

Wu's Spacetime Equation and Wu's Spacetime Constant

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[Abstract]

Wu's Spacetime Equation $t_{yy} = \gamma l_{yy}$ is derived between Wu's Unit Length l_{yy} (diameter) and Wu's Unit Time t_{yy} (period) of Wu's Pairs, a pair of superfine antimatter circulating Yangton and Yington particles as the building blocks of the universe. Both Wu's Spacetime constant $\gamma = \pi (2K)^{-1/2}$ and Wu's constant $K = V^2 r = \frac{1}{2} k (q_{yy}^2 / m_{yy})$ (where V is the circulation speed, $r = \frac{1}{2} l_{yy}$, k is Coulomb's Constant, q_{yy} is the charge of Yangton or Yington particle, and m_{yy} is the mass of Wu's Pair) are physical constants composed of a real number and a group of unit quantities with arithmetic operations. Like any other physical constants, they are dependent on gravitational field and aging of the universe. Furthermore, according to Principle of Parallelism, physical properties such as dimension, duration, velocity and acceleration of an object or event can be represented by (transformed to) both Wu's Spacetime constant γ and Wu's Unit Length l_{yy} (of a reference corresponding identical subatomic particle) which are dependent on gravitational field and aging of the universe. It can be used to explain many important physical phenomena including Gravitational Redshift, Cosmological Redshift, General Relativity, Gravitational Time Dilation, Hubble's Law, Spacetime Reverse Expansion (Universe Expansion), Deflection of Light, Perihelion Precession of Mercury and Einstein's Field Equations, etc.

[Keywords]

Arithmetic Operations, Wu's Constant, Wu's Spacetime Equation, Wu's Spacetime Constant, Redshift, Gravitational Redshift, Cosmological Redshift, General Relativity, Gravitational Time Dilation, Hubble's Law, Universe Expansion, Deflection of Light, Perihelion Precession of Mercury, Einstein's Field Equations.

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I. Introduction

Wu's Spacetime Equation $t_{yy} = \gamma l_{yy}$ and Wu's Spacetime Constant γ have been used successfully in the interpretations of many important physical phenomena including Gravitational Redshift, Cosmological Redshift, General Relativity, Gravitational Time Dilation, Hubble's Law, Spacetime Reverse Expansion (Universe Expansion), Deflection of Light, Perihelion Precession of Mercury and Einstein's Field Equations, etc. Where γ like any other physical constant, is assumed as a real number "constant", doesn't change with anything at all.

However, in this paper, Wu's Spacetime Equation and Wu's Spacetime Constant are carefully studied and analyzed. Also γ containing unit quantities with arithmetic operations, is considered as a variable which is dependent on gravitational field and aging of the universe. As a result, instead of dependent only on l_{yy} as claimed in my previous publication [1], all the properties of an object or event such as dimension, duration, velocity and acceleration are actually dependent on both γ and l_{yy} . Accordingly, all the above physical phenomena are once again very well explained.

II. Yangton and Yington Theory

"Yangton and Yington Theory" [2] is a hypothetical theory based on a pair of super fine antimatter circulating Yangton and Yington particles with an inter-attractive Force of Creation forming a permanent circulating particle pair named "Wu's Pair" proposed as the fundamental building blocks of the universe. It explains the formation of all the subatomic particles and unified field theory in the universe as well as the correlations between space, time, energy and matter. It is basically a theory of everything.

III. Equation of Wu's Pairs and Wu's Constant

The circulation of Yangton and Yington Antimatter particles in Wu's Pairs can be illustrated as the revolution of Yangton and Yington particles in a circulation orbit around an axis. Fig. 1 is a schematic diagram of the circulation.

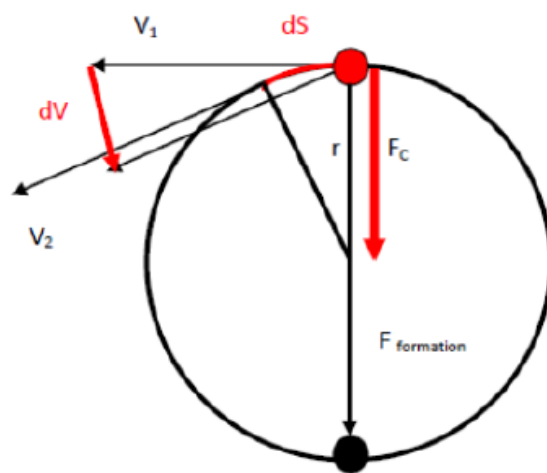


Fig. 1 Schematic diagram of a Wu's Pair.

Because of the circulation motion, the central acceleration (a_c) can be derived as follows:

$$a_c = dV/dt = (VdS/r)/dt = V(dS/dt)/r = V^2/r$$

And the central force can be represented as follows:

$$F_c = \frac{1}{2} m_{yy} a_c = \frac{1}{2} m_{yy} V^2/r$$

Where m_{yy} is the mass of a single Wu's Pair.

Also, because of Coulomb's Law of Electrical Force,

$$F_{\text{attraction}} = k q_{yy}^2 / (2r)^2$$

Where k is Coulomb's Constant and q_{yy} is the charge of either a Yangton particle or a Yington particle (opposite charges).

And

$$F_c = F_{\text{attraction}}$$

Therefore,

$$\frac{1}{2} m_{yy} V^2/r = k q_{yy}^2 / (2r)^2$$

$$V^2 r = \frac{1}{2} k (q_{yy}^2 / m_{yy})$$

Given

$$K = \frac{1}{2} k (q_{yy}^2 / m_{yy})$$

Therefore,

$$V^2 r = K$$

Where K is a constant (named Wu's Constant), V is the speed of circulation and r is the radius of the circulation orbit. This equation is named "Equation of Wu's Pairs" [1].

IV. Wu's Spacetime Equation and Wu's Spacetime Constant

Furthermore, the period (t_{yy}) and the diameter (l_{yy}) of the circulation orbit of Wu's Pairs (Fig. 1) are correlated to each other as follows:

Because

$$V^2 r = K$$

Where K is Wu's Constant, V is the speed of circulation and r is the radius of the circulation orbit.

$$T = 2\pi r/V$$

$$T^2 = 4\pi^2 r^2/V^2 = 4\pi^2 r^3/V^2 r = 4\pi^2 r^3/K$$

$$T = 2\pi K^{-1/2} r^{3/2} = \pi (2K)^{-1/2} d^{3/2}$$

Given

$$\gamma = \pi (2K)^{-1/2}$$

Because

$$T = t_{yy}$$

$$d = l_{yy}$$

Therefore,

$$t_{yy} = \gamma l_{yy}^{3/2}$$

Where K is Wu's Constant, t_{yy} is the circulation period of Wu's Pairs, named "Wu's Unit Time", l_{yy} is the diameter of the circulation orbit of Wu's Pairs, named "Wu's Unit Length", and γ is Wu's Spacetime Constant. This equation is named "Wu's Spacetime Equation" [1].

Wu's Spacetime Equation gives the correlation between Wu's Unit Time and Wu's Unit Length in a Wu's Pair. In addition, based on Principle of Parallelism, the quantities of the properties of an object or event such as dimension, duration, velocity and acceleration can be represented by (transformed to) Wu's Spacetime constant and Wu's Unit Length (of a reference corresponding identical subatomic particle) which are dependent on local gravitational field and aging of the universe. It can be used to explain many important physical phenomena including Gravitational Redshift [3], Cosmological Redshift [3], General Relativity [4], Gravitational Time Dilation [5], Hubble's Law [6], Spacetime Reverse Expansion (Universe Expansion)[7], Deflection of Light [8], Perihelion Precession of Mercury [8], and Einstein's Field Equations [9], etc.

V. Thermal Equilibrium and Subatomic Equilibrium

At "Thermal Equilibrium", all atoms and subatomic particles (macrostructures) are stabilized and have a fixed structure at a constant temperature and pressure. Simultaneously at "Subatomic Equilibrium" (co thermal and subatomic equilibriums), all Wu's Pairs (microstructures) the building blocks of the universe, in the subatomic particles are stabilized and have a fixed structure at a fixed gravitational field and aging of the universe [10]. Where fixed gravitational field is established by the bombardment of gravitons based on Gravity Affected Wu's Spacetime Shrinkage Theory [3] in compliance with Graviton Radiation and Contact Interaction [11]. Also fixed aging of the universe is due to the built-in attractive Force of Creation in Wu's Pairs [2] based on Aging Affected Wu's Spacetime Shrinkage Theory in according to CMB radiation [12].

In fact, only at a fixed temperature and pressure having fixed atomic and subatomic particle structures (thermal equilibrium), also simultaneously at a fixed gravitational field and aging of the universe having fixed Wu's Pair structures (subatomic equilibrium), in other words only at co-thermal and subatomic equilibriums, all properties of an object or event should each have a fixed associated quantity. This is named "Principle of Equilibrium" [13].

As a result, co-thermal and subatomic equilibriums at a fixed temperature, pressure, gravitational field and aging of the universe is a pre-condition for all physical quantities, physical constants and equations of physical laws.

VI. Wu's Spacetime Shrinkage Theory

When the universe becomes older, according to Cosmic Microwave Background Radiation (CMB) and Five Principles of The Universe [14], Wu's Unit Length l_{yy} (the diameter of Wu's Pair) is getting smaller through aging of the universe due to the built-in attractive Force of Creation in Wu's Pairs, and eventually Yangton will recombine with Yington to destroy each other such that everything will go back to Nothing. This phenomenon is named "Aging Affected Wu's Spacetime Shrinkage Theory" [3]. Furthermore, in compliance with Principle of Parallelism, wavelength should decrease with aging of the universe ($\lambda \propto l_{yy}$), such that Cosmological Redshift can be observed.

On the other hand, under large gravitational field, because of the heavy graviton bombardment, the speed of Yangton and Yington circulation is getting slower and Wu's Unit Length is getting bigger. This phenomenon is named "Gravity Affected Wu's Spacetime Shrinkage Theory" [3]. Furthermore, in compliance with Principle of Parallelism [13], wavelength should increase at massive gravitational field ($\lambda \propto l_{yy}$), such that Gravitational Redshift can be observed.

According to Aging Affected Wu's Spacetime Shrinkage Theory, the diameter of Wu's Pairs l_{yy} (Wu's Unit Length) and the period of the Wu's Pairs t_{yy} (Wu's Unit Time) on the present earth are smaller than that of the stars 5 billion years ago. Based on Principle of Parallelism, normal unit length (meter) and normal unit time (second) also become smaller on the present earth. However, the universe on the contrary becomes bigger (reverse expansion) as measured by the shrinking normal unit length on the present earth. This is named "Wu's Spacetime Reverse Expansion Theory" [15]. In addition, the photon emitted from a star a few billion light years away, has a lower light speed (Absolute Light Speed) and lower frequency but longer wavelength than that of the present earth. These preserved ancient properties (longer wavelength means redder spectrum) can be observed as the photon quenches onto the present earth. This phenomenon is called Cosmological Redshift.

VII. Effects of Gravitational Field and Aging of the Universe on Wu's Spacetime Constant

For the same reason that associated quantities in equations of physical laws have fixed values at co-thermal and subatomic equilibriums [10], physical constants should also be stabilized and have fixed values under co-thermal and subatomic equilibriums. On the other hand, like all associated quantities of the properties of an object or event, the unit quantities in physical constants, as the associated quantities of the specific property of a standard object or event, should also have fixed associated quantities at co-thermal and subatomic equilibriums. As a result, like all associated quantities of the properties of an object or event, physical constants are also dependent on gravitational field and aging of the universe [10].

For examples, Gravitational Constant $6.674 \times 10^{11} \text{ N m}^2 \text{ Kg}^{-2}$ contains a real number 6.674×10^{11} and a group of unit quantities with arithmetic operations $\text{N m}^2 \text{ Kg}^{-2}$ which is dependent on gravitational field and aging of the universe. Similarly, Planck's Constant $6.626 \times 10^{-34} \text{ m}^2 \text{ Kg/s}$ contains a real number 6.626×10^{-34} and a group of unit quantities with arithmetic operations $\text{m}^2 \text{ Kg/s}$ which is also dependent on gravitational field and aging of the universe.

Like all other physical constants, both Wu's Constant and Wu's Spacetime Constant are composed of a real number and a group of unit quantities with arithmetic operations, therefore, both K and γ are also dependent on gravitational field and aging of the universe.

Furthermore, because of Wu's Constant K [1],

$$K = V^2 r = \frac{1}{2} k \frac{q_{yy}^2}{m_{yy}}$$

$$k = 8.99 \times 10^9 \text{ Kg m}^3 \text{ s}^{-2} \text{ C}^{-2}$$

Where V is Wu's Pair's circulation speed, r is the radius of Wu's Pair ($r = \frac{1}{2} l_{yy}$), k is Coulomb's Constant, q_{yy} (a physical constant $q_0 \text{ C}$) is the charge of Yangton or Yington, and m_{yy} (a physical constant $m_0 \text{ Kg}$) is the mass of Wu's Pair.

Also, Wu's Spacetime Constant γ [1],

$$\gamma = \pi (2K)^{-1/2} = \pi k^{-1/2} (q_{yy}^2 / m_{yy})^{-1/2}$$

Therefore,

$$\gamma = \pi (8.99 \times 10^9)^{-1/2} q_0^{-1} m_0^{1/2} (\text{m}^3 \text{ s}^{-2})^{-1/2} = \pi (8.99 \times 10^9)^{-1/2} q_0^{-1} m_0^{1/2} (\text{m/s})^{-1} \text{m}^{-1/2}$$

$$\gamma = 3.313 \times 10^{-5} q_0^{-1} m_0^{1/2} (\text{m/s})^{-1} \text{m}^{-1/2}$$

Where q_0 is the amount of charges in Yangton or Ying particle and m_0 is the amount of mass in Wu's Pair. Both of them are real number constants.

At massive gravitational field or early age of the universe, l_{yy} is bigger and V is smaller ($V^2 r = K$).

Consequently, according to Principle of Parallelism, m is bigger and m/s is smaller. Also, $(m/s)^{-1}$ is bigger and

$m^{-1/2}$ is smaller. However $(m/s)^{-1}$ is getting bigger at a rate twice order faster than that $m^{-1/2}$ is getting smaller. As a result, at massive gravitational field or early age of the universe, l_{yy} is bigger and also γ is larger.

VIII. Physical Quantities and Unit Transformations

The quantities of the properties of an object or event such as dimension L , duration T , velocity V and acceleration A , can be represented (measured) by the normal unit length l_s and normal unit time t_s of a reference object or event (Normal Cartesian System), and also be represented (measured) by Wu's Unit Length l_{yy} and Wu's Unit Time t_{yy} of a reference subatomic particle (Wu's Cartesian System) at the same gravitational field and aging of the universe as follows [1]:

$$L = l l_s = l m l_{yy}$$

$$T = t t_s = t n t_{yy}$$

$$V = v (l_s/t_s) = v m n^{-1} (l_{yy}/t_{yy})$$

$$A = a (l_s/t_s^2) = a m n^{-2} (l_{yy}/t_{yy}^2)$$

Where m is reference-dependent real number constant of normal unit length and n is reference-dependent real number constant of normal unit time. l is the amount of normal unit length, l_s is normal unit length and l_{yy} is Wu's Unit Length. t is the amount of normal unit time, t_s is normal unit time and t_{yy} is Wu's Unit Time. v is the amount of normal unit velocity, l_s/t_s is normal unit velocity and l_{yy}/t_{yy} is Wu's Unit Velocity. a is the amount of normal unit acceleration, l_s/t_s^2 is normal unit acceleration and l_{yy}/t_{yy}^2 is Wu's Unit Acceleration.

For corresponding identical objects or events [13] (same object or event at different locations, or at different gravitational fields or aging of the universe, under co-thermal and subatomic equilibriums), according to Principle of Parallelism [13], l , t , v and a are all real number constants, also m and n are reference-dependent real number constants, therefore L , T , V and A are functions of l_s , t_s , l_{yy} and t_{yy} . Furthermore, because l_s , t_s , l_{yy} and t_{yy} are dependent on gravitational field and aging of the universe, therefore, all quantities of the properties of an object or event are also dependent on gravitational field and aging of the universe.

IX. Wu's Spacetime Constant and Unit Transformations

Based on Wu's Spacetime Equation:

$$t_{yy} = \gamma l_{yy}^{3/2}$$

The quantities of the properties of an object or event such as dimension L , duration T , velocity V and acceleration A , can be represented by Wu's Spacetime Constant γ and Wu's Unit Length l_{yy} of the reference subatomic particle at the same gravitational field and aging of the universe as follows [1]:

$$L = l m l_{yy}$$

$$T = t n \gamma l_{yy}^{3/2}$$

$$V = v m n^{-1} \gamma^{-1} l_{yy}^{-1/2}$$

$$A = a m n^{-2} \gamma^{-2} l_{yy}^{-2}$$

Where l_{yy} is Wu's Unit Length of the reference subatomic particle, m is reference-dependent real number constant of normal unit length and n is reference-dependent real number constant of normal unit time, l is the amount of normal unit length, t is the amount of normal unit time, v is the amount of normal unit velocity, a is the amount of normal unit acceleration, γ is Wu's Spacetime Constant.

For a corresponding identical object or event, according to Principle of Parallelism, l , t , v and a are real number constants. Also, m and n are reference-dependent real number constants. However, γ is a physical constant which can change with gravitational field and aging of universe. Therefore, instead of dependent only on l_{yy} as claimed in my previous publication [1], L , T , V and A are actually dependent on both γ and l_{yy} as follows:

A. Dimension

$$L \propto l_{yy}$$

When l_{yy} increases, dimension L is getting bigger.

B. Duration

$$T \propto \gamma l_{yy}^{3/2}$$

When l_{yy} increases, both γ and $l_{yy}^{3/2}$ are getting bigger, therefore duration T is getting bigger.

C. Velocity

$$V \propto \gamma^{-1} l_{yy}^{-1/2}$$

When l_{yy} increases, γ is getting bigger and both γ^{-1} and $l_{yy}^{-1/2}$ are getting smaller, therefore velocity V is getting smaller.

D. Acceleration

$$A \propto \gamma^{-2} l_{yy}^{-2}$$

When l_{yy} increases, γ is getting bigger and both γ^{-2} and l_{yy}^{-2} are getting smaller, therefore acceleration A is getting smaller.

According to Wu's Spacetime Shrinkage Theory, at massive gravitational field and early age of the universe, Wu's Unit Length is bigger, therefore dimension and duration is bigger, but velocity and acceleration are smaller. These results can be used to interpret many phenomena such as Cosmological Redshift, Gravitational Redshift, General Relativity, Gravitational Time Dilation, Deflection of Light, Perihelion Precession of Mercury, Einstein's Field Equations and Wu's Spacetime Field Equations, as well as Hubble's Law and Wu's Spacetime Reverse Expansion Theory.

For examples, on the present earth, because of the smaller Wu's Unit Length l_{yy} of the light source, photon also has a smaller wavelength and larger frequency than that coming from a star at far distance (young universe). It is this reason to cause cosmological Redshift and universe expansion (actually spacetime shrinkage). Also, at a massive star, because of the larger Wu's Unit Length of the light source, photon also has a larger wavelength than that on earth. As a consequence, the photon emitted from a massive star then quenched onto earth has a larger wave length and smaller frequency than that on earth which can cause Gravitational Redshift.

Furthermore, in comparison of Einstein's Field Equation with Wu's Spacetime Field Equation, Einstein's Field Equation is an equation between the derivative of potential energy and the distribution of mass and energy with a transformation from Geodesic System on star to Cartesian System on earth. In contrast, Wu's Spacetime Field Equation is an equation between acceleration and gravitational field with a transformation from Normal Cartesian System on star to Wu's Cartesian System on earth and a conversion from Wu's Unit Time and Wu's Unit Length to Wu's Unit Length and Wu's Spacetime Constant.

X. Conclusions

Wu's Spacetime Equation $t_{yy} = \gamma l_{yy}$ is derived between Wu's Unit Length l_{yy} (diameter) and Wu's Unit Time t_{yy} (period) of Wu's Pairs, a pair of superfine antimatter circulating Yangton and Yington particles as the building blocks of the universe. Both Wu's Spacetime constant $\gamma = \pi (2K)^{-1/2}$ and Wu's constant $K = V^2 r = \frac{1}{2} k (q_{yy}^2 / m_{yy})$ (where V is the circulation speed, $r = \frac{1}{2} l_{yy}$, k is Coulomb's Constant, q_{yy} is the charge of Yangton or Yington particle, and m_{yy} is the mass of Wu's Pair) are physical constants composed of a real number and a group of unit quantities with arithmetic operations. Like any other physical constants, they are dependent on gravitational field and aging of the universe. Furthermore, according to Principle of Parallelism, physical properties such as dimension, duration, velocity and acceleration of an object or event can be represented by (transformed to) both Wu's Spacetime constant γ and Wu's Unit Length l_{yy} (of a reference corresponding identical subatomic particle) which are dependent on gravitational field and aging of the universe. It can be used to explain many important physical phenomena including Gravitational Redshift, Cosmological Redshift,

General Relativity, Gravitational Time Dilation, Hubble's Law, Spacetime Reverse Expansion (Universe Expansion), Deflection of Light, Perihelion Precession of Mercury and Einstein's Field Equations, etc.

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