbb{C}^4\right)\right)=\pi_5\left(S^5\right)=\mathbb{Z}$$

Furthermore, we know, that the space

$$SG_k\left(F^n\right)=SO(n)/SO(k)\times SO(n-k)$$

is the base of the principal bundle according to the theorem 1

Theorem 1 *The bundle ($V_k(F^n)$, p, $G_k(F^n)$) is a principal $U_F(k)$-bundle and the bundle ($V_k(F^n)$, p, $SG_k(F^n)$) is a principal $SU_F(k)$-bundle.*

In our cases for $F = \mathbb{R}$ the base spaces for principal bundles ($V_2(R^5)$, p, $G_2(R^5)$) and ($V_2(R^6)$, p, $G_2(R^6)$) are

$$G_2\left(\mathbb{R}^5\right)=O(5)/O(2)\times O(3)$$
$$G_2\left(\mathbb{R}^6\right)=O(6)/O(2)\times O(4)$$

and for $F = \mathbb{C}$ the base spaces for principal bundles ($V_2(C^3)$, p, $G_2(C^3)$) and ($V_2(C^4)$, p, $G_2(C^4)$) are

$$G_2\left(\mathbb{C}^3\right)=U(3)/U(2)\times U(1)$$
$$G_2\left(\mathbb{C}^4\right)=U(4)/U(2)\times U(2)$$

Using the theorem 2

Theorem 2 *For $U_F(k)$, the principal bundle $V_k(F^{k+m})\rightarrow G_k(F^{k+m})$ is universal in dimensions $\leq c(m+1)-2$, and $V_k(F^{\infty})\rightarrow G_k(F^{\infty})$ is a universal bundle. For $SU_F(k)$, the principal bundle $V_k(F^{k+m})\rightarrow SG_k(F^{k+m})$ is universal in dimensions $\leq c(m+1)-2$, and $V_k(F^{\infty})\rightarrow SG_k(F^{\infty})$ is a universal bundle.*

we can conclude that principal bundles $V_2(R^5)\rightarrow G_2(R^5)$ and $V_2(R^6)\rightarrow G_2(R^6)$ are universal in dimensions $d \leq 1(3+1)-2$ or in d = 1,2 and $d \leq 1(4+1)-2$ or in d = 1,2,3 correspondingly.

Principal bundles $V_2(C^3)\rightarrow G_2(C^3)$ and $V_2(C^4)\rightarrow G_2(C^4)$ are universal in dimensions $d \leq 2(1+1)-2$ or in d = 1,2 and $d \leq 2(2+1)-2$ or in d = 1,2,3,4 correspondingly. Using the

Proposition 1 *The bundle morphism f: $\alpha_{\boldsymbol{k}}^{\boldsymbol{n}}\left[\boldsymbol{F}^{\boldsymbol{k}}\right]\rightarrow\gamma_k{}^n$ is a vector bundle isomorphism.*

We conclude that the bundles

$$\alpha_2^5\left(V_2\left(\mathbb{R}^5\right),p,G_k\left(\mathbb{R}^5\right)\right)$$
$$\alpha_2^6\left(V_2\left(\mathbb{R}^6\right),p,G_k\left(\mathbb{R}^6\right)\right)$$

and

$$\alpha_2^3\left(V_2\left(\mathbb{C}^3\right),p,G_k\left(\mathbb{C}^3\right)\right)$$
$$\alpha_2^4\left(V_2\left(\mathbb{C}^4\right),p,G_k\left(\mathbb{C}^4\right)\right)$$

are vector bundles.

III. Conclusion

We considered two universal bundles $\alpha_2^5:\left(\boldsymbol{V}_2(\boldsymbol{R}^5),\boldsymbol{p},\boldsymbol{G}_2(\boldsymbol{R}^5)\right)$, $\alpha_2^6:\left(\boldsymbol{V}_2(\boldsymbol{R}^6),\boldsymbol{p},\boldsymbol{G}_2(\boldsymbol{R}^6)\right)$ and $\alpha_2^3:\left(\boldsymbol{V}_2(\boldsymbol{C}^3),\boldsymbol{p},\boldsymbol{G}_2(\boldsymbol{C}^3)\right)$, $\alpha_2^4:\left(\boldsymbol{V}_2(\boldsymbol{C}^4),\boldsymbol{p},\boldsymbol{G}_2(\boldsymbol{C}^4)\right)$ which are isomorphic to vector bundles, γ_2^5,γ_2^6 and γ_2^3,γ_2^4 correspondingly.

Using the isomorphism of homotopic groups for $F = \mathbb{R}$, k = 1, c = 1, n = 4,5

$$\pi_2\left(V_1\left(\mathbb{R}^4\right)\right)=\pi_2\left(V_2\left(\mathbb{R}^5\right)\right)=\mathbb{Z}$$
$$\pi_3\left(V_1\left(\mathbb{R}^5\right)\right)=\pi_3\left(V_2\left(\mathbb{R}^6\right)\right)=\mathbb{Z}$$

and for F=C, k = 1, c = 2, n = 2,3

$$\pi_3\left(V_1\left(\mathbb{C}^2\right)\right)=\pi_3\left(V_2\left(\mathbb{C}^3\right)\right)=\pi_3\left(S^3\right)=\mathbb{Z}$$
$$\pi_5\left(V_1\left(\mathbb{C}^3\right)\right)=\pi_5\left(V_2\left(\mathbb{C}^4\right)\right)=\pi_5\left(S^5\right)=\mathbb{Z}$$

and the fact that $V_1(F^{n+1}) = S^{c(n+1)-1}$ we can do the following physical conclusion:

D-branes can be represented as a vector bundles with a base - a sphere and using the embedding of spheres, $S^4 \subset S^5$ for the real space and $\mathbb{C}P^2 \subset \mathbb{C}P^3$ for the complex spaces, we observe a transition from one solitonic state in the form of D-brane to another one with the corresponding equidistant set of energy levels. The obtained result signals about the existence of the vacuum states in every minimum of potential energy and about the possibility of phase transitions in early Universe.

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