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Quantum Teleportation of Three-Qubits Entangled States Via Ghz-Like States

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Abstract:
Background*: A three-qubit entangled quantum teleportation protocol via GHZ-like state channel with a description of its combined state, measurement of Alice's projection and Bob's unitary transformation in two stages, calculation of the total probability of success and its fidelity*.
Materials and Methods*: Teleportation protocols over GHZ-like three-qubit entanglement state channels have been widely proposed. Likewise, the use of GHZ-like state channels in the implementation of quantum communication protocols. From many quantum teleportation protocols over a three-qubit GHZ-like state channel that has been proposed. It is observed that to transmit an arbitrary n-qubit unentangled state need n-channels of GHZ-like states*
Results*: Quantum teleportation protocol to transmit an unentangled three-qubit quantum state over a GHZ-Like three-qubit state channel. Alice's projection measurements and Bob's unitary transformation involve Bob's additional particles. The probability and fidelity value of this protocol reaches one*
Key Word*: fidelity, projection measurement, quantum teleportation*.

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I. Introduction

Many research developments of the quantum states transmission have been carried out. In the field of quantum cryptography, Ekert [1] uses the two-qubit state of the EPR (Einstein, Podolsky, Rosen) as a state of the tightness tester and in the communication protocol of Bennet [2], this EPR is shared via one-and two-particle operators. In 1993 Bennet et al. [3] for the first time proposed a theoretical protocol for one-qubit state quantum teleportation via the EPR channel. Where quantum teleportation is the process of sending an arbitrary number of unknown a qubit-state to a separate place between the sender (Alice) and the recipient (Bob) through the splitting of the quantum entanglement state and the classical state by involving many non-local measurements. In general, non-local measurements in Alice use projective measurements, and in Bob are unitary operations. There are also protocols whose non-local measurements were implemented via the methods of Aharanov and Albert [4], non-linear interactions in the experiments by Kim et al [5] , and in the work of Cardoso et al [6] which used resonance between the state source cavity and the channel source. The selection of quantum channels for arbitrary two-qubit entanglement states is obtained through the Schmidt decomposition test [7] and in multiqubit is through the composition of the reduced density matrix rank values [8].

Proposed quantum teleportation protocols over GHZ-like three-qubit entanglement state channels have been widely proposed [9-12]. Likewise, the use of GHZ-like state channels in the implementation of quantum communication protocols [11][13,14][15]. From many quantum teleportation protocols over a three-qubit GHZ-like state channel that has been proposed [9][16], it is observed that to transmit an arbitrary n-qubit unentangled state need n-channels of GHZ-like states. In the work of Tsai et al [10] and Nandhi et al [17], it was observed that the transmission of a two-qubit state in a special form requires only one three-qubit GHZ-like state.

In this study, we propose a quantum teleportation protocol scheme for transmitting a quantum state, i.e., a three-qubit entangled state via a three-qubit GHZ-like state. This teleportation scheme is supported by Bob's additional particle [18] and Alice's projective measurement process and Bob's unitary transformation are carried out in two stages. This protocol was minimized a quantum channel as much as one three-qubit GHZ-like state compared to Wen Yuan's work [19] and one-qubit to more states than Tsai et al. [10] and Nandhi et al. [17].

Furthermore, the structure of this article in Section 2 is to describe a three-qubit entangled quantum teleportation protocol via GHZ-like state channel with a description of its combined state, measurement of Alice's projection and Bob's unitary transformation in two stages, calculation of the total probability of success and its fidelity. In the last section or section 3 is the conclusion of our work.

II. Quantum Teleportation of Three- qubit Entangled State via the GHZ-Like

In this protocol scheme, all the measurement steps are projections on Alice's particle (A, 1, 2, B, 4, 5, C) and the unitary transformation on Bob's particle (3,6,7) in transmitting a three-qubit entangled state,

$$|\varphi\rangle_{ABC} = (a|001\rangle + b|010\rangle + c|100\rangle + d|111\rangle)_{ABC} \tag{1}$$

with $|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$.
Consider the two-GHZ-like state quantum channel,

$$|\psi\rangle_{123} = \frac{1}{2}(|000\rangle + |011\rangle + |101\rangle + |110\rangle)_{123} \tag{2}$$

$$|\psi\rangle_{456} = \frac{1}{2}(|000\rangle + |011\rangle + |101\rangle + |110\rangle)_{456} \tag{3}$$

and the following Alice's measurement bases

$$|\eta^{\pm}\rangle_{ijk} = \frac{1}{2}[(|000\rangle + |011\rangle) \pm (|101\rangle + |110\rangle)]_{ijk} \tag{4}$$

$$|\xi^{\pm}\rangle_{ijk} = \frac{1}{2}[(|100\rangle + |111\rangle) \pm (|001\rangle + |010\rangle)]_{ijk} \tag{5}$$

From equations (1) untill (5), and additional Bob's particle $|0\rangle_7$we will construct a joint state

$$\begin{aligned}
|\Gamma\rangle_{ABC1234567} &= |\varphi\rangle_{ABC}\otimes|\psi\rangle_{123}\otimes|\psi\rangle_{456}\otimes|0\rangle_7 \\
&= \frac{1}{4}|\eta^+\rangle_{A12}|\eta^+\rangle_{B45}\{a|0001\rangle + b|0100\rangle + c|1000\rangle + d|1101\rangle\}_{367C} \\
&+\frac{1}{4}|\eta^+\rangle_{A12}|\eta^-\rangle_{B45}\{a|0001\rangle - b|0100\rangle + c|1000\rangle - d|1101\rangle\}_{367C} \\
&+\frac{1}{4}|\eta^-\rangle_{A12}|\eta^+\rangle_{B45}\{a|0001\rangle + b|0100\rangle - c|1000\rangle - d|1101\rangle\}_{367C} \\
&+\frac{1}{4}|\eta^-\rangle_{A12}|\eta^-\rangle_{B45}\{a|0001\rangle - b|0100\rangle - c|1000\rangle + d|1101\rangle\}_{367C} \\
&+\frac{1}{4}|\eta^+\rangle_{A12}|\xi^+\rangle_{B45}\{a|0101\rangle + b|0000\rangle + c|1100\rangle + d|1001\rangle\}_{367C} \\
&+\frac{1}{4}|\eta^+\rangle_{A12}|\xi^-\rangle_{B45}\{-a|0101\rangle + b|0000\rangle - c|1100\rangle + d|1001\rangle\}_{367C} \\
&+\frac{1}{4}|\eta^-\rangle_{A12}|\xi^+\rangle_{B45}\{a|0101\rangle + b|0000\rangle - c|1100\rangle - d|1001\rangle\}_{367C} \\
&+\frac{1}{4}|\eta^-\rangle_{A12}|\xi^-\rangle_{B45}\{-a|0101\rangle + b|0000\rangle + c|1100\rangle - d|1001\rangle\}_{367C} \\
&+\frac{1}{4}|\xi^+\rangle_{A12}|\eta^+\rangle_{B45}\{a|1001\rangle + b|1100\rangle + c|0000\rangle + d|0101\rangle\}_{367C} \\
&+\frac{1}{4}|\xi^+\rangle_{A12}|\eta^-\rangle_{B45}\{a|1001\rangle - b|1100\rangle + c|0000\rangle - d|0101\rangle\}_{367C} \\
&+\frac{1}{4}|\xi^-\rangle_{A12}|\eta^+\rangle_{B45}\{-a|1001\rangle - b|1100\rangle + c|0000\rangle + d|0101\rangle\}_{367C} \\
&+\frac{1}{4}|\xi^-\rangle_{A12}|\eta^-\rangle_{B45}\{-a|1001\rangle + b|1100\rangle + c|0000\rangle - d|0101\rangle\}_{367C} \\
&+\frac{1}{4}|\xi^+\rangle_{A12}|\xi^+\rangle_{B45}\{a|1101\rangle + b|1000\rangle + c|0100\rangle + d|0001\rangle\}_{367C} \\
&+\frac{1}{4}|\xi^+\rangle_{A12}|\xi^-\rangle_{B45}\{-a|1101\rangle + b|1000\rangle - c|0100\rangle + d|0001\rangle\}_{367C} \\
&+\frac{1}{4}|\xi^-\rangle_{A12}|\xi^+\rangle_{B45}\{-a|1101\rangle - b|1000\rangle + c|0100\rangle + d|0001\rangle\}_{367C} \\
&+\frac{1}{4}|\xi^-\rangle_{A12}|\xi^-\rangle_{B45}\{a|1101\rangle - b|1000\rangle - c|0100\rangle + d|0001\rangle\}_{367C}.\#
\end{aligned} \tag{6}$$

Furthermore, the Alice's measurement results were obained

$$
\begin{aligned}
\left|\varphi_0^{1,2,3,4}\right\rangle_{367} &= \frac{1}{4}\left(+^1-^2+^3-^4 b|010\rangle+^1+^2-^3-^4 c|100\rangle\right)_{367},\\
\left|\varphi_1^{1,2,3,4}\right\rangle_{367} &= \frac{1}{4}\left(+^1+^2+^3+^4 a|001\rangle+^1-^2-^3+^4 d|111\rangle\right)_{367},\\
\left|\varphi^{1,2,3,4}\right\rangle_{367} &= \left|\varphi_0^{1,2,3,4}\right\rangle_{367}+\left|\varphi_1^{1,2,3,4}\right\rangle_{367}\\
&= \frac{1}{4}\left(+^1+^2+^3+^4 a|001\rangle+^1-^2+^3-^4 b|010\rangle\right.\\
&\quad \left.+^1+^2-^3-^4 c|100\rangle+^1-^2-^3+^4 d|111\rangle\right)_{367},\\
\left|\varphi_0^{5,6,7,8}\right\rangle_{367} &= \frac{1}{4}\left(+^5+^6+^7+^8 b|000\rangle+^5-^6-^7+^8 c|110\rangle\right)_{367},\\
\left|\varphi_1^{5,6,7,8}\right\rangle_{367} &= \frac{1}{4}\left(+^5-^6+^7-^8 a|011\rangle+^5+^6-^7-^8 d|101\rangle\right)_{367},\\
\left|\varphi^{5,6,7,8}\right\rangle_{367} &= \left|\varphi_0^{5,6,7,8}\right\rangle_{367}+\left|\varphi_1^{5,6,7,8}\right\rangle_{367}\\
&= \frac{1}{4}\left(+^5-^6+^7-^8 a|011\rangle+^5+^6+^7+^8 b|000\rangle\right.\\
&\quad \left.+^5-^6-^7+^8 c|110\rangle+^5+^6-^7-^8 d|101\rangle\right)_{367},\\
\left|\varphi_0^{9,10,11,12}\right\rangle_{367} &= \frac{1}{4}\left(+^9-^{10}-^{11}+^{12} b|110\rangle+^9+^{10}+^{11}+^{12} c|000\rangle\right)_{367},\\
\left|\varphi_1^{9,10,11,12}\right\rangle_{367} &= \frac{1}{4}\left(+^9+^{10}-^{11}-^{12} a|101\rangle+^9-^{10}+^{11}-^{12} d|011\rangle\right)_{367},\\
\left|\varphi^{9,10,11,12}\right\rangle_{367} &= \left|\varphi_0^{9,10,11,12}\right\rangle_{367}+\left|\varphi_1^{9,10,11,12}\right\rangle_{367}\\
&= \frac{1}{4}\left(+^9+^{10}-^{11}-^{12} a|101\rangle+^9-^{10}-^{11}+^{12} b|110\rangle\right.\\
&\quad \left.+^9+^{10}+^{11}+^{12} c|000\rangle+^9-^{10}+^{11}-^{12} d|011\rangle\right)_{367},\\
\left|\varphi_0^{13,14,15,16}\right\rangle_{367} &= \frac{1}{4}\left(+^{13}+^{14}-^{15}-^{16} b|100\rangle+^{13}-^{14}+^{15}-^{16} c|010\rangle\right)_{367},\\
\left|\varphi_1^{13,14,15,16}\right\rangle_{367} &= \frac{1}{4}\left(+^{13}-^{14}-^{15}+^{16} a|111\rangle+^{13}+^{14}+^{15}+^{16} d|001\rangle\right)_{367},\\
\left|\varphi^{13,14,15,16}\right\rangle_{367} &= \left|\varphi_0^{13,14,15,16}\right\rangle_{367}+\left|\varphi_1^{13,14,15,16}\right\rangle_{367}\\
&= \frac{1}{4}\left(+^{13}-^{14}-^{15}+^{16} a|111\rangle+^{13}+^{14}-^{15}-^{16} b|100\rangle\right.\\
&\quad \left.+^{13}-^{14}+^{15}-^{16} c|010\rangle+^{13}+^{14}+^{15}+^{16} d|001\rangle\right)_{367}.
\end{aligned}
\tag{7}
$$

Bob performs a unitary operation using the following operators,

$$
\begin{aligned}
V^{1,2,3,4} &= +^1+^2+^3+^4|001\rangle\langle001|+^1-^2+^3-^4|010\rangle\langle010|\\
&\quad +^2-^3-^4|100\rangle\langle100|+^1-^2-^3+^4|111\rangle\langle111|\\
&\quad +^1+^2+^3+^4|000\rangle\langle000|+^1+^2+^3+^4|011\rangle\langle011|\\
&\quad +^1+^2+^3+^4|101\rangle\langle101|+^1+^2+^3+^4|110\rangle\langle110|,\\
V^{5,6,7,8} &= +^5-^6+^7-^8|001\rangle\langle011|+^5+^6+^7+^8|010\rangle\langle000|\\
&\quad +^5-^6-^7+^8|100\rangle\langle110|+^5+^6-^7-^8|111\rangle\langle101|\\
&\quad +^5+^6+^7+^8|000\rangle\langle001|+^5+^6+^7+^8|011\rangle\langle010|\\
&\quad +^5+^6+^7+^8|101\rangle\langle100|+^5+^6+^7+^8|110\rangle\langle111|,\\
V^{9,10,11,12} &= +^9+^{10}-^{11}-^{12}|001\rangle\langle101|+^9+^{10}+^{11}+^{12}|010\rangle\langle110|\\
&\quad +^9+^{10}+^{11}+^{12}|100\rangle\langle000|+^9-^{10}+^{11}-^{12}|111\rangle\langle011|\\
&\quad +^9+^{10}+^{11}+^{12}|000\rangle\langle001|+^9+^{10}+^{11}+^{12}|011\rangle\langle010|\\
&\quad +^9+^{10}+^{11}+^{12}|101\rangle\langle100|+^9+^{10}+^{11}+^{12}|110\rangle\langle111|,\\
V^{13,14,15,16} &= +^{13}-^{14}-^{15}+^{16}|001\rangle\langle111|+^{13}+^{14}-^{15}-^{16}|010\rangle\langle100|\\
&\quad +^{13}-^{14}+^{15}-^{16}|100\rangle\langle010|+^{13}+^{14}+^{15}+^{16}|111\rangle\langle001|\\
&\quad +^{13}+^{14}+^{15}+^{16}|000\rangle\langle000|+^{13}+^{14}+^{15}+^{16}|011\rangle\langle011|
\end{aligned}
$$

$$+^{13}+^{14}+^{15}+^{16}|101\rangle\langle 101|+^{13}+^{14}+^{15}+^{16}|110\rangle\langle 110|. \quad (8)$$

The complete results of Alice's and Bob's measurements are presented in Table. 1

Table. 1. Alice's and Bob's Measurement Results

Alice's Measurement Results	**Bob's Unitary Transformation** $(i = 0,1)$, $(j = 1,2,\dots,16)$		
	$\lvert\varphi_i^j\rangle_{367}$	$\lvert\varphi^j\rangle_{367}$	V^j
$\lvert\eta^{+^1+^2-^3-^4}\rangle_{A12}\lvert\eta^{+^1-^2+^3-^4}\rangle_{B45}\lvert 0\rangle_C$	$\lvert\varphi_0^{1,2,3,4}\rangle_{367}$	$\lvert\varphi^{1,2,3,4}\rangle_{367}$	$V^{1,2,3,4}$
$\lvert\eta^{+^1+^2-^3-^4}\rangle_{A12}\lvert\eta^{+^1-^2+^3-^4}\rangle_{B45}\lvert 1\rangle_C$	$\lvert\varphi_1^{1,2,3,4}\rangle_{367}$		
$\lvert\eta^{+^5+^6-^7-^8}\rangle_{A12}\lvert\xi^{+^5-^6+^7-^8}\rangle_{B45}\lvert 0\rangle_C$	$\lvert\varphi_0^{5,6,7,8}\rangle_{367}$	$\lvert\varphi^{5,6,7,8}\rangle_{367}$	$V^{5,6,7,8}$
$\lvert\eta^{+^5+^6-^7-^8}\rangle_{A12}\lvert\xi^{+^5-^6+^7-^8}\rangle_{B45}\lvert 1\rangle_C$	$\lvert\varphi_1^{5,6,7,8}\rangle_{367}$		
$\lvert\xi^{+^9+^{10}-^{11}-^{12}}\rangle_{A12}\lvert\eta^{+^9-^{10}+^{11}-^{12}}\rangle_{B45}\lvert 0\rangle_C$	$\lvert\varphi_0^{9,10,11,12}\rangle_{367}$	$\lvert\varphi^{9,10,11,12}\rangle_{367}$	$V^{9,10,11,12}$
$\lvert\xi^{+^9+^{10}-^{11}-^{12}}\rangle_{A12}\lvert\eta^{+^9-^{10}+^{11}-^{12}}\rangle_{B45}\lvert 1\rangle_C$	$\lvert\varphi_1^{9,10,11,12}\rangle_{367}$		
$\lvert\xi^{+^{13}+^{14}-^{15}-^{16}}\rangle_{A12}\lvert\xi^{+^{13}-^{14}+^{15}-^{16}}\rangle_{B45}\lvert 0\rangle_C$	$\lvert\varphi_0^{13,14,15,16}\rangle_{367}$	$\lvert\varphi^{13,14,15,16}\rangle_{367}$	$V^{13,14,15,16}$
$\lvert\xi^{+^{13}+^{14}-^{15}-^{16}}\rangle_{A12}\lvert\xi^{+^{13}-^{14}+^{15}-^{16}}\rangle_{B45}\lvert 1\rangle_C$	$\lvert\varphi_1^{13,14,15,16}\rangle_{367}$		

The calculation of the probability of success in sending information from Alice to Bob is$P_{Succses} = (1/16) \times 16 = 1$.Also, the fidelity of this protocol is$\mathcal{F}_{Total}\left(\rho_{in}, \rho_{(out)}\right) = (1/16) \times 16 = 1$.

III. Conclusion

We have demonstrated that GHZ-like states can be used as a quantum channel to realize quantum state transmission in the form of an entangled three-qubit state. The main result of this protocol is that the total probability of success and the channel fidelity to transmit this quantum state are respectively one. We hope that these two protocols can be realized in experiments that use photons as the source.

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