IOSR Journal Of Applied Physics (IOSR-JAP)
e-ISSN: 2278-4861.Volume 14, Issue 5 Ser. I (Sep. – Oct. 2022), PP 48-51
www.Iosrjournals.Org

THE UNITARY VECTOR FIELD$\boldsymbol{u^i}(\boldsymbol{x}(\boldsymbol{s}))$ AS A PARALLEL TRANSPORTED FIELD

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Abstract:
*In a 4- dimensional continuum with asymmetrical affine connection$L^i_{jk}(x)$, the covariant derivative of the unitary vector field $\boldsymbol{u^i}(\boldsymbol{x}(\boldsymbol{s}))$, tangent to the curve $\boldsymbol{x}(\boldsymbol{s})$, i.e. $\nabla_j u^i = 0$, with respect to the connection $L^i_{jk}(x)$, defines a parallel vector field $\boldsymbol{u^i}(\boldsymbol{x}(\boldsymbol{s}))$, and at the same time the equation of motion of a charged particle in both gravitational and electromagnetic field.***
Key Word: *Unitary vector, Transporter, Torsion Tensor, Gravitation, Electromagnetic field Tensor*.

Date of Submission: 14-09-2022 Date of Acceptance: 29-09-2022

I. Introduction

It is desirable to have a single equation to describe the motion of a charged particle in the presence of the gravitational and electromagnetic fields.

As is well Known, in a system of coordinate X, a covariant parallel vector field satisfies a system of differentials equations. The transporter or affine connection $L^i_{jk}(x) = \Gamma^i_{jk}(x) + \Omega^i_{jk}(x)$ [1] in a 4-dimensional continuum, allow us to construct such systems of differentials equations. Where $\Gamma^i_{jk}(x)$ are the well known Christoffel Symbols and $\Omega^i_{jk}(x)$ is the torsion tensor.

In section II we propose a single equation to describe four equations of motion. In section III we construct Maxwell equations in absence of charge andcurrent.

II. A single equation

A parallel vector field$u_i(x)$, with $x = x(s)$,satisfies the system of differentials equations [1]:

$$\nabla_l u_i = \frac{\partial u_i}{\partial x^l} - L^h_{il} u_h = 0 \qquad \text{.............................. (2.1)}$$

Multiplication by u^l , gives:

$$u^l \frac{\partial u_i}{\partial x^l} - L^h_{jl} u_h u^l = 0 \qquad \text{.............................. (2.2)}$$

Using the chain- rule relation:

$$\frac{d}{ds} = \frac{dx^j}{ds}\frac{\partial}{\partial x^j} = u^j \frac{\partial}{\partial x^j}$$

We have, [1]:

$$\frac{du_i}{ds} - L^k_{ij} u_k u^j = 0 \qquad \text{.............................. (2.3)}$$

Explicitly

$$\frac{du_i}{ds} - \Gamma^k_{ij} u_k u^j - \Omega^k_{ij} u_k u^j = 0 \qquad \text{.............................. (2.4)}$$

(2.4) is a unique equation containing 4 equations of motion.

1) If$\Gamma^k_{ij} = 0$and $\Omega^k_{ij} = 0$:

$$\frac{du_i}{ds} = 0 \qquad \text{.............................. (2.5)}$$

Equation (2.5) is the Law of inertia if u_i is the covariant component of the four velocities$u$.

2)If$\Omega^k_{ij} = 0$

$$\frac{du_i}{ds} - \Gamma^k_{ij} u_k u^j = 0 \qquad \text{.............................. (2.6)}$$

Is the geodesic equation of motion of a particle in a gravitational field.

3)If $\Gamma_{ij}^{k} = 0$

$$\frac{du_i}{ds} - \Omega_{ij}^{k} u_k u^j = 0 \quad \text{………………………… (2.7)}$$

Is the famous Lorentz equation of motion of a charged particle.
To see this, let us define:

$$u_m \Omega_{ij}^{m} = \left(\frac{e}{mc^2}\right) f_{ij} \quad \text{………………………… (2.8)}$$

Where f_{ij} is the electromagnetic field tensor.Substitution of (2.8) in(2.7) gives [2]:

$$\frac{du_i}{ds} - \left(\frac{e}{mc^2}\right) f_{ij} u^j = 0 \quad \text{………………………… (2.9)}$$

Which is immediately recognizable as the Lorentz equation of motion of a charged particle. m is the mass of the particle; e is the electrical charge and cis light velocity.
Definition (2.8) is very important because it gives a physical meaning to the torsion tensor.

$$\Omega_{ij}^{m} = \left(\frac{e}{mc^2}\right) u^m f_{ij} \quad \text{………………………… (2.10)}$$

4) If Γ_{ij}^{k} andΩ_{ij}^{k} are different from zero, then:

$$\frac{du_i}{ds} - \Gamma_{ij}^{k} u_k u^j - \Omega_{ij}^{k} u_k u^j = 0 \quad \text{………………………… (2.11)}$$

Is the equation of motion of a charged particle in both gravitational and electromagnetic field[2].
Now, the condition of integrability of (2.1) is:

$$u_i L_{jkl}^{i} = 0 \quad \text{………………………… (2.12)}$$

Or simply as:

$$L_{jkl}^{i} = 0 \quad \text{………………………… (2.13)}$$

Where:

$$L_{jkl}^{i} = \frac{\partial L_{jl}^{i}}{\partial x^k} - \frac{\partial L_{jk}^{i}}{\partial x^l} + L_{hk}^{i} L_{jl}^{h} - L_{hl}^{i} L_{jk}^{h} \quad \text{………………………… (2.14)}$$

But:$L_{jk}^{i} = \Gamma_{jk}^{i} + \Omega_{jk}^{i}$ then:

$$L_{jkl}^{i} = R_{jkl}^{i} + \Omega_{jkl}^{i} \quad \text{………………………… (2.15)}$$

Where:

$$R_{jkl}^{i} = \frac{\partial \Gamma_{jl}^{i}}{\partial x^k} - \frac{\partial \Gamma_{jk}^{i}}{\partial x^l} + \Gamma_{hk}^{i} \Gamma_{jl}^{h} - \Gamma_{hl}^{i} \Gamma_{jk}^{h} \quad \text{………………………… (2.16)}$$

$$\Omega_{jkl}^{i} = \Omega^{i}{}_{j\,l|k} - \Omega_{j\,k|l}^{i} + \Omega_{hl}^{i} \Omega_{jk}^{h} - \Omega_{hk}^{i} \Omega_{jl}^{h} - 2\Omega_{jh}^{i} \Omega_{kl}^{h} \quad \text{……… (2.17)}$$

A solidus followed by an index indicates covariant differentiation with respect to the transporter or affine connection L_{jk}^{i} . $\Omega^{i}{}_{jlk}$ is the so called torsional tensor or Cartan's tensor.

R_{jkl}^{i} as defined by (2.16) are the components of the Riemann curvature tensor of the symmetric connection coefficient Γ_{jk}^{i} ,well known as the Christoffel symbols, they satisfy the identities:

$$R_{jkl}^{i} + R_{jlk}^{i} = 0 \quad \text{………………………… (2.18)}$$

$$R_{jkl}^{i} + R_{klj}^{i} + R_{ljk}^{i} = 0 \quad \text{………………………… (2.19)}$$

From parallelism condition:

$$L_{jkl}^{i} = 0 \quad \text{………………………… (2.20)}$$

We have:

$$R_{jkl}^{i} + \Omega_{jkl}^{i} = 0 \quad \text{………………………… (2.21)}$$

And from (2.21):

$$\Omega_{jkl}^{i} + \Omega_{klj}^{i} + \Omega_{ljk}^{i} = 0 \quad \text{………………………… (2.22)}$$

Permutation of the indexes jkl in (2.22) and then add,according to (2.22),gives [3]:

$$\Omega_{j\,l|k}^{i} + \Omega_{lk|j}^{i} + \Omega_{kj|l}^{i} + 2\left(\Omega_{jl}^{h} \Omega_{kh}^{i} + \Omega_{lk}^{h} \Omega_{jh}^{i} + \Omega_{kj}^{h} \Omega_{lh}^{i}\right) = 0 \quad \text{……… (2.23)}$$

This identity discovered by Einstein in 1929,can it be obtained from Ricci and Jacobi identities.
Let us consider Einstein demand; e.i. the product:

$$\Omega_{jl}^{h} \Omega_{kh}^{i} + \Omega_{lk}^{h} \Omega_{jh}^{i} + \Omega_{kj}^{h} \Omega_{lh}^{i} = 0 \quad \text{………………………… (2.24)}$$

Multiplication by $u_h u_i$ gives:

$$u_h u_i (\Omega^h_{jl}\Omega^i_{kh} + \Omega^h_{lk}\Omega^i_{jh} + \Omega^h_{kj}\Omega^i_{lh}) = 0 \qquad \text{(2.25)}$$

From (2.8) and (2.25):

$$f_{jk} f_{mh} + f_{km} f_{jh} f_{mj} f_{kh} = 0 \qquad \text{(2.26)}$$

But: $f_{ii} = 0$ for i=1,2,3,4.

$$f_{12} = -f_{21} = H_z \qquad f_{14} = -f_{41} = -iE_x$$
$$f_{13} = -f_{31} = H_y \qquad f_{24} = -f_{42} = -iE_y$$
$$f_{23} = -f_{32} = H_x \qquad f_{34} = -f_{43} = -iE_z$$

and if we substitute: j=1 k = 2, m =3, h = 4 in (2.26), we have:

$$f_{12}f_{34} + f_{23}f_{14} + f_{31}f_{24} = 0$$
$$H_z(-iE_z) + H_x(-iE_x) + H_y\big(-iE_y\big) = 0$$

Which is obviously:

$$E \bullet H = 0 \qquad \text{(2.27)}$$

Other products cancel each other, and Einstein demand (2.24) means that the Scalar product of E and H is zero.

III. Maxwell equations

From (2.23) and (2.24),we have:

$$\Omega^i_{jk|l} + \Omega^i_{kl|j} + \Omega^i_{lj|k} = 0 \qquad \text{(3.1)}$$

Multiplication by u_i and remembering that $\nabla_j u_i = 0$ we have:

$$f_{jk|l} + f_{kl|j} + f_{lj|k} = 0 \qquad \text{(3.2)}$$

Using the definition of the covariant derivative with respect to L^i_{jk} and (2.26), we have:

$$\frac{\partial f_{jk}}{\partial x^l} + \frac{\partial f_{kl}}{\partial x^j} + \frac{\partial f_{lj}}{\partial x^k} = 0 \qquad \text{(3.3)}$$

Which correspond to:

$$\nabla \times \boldsymbol{E} = -\frac{1}{c}\frac{\boldsymbol{\partial H}}{\boldsymbol{\partial t}} \qquad \text{(3.4)}$$

$$\nabla \bullet \boldsymbol{H} = \boldsymbol{0} \qquad \text{(3.5)}$$

Now from (2.17) and (3.1), we have:

$$\Omega^i_{jkl} = \Omega^i{}_{kl|j} - \Omega^i_{jh}\Omega^h_{kl} \qquad \text{(3.6)}$$

And from parallelism condition:

$$R^i_{jkl} + \Omega^i_{jkl} = 0 \qquad \text{(3.7)}$$

And (3.6) we have:

$$R^i_{jkl} + \Omega^i_{kl|j} - \Omega^i_{jh}\Omega^h_{kl} = 0 \qquad \text{(3.8)}$$

Multiplication by u_i gives:

$$f_{kl|j} - \Omega^h_{kl} f_{jh} + \frac{mc^2}{e} u_i R^i_{jkl} = 0 \qquad \text{(3.9)}$$

Explicitly:

$$\frac{\partial f_{kl}}{\partial x^j} - \Gamma^h_{kj} f_{hl} - \Omega^h_{kj} f_{hl} - \Gamma^h_{lj} f_{kh} - \Omega^h_{lj} f_{kh} - \Omega^h_{kl} f_{jh} + \frac{mc^2}{e} u_i R^i_{jkl} = 0 \qquad \text{(3.10)*}$$

If in (3.10) we put $l = j$, we have [5], [6]:

$$\frac{\partial f_{kj}}{\partial x^j} = \Gamma^h_{kj} f_{hl} + \Gamma^h_{jj} f_{kh} + \frac{mc^2}{e} u_i R^i_{jkj} \qquad \text{(3.11)}$$

That is, the divergences of the electromagnetic field tensor are no more equal to zero. The first pair of Maxwell equations are modified.

(3.11) becomes:

$$\frac{\partial f_{kj}}{\partial x^j} = 0, \text{ if an only if } \Gamma^i_{jk} = 0$$

*Unfortunately, in a previous article we omitted accidentally the term $\Omega^h_{kl} f_{jh}$.

IV. Conclusion

The affine connection $L^i_{jk} = \Gamma^i_{jk} + \Omega^i_{jk}$ unify gravitation and electromagnetism.

Equation (2.3) is a fundamental equation of motion for gravitation and electromagnetic fields.

Identity (2.19) is equivalent to Jacobi identity both gives (2.23).
It is surprising that Einstein claim is nothing other than the perpendicularity of the electric and magnetic fields.
From (3.8) if the electromagnetic tensor f_{ij} is a parallel field$\left(\nabla_k f_{ij} = 0\right)$,then$R^i_{jkl} = \Omega^i_{jh}\Omega^h_{kl}$, or$R^i_{jkl} = \left(\frac{e}{mc^2}\right)^2 u^i u^h f_{jh} f_{kl}$, which means **that the curvature of the space is generated by the electromagnetic field. Or that Gravitation is generated by the Electromagnetic field.**

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Alcantara Montes Samuel, et. al. “THE UNITARY VECTOR FIELDu^i (x(s)) AS A PARALLEL TRANSPORTED FIELD.” *IOSR Journal of Applied Physics (IOSR-JAP)*, 14(5), 2022, pp. 48-51.