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On polynomials of reduced topological indices of TUC_4C_8[S] carbon nanotubes

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Abstract
Degree based topological indices of chemical graphs are computed from classical formula and M-polynomial. M-polynomial of nine basic topological indices are investigated by many researchers for nanostructures. Reduced topological indices are derived from classical formula of topological indices. In this paper some reduced topological indices of TUC_4C_8[m,n] carbon nanotubes are investigated by M-polynomial.
Keywords: *Carbon nanotube, M-polynomial, reduced topological index, reduced Zagreb indices, topological index, Zagreb index.*

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I. Introduction

A molecular graph is a simple graph such that its vertices correspond to the atoms and the edges to the bonds. A topological index is a numeric quantity from the structural graph of a molecule and is invariant on the automorphism of the graph. Let G be a graph with vertex set V(G) and edge set E(G).For any vertex v, the degree d_v is the number of vertices adjacent to v. We can represent an edge by e = uv ∈ E(G) which connects vertex u and v. D.Y.Shin added the study of SK indices by M-polynomial to the classical topological indices by M-polynomial of molecular graph [1].The degree based topological indices of dendrimer nanostars and and carbon nanotubes are determined by Z.Raza with graphical representation [2]. Reduced version of Sombor index is studied by I.Gutman [3].Topological indices TUC_4C_8(S) nanotorus are determined by J.Asadpour et.al.[4].The third redefined Zagreb index and sum connectivity index in terms of M-polynomial is investigated by [5].Redefined first and second Zagreb indices are studied by M.A.Rashid et al.,[6].Zagreb first, second and third polynomials and indices of vitamin D_3 are studied by M.R.R.Kanna [7].The Zagreb group indices and Zagreb polynomials are studied by N.K.Raut [8].Topological indices of nanostructure is summarized in detail by S.Pandit [9].Some reduced multiplicative topological indices are investigated by V.R.Kulli [10].Different versions of harmonic index for nanotubes are studied by S.Ediz et.al.,[11]. The hyper Zagreb index of $TUSC_4C_8$(S) nanotubes is studied by M.R.Farahani [12].M-polynomials of degree based topological indices for many molecular graphs are investigated by [13-22].

The sum connectivity index of nanostructures are studied by S.Hayat, A.R.Ashraphi [23-24].GA_5 index of $TURC_4C_8$(S) is studied by M.R.Farahani [25]. In this paper reduced reciprocal Randic index, reduced second Zagreb index, reduced modified first Zagreb index, reduced sum connectivity index, reduced hyper second Zagreb index, reduced modified second index, reduced forgotten index, reduced Gourava first index, reduced product connectivity index and reduced redefined second Zagreb index are investigated by M-polynomial for TUC_4C_8[m,n] carbon nanotubes. The notations used in this paper are mainly taken from standard books of graph theory [26-30].Let us define some reduced topological indices.

The reduced reciprocal Randic index is defined as [31]

$$RRR(G) = \sum_{u,v\epsilon E(G)} \sqrt{(d_u - 1)(d_v - 1)}$$

and in M-polynomial as $RRR(G) = D_x^{1/2} D_y^{1/2} Q_{x(-1)}Q_{y(-1)}(M(G;x,y))|_{x = y = 1}$.

The reduced second Zagreb index is defined as [32]

$$RM_2(G) = \sum_{u,v\in E(G)} (d_u - 1)(d_v - 1)$$

$$RM_2(G) = D_x D_y Q_{x(-1)}Q_{y(-1)}(M(G;x,y)|_{x = y = 1}.$$

The reduced modified first Zagreb index is defined as

$$R^mM_1(G) = \sum_{u,v\in E(G)} \frac{1}{(d_u-1)+(d_v-1)}$$

$$R^mM_1(G) = S_xJQ_{x(-1)}Q_{y(-1)} (M(G;x,y))|_{x=y=1}.$$

The reduced sum connectivity index is defined as

$RSCI(G) = \sum_{u,v\in E(G)} \frac{1}{\sqrt{(d_u-1)+(d_v-1)}}$

and $RSCI(G) = S_x^{1/2} J Q_{x(-1)} Q_{y(-1)} (M(G;x,y))|_{x=y=1}$.

The reduced hyper second Zagreb index is defined as

$RHM_2(G) = \sum_{u,v\in E(G)} ((d_u-1)(d_v-1))^2$

and its M-polynomial version is

$RHM_2(G) = D_x^2 D_y^2 Q_{x(-1)} Q_{y(-1)} (M(G;x,y))|_{x=y=1}$.

The reduced second modified Zagreb index is defined as

$R^mM_2(G) = \sum_{u,v\in E(G)} \frac{1}{(d_u-1)(d_v-1)}$

$R^mM_2(G) = (D_x D_y)^{-1} Q_{x(-1)} Q_{y(-1)} (M(G;x,y))|_{x=y=1}$.

The reduced forgotten topological index is defined as [33]

$RF(G) = \sum_{u,v\in E(G)} ((d_u-1)^2 + (d_v-1)^2)$

$RF(G) = (D_x^2 + D_y^2) Q_{x(-1)} Q_{y(-1)} (M(G;x,y))|_{x=y=1}$.

The reduced first Gourava index is defined as [34]

$RGO_1(G) = \sum_{u,v\in E(G)} \{((d_u-1)+(d_v-1)) + ((d_u-1)(d_v-1))\}$

$RGO_1(G) = ((D_x + D_y) + (D_x D_y)) Q_{x(-1)} Q_{y(-1)} (M(G;x,y))|_{x=y=1}$.

The reduced product connectivity index is defined as [35-36]

$RPCI(G) = \sum_{u,v\in E(G)} \frac{1}{\sqrt{(d_u-1)(d_v-1)}}$

$RPCI(G) = S_x^{1/2} S_y^{1/2} Q_{x(-1)} Q_{y(-1)} (M(G;x,y))|_{x=y=1}$.

The reduced redefined second Zagreb index is defined as

$RReM_2(G) = \sum_{u,v\in E(G)} \frac{(d_u-1)(d_v-1)}{(d_u-1)+(d_v-1)}$

$RReM_2(G) = S_x J D_x D_y Q_{x(-1)} Q_{y(-1)} (M(G;x,y))|_{x=y=1}$.

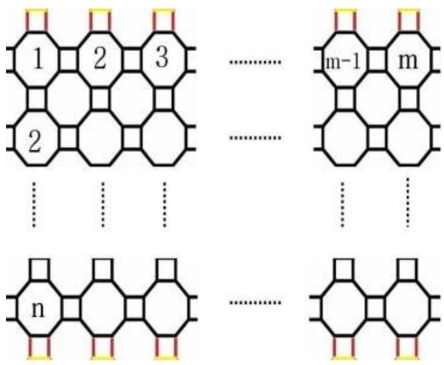

Figure 1. The graph of 2-*D* lattice of $TUC_4C_8[m,n]$.

II. Materials and Method

A molecular graph is constructed by representing each atom of a molecule by a vertex and bonds between atoms by edges. The number of vertices of G adjacent to a given vertex v, is the degree of this vertex and will be denoted by d_v. The 2-dimensional graph of $TUC_4C_8[m,n]$ is shown in figure (1).We denote molecular graph G = $TUC_4C_8(S)$ nanotube. It is clear that $TUC_4C_8[S]$ has 8mn + 4m vertices and 12mn + 4m edges. There are three partitions of edge set corresponding to their degrees of end vertices E_1, E_2 and E_3.The number of edges in E_1, E_2 and E_3 are 2m, 4m and 12mn-2m respectively.In $TUC_4C_8[m,n]$ for any m,n ∈ N, m is the number of octagons C_8 in first row and n is the number of octagons C_8 in the first column.And also $|E_4|$ = 2m = (2,2),$|E_5|$ = 4m = (2,3) and $|E_6|$ = 12mn-2m = (3,3).The classical formulas for topological indices are used to test the correctness of indices in computation of topological indices by M-polynomial. The M-polynomial is represented by M(G;x,y).The function used in M-polynomial derivation is f(u,v) = f(x,y).In degree based topological indices computation-degree of vertices and edges between them is very important. Mostly studied degree based indices are Zagreb indices which almost appear in many research papers. The derivational operators are D_x, D_y. In this paper some degree based reduced topological indices such as RRR(G), $RM_2(G)$, $R^mM_1(G)$, RSCI(G), $RHM_2(G)$, $R^mM_2(G)$, RF(G), $RGO_1(G)$, RPCI(G) and $RReM_2(G)$ are defined and investigated by M-polynomials for $TUC_4C_8[m,n]$ carbon nanotubes.

III. Results And Discussion

The two dimensional lattice of $TUC_4C_8[m,n]$ carbon nanotubes has m number octagons C_8 in the first row and n number of octagons C_8 in the first column.The number of vertices in this molecular graph of $TUC_4C_8[m,n]$ nanotubes is equal to 8mn + 4m and the number of edges in G are 12mn + 4m [37-42].Every vertex in G has degree either 2 or 3. Let G be graph with vertex set V(G) and edge set E(G).The number of edges with frequency for degrees d_u and d_v are represented in table 1 and derivational formulas of reduced topological indices by M-polynomial in table 2.There are three partitions of edge set corresponding to the degree of end vertices which are

$E_1 = \{uv \in E_G(G) \mid d_u=2, d_v=2\}$,

$E_2 = \{uv \in E_G(G) \mid d_u=2, d_v=3\}$,and

$E_3 = \{uv \in E_G(G) \mid d_u=3, d_v=3\}$.

The number of edges in E_1, E_2 and E_3 are $|E_{(2,2)}| = 2m$, $|E_{(2,3)}| = 4m$ and $|E_{(3,3)}| = 12mn-2m$.
The M-polynomial of G is defined as

$$M(G;x,y) = \sum_{\delta \le i \le j \le \Delta} m_{ij}(G)\, x^i y^j,$$

where $\delta = \min\{d_v \mid v \in V(G)\}$, $\Delta = \max\{d_v \mid v \in V(G)\}$, and $m_{ij}(G)$ is the edge vu $\in$ E(G) such that $i, j \ge 1$, with $D_x(f(x,y))= x\frac{\partial f(x,y)}{\partial x}$, $D_y(f(x,y))= y\frac{\partial f(x,y)}{\partial y}$, $S_x = \int_0^x \frac{f(t,y)}{t}\,dt$, $S_y = \int_0^y \frac{f(x,t)}{t}\,dt$, $J(f(x,y)) = f(x,x)$, $Q_\alpha(f(x,y)) = x^\alpha f(x,y)$.
Using notation f(u,v) = f(x,y) in the M-polynomial.The M-polynomial for $TUC_4C_8[m,n]$ is written as

$M(G;x,y) = \sum_{i \le j} m_{ij}(G)\, x^i y^j = \sum_{2 \le 2} m_{22}(G)\, x^2y^2 + \sum_{2 \le 3} m_{23}(G)\, x^2y^3 + \sum_{3 \le 3} m_{33}(G)\, x^3y^3$.
$= |E_{(2,2)}|x^2y^2 + |E_{(2,3)}|x^2y^3 + |E_{(3,3)}|x^3y^3$.
$= 2mx^2y^2 + 4mx^2y^3 + (12mn-2m)\, x^3y^3$.
This equation is used in the determination of reduced topological indices by M-polynomial.

Table 1.The edge partition of $TUC_4C_8[m,n]$ carbon nanotubes.

d_u, d_v	(2,2)	(2,3)	(3,3)
Number of edges	2m	4m	12mn -2m

Table 2. Derivation of topological indices from M-polynomial.

Topological index	Derivation from M(G;x,y)
Reduced reciprocal Randic index RRR(G)	$D_x^{1/2}D_y^{1/2}Q_{x(-1)}Q_{y(-1)}(M(G;x,y))\vert_{x=y=1}$
Reduced second Zagreb index $RM_2(G)$	$D_xD_yQ_{x(-1)}Q_{y(-1)}(M(G;x,y))\vert_{x=y=1}$
Reduced modified first Zagreb index $R^mM_1(G)$	$S_xJQ_{x(-1)}Q_{y(-1)}(M(G;x,y))\vert_{x=y=1}$
Reduced sum connectivity index RSCI(G)	$S_x^{1/2}JQ_{x(-1)}Q_{y(-1)}(M(G;x,y))\vert_{x=y=1}$
Reduced hyper second Zagreb index $RHM_2(G)$	$D_x^2D_y^2Q_{x(-1)}Q_{y(-1)}(M(G;x,y))\vert_{x=y=1}$
Reduced second modified Zagreb index $R^mM_2(G)$	$(D_xD_y)^{-1}Q_{x(-1)}Q_{y(-1)}(M(G;x,y))\vert_{x=y=1}$
Reduced forgotten index RF(G)	$(D_x^2+D_y^2)Q_{x(-1)}Q_{y(-1)}(M(G;x,y))\vert_{x=1}$
Reduced first Gourava index $RGO_1(G)$	$((D_x+D_y)+(D_x\ D_y))\ Q_{x(-1)}Q_{y(-1)}(M(G;x,y))\vert_{x=y=1}$
Reduced product connectivity index RPCI(G)	$S_x^{1/2}S_y^{1/2}Q_{x(-1)}Q_{y(-1)}(M(G;x,y))\vert_{x=y=1}$
Reduced redefined second Zagreb index $RReM_2(G)$	$S_xJD_xD_yQ_{x(-1)}Q_{y(-1)}(M(G;x,y))\vert_{x=y=1}$

Theorem 3.1. Reduced reciprocal Randic index of graph G of TUC_4C_8 is 24mn+4m.
Proof. Consider a molecular graph of $TUC_4C_8[m,n]$ as shown in figure 1.From table 1,2 and formula of reduced reciprocal Randic index as

$RRR(G) = \sum_{u,v \in E(G)} \sqrt{(d_u-1)(d_v-1)}$

$RRR(G) = D_x^{1/2} D_y^{1/2} Q_{x(-1)}Q_{y(-1)}(M(G;x,y))|_{x=y=1}$

$f(x,y)= 2mnx^2 y^2 + 4mx^2y^3 + (12mn-2m) x^3 y^3$

$D_x^{1/2} Q_{x(-1)}Q_{y(-1)}= 2mnx y + 4mxy^2 + \sqrt{2} (12mn-2m) x^2 y^2$

$D_x^{1/2} D_y^{1/2} Q_{x(-1)}Q_{y(-1)}= 2mnx y + 4\sqrt{2}mxy^2 +2(12mn-2m) x^2 y^2$

$RRR(G) = 2mn + 4\sqrt{2}m +2(12mn-2m)$

$= 24mn + 4m.$

Theorem 3.2. Reduced second Zagreb index of graph G of TUC_4C_8 is 48mn+2m.

Proof. Consider a molecular graph of $TUC_4C_8[m,n]$ as shown in figure 1.From table 1,2 and formula of Reduced second Zagreb index as

$RM_2(G) = \sum_{u,v\in E(G)}(d_u-1)(d_v-1)$

$RM_2(G) = D_x D_y Q_{x(-1)}Q_{y(-1)}(M(G;x,y))|_{x=y=1}$

$f(x,y)= 2mx^2 y^2 + 4mx^2y^3 + (12mn-2m) x^3 y^3$

$Q_{y(-1)}= 2mx^2 y + 4mx^2y^2 + (12mn-2m) x^3 y^2$

$Q_{x(-1)}Q_{y(-1)}= 2mxy + 4mxy^2 + (12mn-2m) x^2 y^2$

$D_y Q_{x(-1)}Q_{y(-1)}= 2mxy + 8mxy^2 + 2(12mn-2m) x^2 y^2$

$D_x D_y Q_{x(-1)}Q_{y(-1)}= 2mxy + 8mxy^2 + 4(12mn-2m) x^2 y^2$

$RM_2(G) = 48mn +2m.$

Theorem 3.3. Reduced modified first Zagreb index of graph G of $TUC_4C_8[m,n]$ is 3mn + 2m.

Proof. Consider a molecular graph of $TUC_4C_8[m,n]$ as shown in figure 1.From table 1,2 and formula of reduced modified first Zagreb index as

$R\,^mM_1(G) = \sum_{u,v\in E(G)}\frac{1}{(d_u-1)+(d_v-1)}$

$R\,^mM_1(G) = S_x J Q_{x(-1)}Q_{y(-1)}(M(G;x,y))|_{x=y=1}$

$f(x,y)= 2mx^2 y^2 + 4mx^2y^3 + (12mn-2m) x^3 y^3$

$Q_{y(-1)}= 2mx^2 y + 4mx^2y^2 + (12mn-2m) x^3 y^2$

$Q_{x(-1)}Q_{y(-1)}= 2mxy + 4mxy^2 + (12mn-2m) x^2 y^2$

$J Q_{x(-1)}Q_{y(-1)}= 2mx^2 + 4mx^3 + (12mn-2m) x^4$

$S_x J Q_{x(-1)}Q_{y(-1)}= mx^2 +\frac{4}{3} mx^3 + \frac{(12mn-2m)}{4} x^4$

$R\,^mM_1(G) = 3mn + 2m.$

Theorem 3.4. Reduced sum connectivity index of graph G of TUC_4C_8 is 6mn+2m.

Proof. Consider a molecular graph of $TUC_4C_8[m,n]$ as shown in figure 1.From table 1,2 and formula of reduced sum connectivity index as

$RSCI(G) = \sum_{u,v\in E(G)}\frac{1}{\sqrt{(d_u-1)+(d_v-1)}}$

$RSCI(G) = S_x^{1/2} J Q_{x(-1)}Q_{y(-1)}(M(G;x,y))|_{x=y=1}.$

$f(x,y)= 2mx^2 y^2 + 4mx^2y^3 + (12mn-2m) x^3 y^3$

$Q_{y(-1)}= 2mx^2 y + 4mx^2y^2 + (12mn-2m) x^3 y^2$

$Q_{x(-1)}Q_{y(-1)}= 2mxy + 4mxy^2 + (12mn-2m) x^2 y^2$

$J Q_{x(-1)}Q_{y(-1)}= 2mx^2 + 4mx^3 + (12mn-2m) x^4$

$S_x^{1/2} J Q_{x(-1)}Q_{y(-1)}= \frac{2}{\sqrt{2}}mx^2 +\frac{4}{\sqrt{3}} mx^3 + \frac{(12mn-2m)}{2} x^4$

$RSCI(G) = 6mn + 2m.$

Theorem 3.5. Reduced hyper second Zagreb index of graph G of TUC_4C_8 is 192mn-14m.

Proof. Consider a molecular graph of $TUC_4C_8[m,n]$ as shown in figure 1.From table 1,2 and formula of reduced hyper second Zagreb index as

$RHM_2(G) = \sum_{u,v\in E(G)}((d_u-1)(d_v-1))^2$

$RHM_2(G) = D_x^2 D_y^2 Q_{x(-1)} Q_{y(-1)}(M(G;x,y))|_{x=y=1}$

$f(x,y) = 2mx^2y^2 + 4mx^2y^3 + (12mn-2m)x^3y^3$

$Q_{y(-1)} = 2mx^2y + 4mx^2y^2 + (12mn-2m)x^3y^2$

$Q_{x(-1)}Q_{y(-1)} = 2mxy + 4mxy^2 + (12mn-2m)x^2y^2$

$D_y^2 Q_{x(-1)}Q_{y(-1)} = 2mxy + 16mxy^2 + 4(12mn-2m)x^2y^2$

$D_x^2 D_y^2 Q_{x(-1)}Q_{y(-1)} = 2mxy + 16mxy^2 + 16(12mn-2m)x^2y^2$

$RHM_2(G) = 192mn - 14m.$

Theorem 3.6. Reduced second modified Zagreb index of graph G of $TUC_4C_8[m,n]$ is $3mn + 3.5m$.

Proof. Consider a molecular graph of $TUC_4C_8[m,n]$ as shown in figure 1. From table 1,2 and formula of reduced second modified Zagreb index as

$R^mM_2(G) = \sum_{u,v\in E(G)} \frac{1}{(d_u-1)(d_v-1)}$

$R^mM_2(G) = (D_x D_y)^{-1} Q_{x(-1)}Q_{y(-1)}(M(G;x,y))|_{x=y=1}$

$f(x,y) = 2mx^2y^2 + 4mx^2y^3 + (12mn-2m)x^3y^3$

$Q_{x(-1)}Q_{y(-1)} = 2mxy + 4mxy^2 + (12mn-2m)x^2y^2$

$D_y^{-1} Q_{x(-1)}Q_{y(-1)} = -2mxy - 8mxy^2 - 2(12mn-2m)x^2y^2$

$D_x^{-1} D_y^{-1} Q_{x(-1)}Q_{y(-1)} = 2mxy + 8mxy^2 + 4(12mn-2m)x^2y^2$

$R^mM_2(G) = 3mn + 3.5m.$

Theorem 3.7. Reduced forgotten index of graph G of $TUC_4C_8[m,n]$ is $96mn + 8m$.

Proof. Consider a molecular graph of $TUC_4C_8[m,n]$ as shown in figure 1. From table 1,2 and formula of reduced forgotten index as

$RF(G) = \sum_{u,v\in E(G)} ((d_u-1)^2 + (d_v-1)^2)$

$RF(G) = (D_x^2 + D_y^2)\ Q_{x(-1)}Q_{y(-1)}(M(G;x,y))|_{x=y=1}$.

$f(x,y) = 2mx^2y^2 + 4mx^2y^3 + (12mn-2m)x^3y^3$

$Q_{x(-1)}Q_{y(-1)} = 2mxy + 4mxy^2 + (12mn-2m)x^2y^2$

$D_y^2\ Q_{x(-1)}Q_{y(-1)} = 2mxy + 16mxy^2 + 4(12mn-2m)x^2y^2$

$D_x^2\ Q_{x(-1)}Q_{y(-1)} = 2mxy + 4mxy^2 + 4(12mn-2m)x^2y^2$

$RF(G) = (D_x^2 + D_y^2)\ Q_{x(-1)}Q_{y(-1)}M(G;x,y)|_{x=y=1} = 96mn + 8m.$

Theorem 3.8. Reduced first Gourava index of graph G of $TUC_4C_8[m,n]$ is $96mn + 10m$.

Proof. Consider a molecular graph of $TUC_4C_8[m,n]$ as shown in figure 1. From table 1,2 and formula of reduced first Gourava index as

$RGO_1(G) = \sum_{u,v\in E(G)} \{((d_u-1) + (d_v-1)) + (d_u-1)(d_v-1))\}$

$RGO_1(G) = (D_x + D_y)(D_x\ D_y)\ Q_{x(-1)}Q_{y(-1)}(M(G;x,y))|_{x=y=1}$

$f(x,y) = 2mx^2y^2 + 4mx^2y^3 + (12mn-2m)x^3y^3$

$Q_{x(-1)}Q_{y(-1)} = 2mxy + 4mxy^2 + (12mn-2m)x^2y^2$

$(D_x + D_y)\ Q_{x(-1)}Q_{y(-1)} = 4mxy + 12mxy^2 + 4(12mn-2m)x^2y^2$

$D_x D_y\ Q_{x(-1)}Q_{y(-1)} = 2mxy + 8mxy^2 + 4(12mn-2m)x^2y^2$

$RGO_1(G) = (D_x + D_y)\ (D_x\ D_y)\ Q_{x(-1)}Q_{y(-1)}M(G;x,y)|_{x=y=1} = 96mn + 10m.$

Theorem 3.9. Reduced product connectivity index of graph G of $TUC_4C_8[m,n]$ is $6mn + 4m$.

Proof. Consider a molecular graph of $TUC_4C_8[m,n]$ as shown in figure 1. From table 1,2 and formula of reduced product connectivity index as

$RPCI(G) = \sum_{u,v\in E(G)} \frac{1}{\sqrt{(d_u-1)(d_v-1)}}$

$RPCI(G) = S_x^{1/2} S_y^{1/2}\ Q_{x(-1)}Q_{y(-1)}(M(G;x,y))|_{x=y=1}$

$f(x,y) = 2mx^2y^2 + 4mx^2y^3 + (12mn-2m)x^3y^3$

$Q_{x(-1)}Q_{y(-1)} = 2mxy + 4mxy^2 + (12mn-2m)\, x^2 y^2$

$S_x^{1/2} Q_{x(-1)}Q_{y(-1)} = 2mxy + 4\, mxy^2 + \frac{1}{\sqrt{2}} (12mn-2m)\, x^2 y^2$

$S_x^{1/2} S_y^{1/2} Q_{x(-1)}Q_{y(-1)} = 2mxy + \frac{4}{\sqrt{2}} mxy^2 + \frac{1}{2} (12mn-2m)\, x^2 y^2$

RPCI(G) = 6mn + 4m.

Theorem 3.10. Reduced redefined second Zagreb index of graph G of $TUC_4C_8[m,n]$ is 12mn + 2m.

Proof. Consider a molecular graph of $TUC_4C_8[m,n]$ as shown in figure 1. From table 1,2 and formula of reduced redefined second Zagreb index as

$RReM_2(G) = \sum_{u,v \in E(G)} \frac{(d_u-1)(d_v-1)}{(d_u-1)+(d_v-1)}$

$RReM_2(G) = S_x J D_x D_y Q_{x(-1)}Q_{y(-1)}(M(G;x,y))|_{x=y=1}$

$f(x,y) = 2mx^2 y^2 + 4mx^2 y^3 + (12mn-2m)\, x^3 y^3$

$Q_{x(-1)}Q_{y(-1)} = 2mxy + 4mxy^2 + (12mn-2m)\, x^2 y^2$

$D_y Q_{x(-1)}Q_{y(-1)} = 2mxy + 8mxy^2 + 2\, (12mn-2m)\, x^2 y^2$

$D_x D_y Q_{x(-1)}Q_{y(-1)} = 2mxy + 8mxy^2 + 4\, (12mn-2m)\, x^2 y^2$

$J D_x D_y Q_{x(-1)}Q_{y(-1)} = 2mx^2 + 8mx^3 + 4(12mn-2m)\, x^4$

$S_x J D_x D_y Q_{x(-1)}Q_{y(-1)} = mx^2 + \frac{8}{3} mx^3 + (12mn-2m)\, x^4$

$RReM_2(G) = 12mn + 2m$.

IV. Conclusion

In this paper some reduced topological indices are defined and studied by M-polynomial for $TUC_4C_8(S)$. The derivation of reduced topological indices by M-polynomial is acceptable if it yields the same result as is obtained by computation of topological indices based on classical formulas.

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