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Scattering, Extinction and Absorption Characteristics of Electromagnetic Waves by a Spherical Particle.

Kurawa, S.M[1],Bunkure Y.I[2], Kashimbila, M.M[3], Shafie, S[4], Ali, M.H[5]

1. Department of Physics, Sa'adatu Rimi College of Education Kumbotso, Kano, Nigeria.
2. Department of Science and Technology Education, Bayero University Kano, Kano, Nigeria.
3. Department of Physics, Bayero University Kano, Kano, Nigeria.
4. Department of Electrical & Electronic Engineering, Faculty of Engineering, Universiti Putra Malaysia
5. Department of Physics, Bayero University Kano, Kano, Nigeria.
Corresponding author: Sabuwa Mustapha Kurawa
Department of Physics, Sa'adatu Rimi College of Education Kumbotso, Kano-Nigeria.

The scattering of electromagnetic wave by a spherical particle was first presented using the Mie formulation. The parameters needed to describe the characteristics of the radiation scattered by the particles were derived. The parameters include the Mie coefficient an and bn, the scattering, absorption and extinction efficiencies, the cross section for radiation pressure and the asymmetry factor <cos >. The behaviour of these parameters was investigated as functions of the dimensionless size of the particle . Experimental value was obtained from radiation pressure measurements on a levitated oil droplet with refractive index of m= 1.29. It was found that the various efficiency factors are related to the optical property of the scattering. For pure dielectric, the efficiency factors for scattering and extinction are equal, since there was no absorption. Superimposed on the extinction curves are the minor oscillations called ripple structures. When the particles were absorbing, the refractive index became complex, it was found that the amplitude of the extinction curve decreased, and the ripple structures gradually disappeared. Similarly, the distance between resonances, also called the period of resonances was found to be related to the refractive index of the particle. Using the Mie theory, the period calculated was 0.750, which was in good agreement with the experimentally quoted value of 0.671.

Keywords: *Scattering, Absorption, Extinction, Mie coefficient*

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I. Introduction

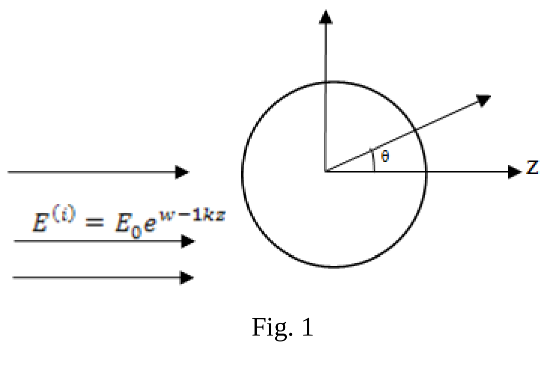

Fig. 1

A particle is irradiated with a beam of linearly plane polarized monochromatic electromagnetic wave. At point 0 there is a particle of radius (a) and of refractive index (m) situated at a distance (r). The particle is assumed to be non-magnetic and isotropic, and the wave is assumed to be of unit amplitude (E_0=1). To evaluate the total power scattered at point p, through an angle θ is done as follows.

The total power scattered at point p is given by the integral of the pointing vector over the surface area.

$$\frac{P=\int(s^{(s)})dA}{(S^{(i)})}=C_{sca.}$$

This is known as the scattering cross-section ($C_{sca.}$). Similarly the same for C_{abs} and $C_{ext.}$

Scattering cross-section C_{sca} and Absorption cross-section C_{abs}

$$C_{sca} = \frac{\iint \langle S^{(s)} \rangle dA}{\langle S^{(i)} \rangle} \qquad C_{abs} = \frac{\iint \langle S^{(s)} \rangle dA}{\langle S^{(i)} \rangle}$$

Extinction cross-section
$C_{ext} = C_{sca} + C_{abs}$

Efficiency Factors for Scattering, Absorption and Extinction
The respective efficiency factors are also given by the expressions:

Scattering efficiency factor $\quad Q_{sca} = \frac{C_{sca}}{\pi a^2}$

Absorption efficiency factor $\quad Q_{abs} = \frac{C_{abs}}{\pi a^2}$

Extinction efficiency factor $\quad Q_{ext} = \frac{C_{ext}}{\pi a^2}$

The Mie Scattering Theory
Solutions to the wave equation

$$\nabla^2 E - \varepsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t} = 0 \qquad (1)$$

Are obtained by by applying the boundary conditions

$$\hat{n} x \vec{E}_1 - \vec{E}_2 = 0$$

$$\hat{n} x \vec{H}_1 - \vec{H}_2 = 0 \qquad (2)$$

If the fields $\vec{E}$ and $\vec{H}$ are oscillatory of the form $e^{i\omega t}$, equation (1) can be written as

$$\nabla^2 \vec{E} + \tilde{K}^2 \vec{E} = 0 \qquad (3)$$

Where $\tilde{K}^2 = \varepsilon \mu \omega^2 + i \sigma \mu \omega$

Solutions to equation (3) can be obtained by solving the associated scalar equation

$$\nabla^2 \psi(r, \theta, \varphi) + K^2 \psi(r, \theta, \varphi) = 0 \qquad (4)$$

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \psi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left(sin\theta \frac{\partial \psi}{\partial \theta} \right) + \frac{1}{r^2 sin\theta} \frac{\partial^2 \psi}{\partial \psi^2} + k^2 \psi = 0 \qquad (5)$$

With $\Psi(r,\theta,\varphi) = R(r)\, \Theta(\theta)\, \Phi(\varphi)$, equation (5) separates into the ordinary differential equations

$$\frac{d^2(rR)}{dr^2} + \left(k^2 - \frac{a}{r^2} \right) rR = 0 \qquad (6)$$

$$\frac{1}{sin\theta} \frac{\partial}{\partial \theta} \left(sin\theta \frac{\partial \Theta}{\partial \Theta} \right) + \left[n(n+1) - \frac{m^2}{sin^2 \theta} = 0 \right] \qquad (7)$$

$$\frac{d^2\Phi}{d\Phi^2} + m^2\Phi = 0 \qquad (8)$$

Equation (6) is solved by

$$z_n(kr) = \sqrt{\frac{\pi}{2kr}}\, Z_n + \frac{1}{2}$$

(7) by

$$\Theta(\theta) = P_n^m(cos\theta)$$

And (8) by

$$\Phi(\varphi) = a_m \cos m\varphi + b_m \sin(m\varphi)$$

The solution of the scalar wave equation for a homogenous medium is therefore

$$\psi(r,\theta,\varphi) =$$

$$\sum_{n=1}^{\infty} Z_n(kr)[a_{n0}P_n(cos\theta) + \sum_{m=1}^{n}(a_{nm}\cos m\varphi + b_{mn}\sin m\varphi)\, P_n^m(cos\theta)]$$

(9)

Stratton showed that the vector wave equation (3) is satisfied by

$$\vec{M} = \vec{\nabla} \text{ X } \vec{r}\psi \,, mk\vec{N} = \vec{\nabla} \text{ X } \vec{M} \qquad (10)$$

Where $\vec{M}$ *and* $\vec{N}$ represent transverse waves, and the electric and magnetic vectors cam be constructed as series of these functions of the type

$$\vec{E} = \sum_n P_n \overrightarrow{M_n} + q_n\vec{N}_n$$

$$\vec{E} = \sum_n q_n\overrightarrow{M_n} + \overrightarrow{P_n}\ \overrightarrow{N_n}$$

Where

$$\overrightarrow{M_{n0}} = \pm\frac{1}{sin\theta} Z_n(kr)P_n^1(cos\theta)_{cos}^{sin}\hat{\theta} - Z_n(kr)\frac{dp_{n1}^1(cos\theta)}{d\theta}\cdot\frac{sin\varphi}{cos\varphi}\hat{\varphi}$$

$$\overrightarrow{N_{n0}} = \pm\frac{n(n+1)}{mkr} Z_n(kr)P_n^1(cos\theta)_{sin\varphi}^{cos\varphi}\,\hat{r} + \frac{1}{mkr}\frac{d(rz_n(kr)dp_n^1 cos\theta)}{d\theta}\cdot_{cos\varphi}^{sin\varphi}\,\hat{\theta}$$

$$\pm\frac{1}{mkr\sin\theta}\frac{d}{dr}(rz_n(kr))P_n^1(\cos\theta)_{cos\varphi}^{sin\varphi}\,\hat{\Phi}$$

The expansion of the incident wave in terms of these function is

$$\vec{E}_{\ inc} = \sum_{n-1}^{\infty} i^n\frac{2n+1}{n(n+1)}\left(M_{0n}^i - iN_{en}^{(i)}\right)$$

The superscript (i) implies that Z_n (kr) = j_n (kr)

The transmitted wave is similarly expanded:

$$\vec{E}_t = \sum_{n-1}^{\infty} i^n \frac{2n+1}{n(n+1)} \left(c_n M_{0n}^i - i d_n N_{en}^{(i)} \right)$$

In this case Z_n (kr) = j_n(mkr).
For the scattered wave

$$\overrightarrow{E_{sca}} = \sum_{n-1}^{\infty} i^n \frac{2n+1}{n(n+1)} \left(a_n M_{0n}^i - i b_n N_{en}^{(s)} \right)$$

Where Z_n (kr) $= h_n^{(1)}$ (kr) $\approx \frac{e^{ikr}}{r}$

The efficiency factors

The efficiency factor for scattering Q_{sca} is

$$Q_{sca} = \frac{1}{2\pi a^2} \int_0^{2\pi} \int_0^{2\pi} Re\left(E_\theta^{(s)} H_\varphi^{*(s)} - E_\varphi^{(s)} H_\theta^{*(s)} \right) r^2 \sin\theta d\theta d\varphi$$

$$= \frac{1}{k^2 a^2} \int_0^{\pi} RE\left[S_2^*(\theta) cos^2\theta + S_1(\theta) S_1^* Sin^2\theta \right] \quad (11)$$

Where $E_\theta^s = H_\varphi^s = \frac{-i}{kr} e^{-ikr+i\omega t} Cos\varphi\, S_2(\theta)$

$$-E_{s\varphi} = -H_\theta^s = \frac{-i}{kr} e^{-ikr+i\omega t} Sin\,\varphi\, S_1(\theta)$$

And

$$S_1(\theta) = \sum_n \frac{2n+1}{n(n+1)} \left[a_n \pi_n(\cos\theta) + b_n \tau_n(\cos\theta) \right]$$

$$S_2(\theta) = \sum_n \frac{2n+1}{n(n+1)} \left[b_n \pi_n(\cos\theta) + a_n \tau_n(\cos\theta) \right]$$

With

$$\pi_n(\cos\theta) = \frac{1}{\sin\theta} P_n^1(\cos\theta), \qquad \tau_n(\cos\theta) = \frac{d}{d\theta}\left[P_n^1(\cos\theta) \right]$$

Evaluation of equation (11) gives

$$Q_{sca} = \frac{2}{x^2} \sum_n (2n+1)\left[|a_n|^2 + |b_n|^2 \right]$$

Where $x = \frac{2\pi a}{\lambda}$

Using the extinction theorem, the extinction efficiency is

$$Q_{ext} = \frac{4}{x^2} Re[S(0)]$$

$$Q_{ext} = \frac{4}{x^2} \sum_n (2n+1) Re(a_n + b_n)$$

INTERPRETATION

The curve for the scattering efficiency factors are shown in figure 1. The upper curve corresponds to the case when there is no absorption, in this case m=1.29 (2, 2, 2-trifluroethanol). Here, the major oscillations and ripple structures are apparent. The second lower curve corresponds to the case when m =1.29 + 0.01i, the lowest corresponds to m=1.29 + 0.1i. These two curves show the effects of absorption, which is damping of the amplitude of the oscillations and the disappearance of the ripple structures. As the size parameter x increases, the curve with the highest absorptivity tends to a limiting value of 1 faster than the rest with complete disappearance of the ripple structures.

The curves for the absorption efficiencies are shown in figure 2. The characteristic behavior of the absorption with efficiency with x is studied for different values of the refractive index m. The imaginary part of x is varied while the real part is kept constant. As we can see from the graphs, as the imaginary part of m is increased, the absorption efficiency also increases. Generally, for any particle $m = m_1 + im_2$. The sign of m_2 can be reversed, Kerker (2017) explained tahat the scattering efficiency factor Q_{sca} is always greater or equal to zero. But Q_{sca} may be negative which implies amplification. The extinction efficiency can have either sign and may be zero. When it is zero it means that the scattering particles radiate an extra amount of energy equal to that which is lost by scattering.

Figure 3 shows the extinction efficiency as a function of particle size x for a refractive index m=1.29. The major oscillations with the ripple structures are apparent,

Fig 4 shows the extinction curve when m is made complex, that is $m = 1.29 + 0.01i$. The effect of this is the decrease in the amplitude of oscillations and the disappearance of ripple structures. When the absorption is increased further as in figure 5, the ripples disappear completely and the extinction curve tends to a limiting value of 2 as the size parameter increases. This shows that from each curve the efficiency is higher for smaller particles.

Figure 6 is obtained by further increase in absorption. It possesses an interference and resonance structures, the slow oscillation of the curve is called interference structure while the sharp resonance peak superimposed upon the smoothly varying curve is known as the ripple structure. It can be seen that the resonance separation (Δx) is found to be 0.8182 from the graph obtained using the Mie formulation and expression by Chylek (2007) while the one obtained analytically is 0.839. The values agree to each other to about 97%.

APPLICATIONS

The scattering of electromagnetic wave by particles is an important problem from the practical point of view. Scattering problems are used in the field of astronomy and meteorology.

In the planetry atmosphere, scattering and extinction measurement can be used to investigate the information about the sizes, concentrations and shapes or chemical compositions of dust particles which are not readily accessible.

Refractive index of materials is also important in scattering problems. The sensitive dependence of sharp ripple resonance features on the size parameter and refractive index render them very suitable as probes for high accuracy determinations of refractive index of a material.

The application of the idea of optical levitation and trapping of small particles with a diameter of several micrometers has been investigated in the micro manipulation of biological cells (Busican 2014). The optical levitation experiments are a modern version of Milikan's drop experiment in which the droplets are supported by the radiation pressure from a laser beam than by static electric field. (Nussenzeig,2000).

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