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Arithmetic Operations and Physical Quantities

Edward T. H. Wu

[Abstract]
Three different physical quantities: Nature Quantities (such as a piece of space or time); Associated Quantities (such as the dimension and duration of an object or event); and Measured Quantities (such as X number of minutes and Y number of meters) are clearly defined and compared. Also, arithmetic operations including addition, subtraction, multiplication and division applied on Associated Quantities (such as D/S and M_1M_2/R^2) and Measured Quantities (such as m/s and Kgm/s^2) are interpreted. Furthermore, commutativity, associativity and distributivity of Associated Quantities and Measured Quantities are derived.

[Keywords]
Arithmetic Operations, Addition, Subtraction, Multiplication, Division, Commutativity, Associativity and Distributivity, Mathematical Physics.

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I. Introduction

Although mathematics is not physics, it has been successfully used in many physical areas such as calculation of physical quantities and interpretation of physical phenomena. For examples, calculation of velocity and acceleration [1] and derivation of Newton's Law of Universal Gravitation [2]. The meanings of the arithmetic operations including addition, subtraction, multiplication and division [3] applied on the physical quantities such as those of Associated Quantities M/S and M_1M_2/R^2, and Measured Quantities m/s and Kgm/s^2 are unclear, even somewhat confusion. It is the purpose of this paper to provide a clear definition and sound interpretation to these issues.

II. Nature Quantities and Associated Quantities

A physical quantity is the quantity of a property (such as dimension and duration) of an object or event in the universe. In terms of mathematics, a physical quantity is the value (quantity) corresponding to a function (property) of an element (object or event) in a domain (universe).

Nature Quantities, such as a piece of Space and Time, don't change with anything at all, no matter of any object or event. However, Associated Quantities, such as the Dimension and Duration of an object or event, are properties of an object or event that are dependent on local gravitational field and aging of the universe [4].

III. Measured Quantities

Measured Quantities are composed of two components "Amount" and "Unit Quantity" of the same property. Amount is a real number. Unit Quantity is a specific Associated Quantity of a standard object or event. Unit Quantities are also dependent on local gravitational field and aging of the universe [4].

IV. Arithmetic Operations and Associated Quantities

In physics, arithmetic operations including Addition, Subtraction, Multiplication and Division are the interactions between two Associated Quantities of the same or two different properties of the same or two different objects or events.

In Addition and Subtraction, the product of the operation is the Associated Quantity of the same property of the combined object or event. In Multiplication and Division, the product of the operation is the Associated Quantities of the induced property generated from the two properties caused by the interactive object or event produced from the interaction between the two objects or events.

Associated Quantities can be represented as Q(A), in which Q is the property and A is the object. For examples, the quantity of Dimension (property Q) of an object (A) can be represented as Q(A) and the quantity of Duration (property P) of an event (B) can be represented as P(B).

A. Addition

1. Definition of Addition

Q(A) and Q(B) are two Associated Quantities of the same property Q of two objects or events (A and B).

Because of the intrinsic structures, Associated Quantity is proportional to the amount of object or event.

$$Q(mA) = mQ(A)$$

Accordingly, Addition is defined as follows:

$$Q(A) + Q(B) \equiv Q(A\psi B)$$

Where AψB is the combined object or event of A and B.

2. Commutativity

Q(A) and Q(B) are two Associated Quantities of the same property Q of two objects or events.

Because

AψB = BψA

Q(AψB) = Q(A) + Q(B)

Q(BψA) = Q(B) + Q(A)

Therefore,

$$Q(A) + Q(B) = Q(B) + Q(A)$$

This is called Commutativity of Addition.

3. Associativity

Q(A), Q(B) and Q(C) are three Associated Quantities of the same property Q of three objects or events.

Because

(AψB)ψC = Aψ(BψC)

Q((AψB)ψC) = Q(AψB) + Q(C) = (Q(A) + Q(B)) + Q(C)

Q(Aψ (BψC)) = Q(A) + Q(BψC) = Q(A) + (Q(B) + Q(C))

Therefore,

$$(Q(A) + Q(B)) + Q(C) = Q(A) + (Q(B) + Q(C))$$

This is called Associativity of Addition.

4. Distributivity

Q(A) and Q(B) are two Associated Quantities of the same property Q of two objects or events.

Because

Q (mA) = mQ(A)

Q(A) + Q(B) = Q(AψB)

Therefore,

m(Q(A) + Q(B)) = mQ(AψB) = Q(m(AψB)) = Q(mA ψ mB) = Q(mA) + Q(mB) = mQ(A) + mQ(B)

Therefore,

$$m(Q(A) + Q(B)) = mQ(A) + mQ(B)$$

This is called Distributivity of Addition.

B. Subtraction

1. Definition of Subtraction

Q(A) and Q(B) are two Associated Quantities of the same property Q of two objects or events (A and B).

If an Associated Quantity Q (M) of the same property Q of an object or event M can satisfy the following:

$$Q(M) + Q(B) = Q(M\psi B) = Q(A)$$

Then Subtraction is defined as:

$$Q(A) – Q(B) = Q(M\psi B) – Q(B) \equiv Q(M)$$

2. Definition of Negative Physical Quantity

If there is an object or event N can satisfy:

$$Q(A) + Q(N) = 0$$

Then the negative physical quantity -Q(A) is defined as:

$$-Q(A) \equiv Q(N)$$

Also,

$$Q(A) + (-Q(A)) = 0$$

3. Theory

If

MψB = A

Then

Q(M) + Q(B) = Q(A)

Q(A) – Q(B) = Q(M)

Also,

Q(A) + (-Q(B)) = (Q(M) + Q(B)) + (-Q(B)) = Q(M)

Therefore,

$$Q(A) – Q(B) = Q(A) + (-Q(B))$$

C. Multiplication

1. Definition of Multiplication

Q(A) is the Associated Quantity of property Q of an object or event A; P(B) is the Associated Quantity of property P of an object or event B; and $\underline{Q\Phi P}$(AΦB) is an Associated Quantity of the induced property $\underline{Q\Phi P}$ generated from the two properties Q and P, caused by the interactive object or event AΦB produced from the interaction between the two object or event A and B (In case of interaction, Q and P can be the same property, but A and B must be considered as two objects or events even they are the same object or event).

Because of the intrinsic structures, Associated Quantity of the induced property is proportional to both amounts of the objects or events.

$$\underline{Q\Phi P}\,(mA\Phi B) = m\,\underline{Q\Phi P}(A\Phi B)$$

$$\underline{Q\Phi P}\,(A\Phi nB) = n\,\underline{Q\Phi P}(A\Phi B)$$

Accordingly, Q(A) x P(B) is defined as follows:

$$Q(A) \times P(B) \equiv \underline{Q\Phi P}(A\Phi B)$$

2. Commutativity

Q(A) is the Associated Quantity of property Q of an object or event A; P(B) is the Associated Quantity of property P of an object or event B; and $\underline{Q\Phi P}$(AΦB) is an Associated Quantity of the induced property $\underline{Q\Phi P}$ generated from the two properties Q and P, caused by the interactive object or event AΦB produced from the interaction between the two object or event A and B.

Because

$A\Phi B = B\Phi A$

$\underline{Q\Phi P}(A\Phi B) = \underline{Q\Phi P}(B\Phi A)$

$\underline{Q\Phi P}(A\Phi B) = Q(A) \times P(B)$

$\underline{Q\Phi P}\,(B\Phi A) = P(B) \times Q(A)$

$$Q(A) \times P(B) = P(B) \times Q(A)$$

This is called Commutativity of Multiplication.

For examples, in case of an ideal gas A (object), $\underline{P\Phi V}$ (AΦA) = P(A) x V(A) = V(A) x P(A) = nRT (Equation of ideal gas PV = VP = nRT). Where AΦA is an induced object from the interaction between object A and object A (where AΦA = A). $\underline{P\Phi V}$ is an induced property from property P (pressure) and property V (volume) of the object or event A.

Also, in case of a moving vehicle A (object), $\underline{M\Phi V}$(AΦA) = M(A) x V(A) = V(A) x M(A) = P (Equation of momentum P = MV = VM). Where $\underline{M\Phi V}$ is an induced property from property M (mass) and property V (speed) of the object or event A.

D. Division

1. Definition of Division

Q(A) is the Associated Quantity of property Q (such as Duration) of an object or event A. R(M) is the Associated Quantity of property R (such as Distance) of an object or event M.

If an Associated Quantity P(B) of property P (such as Speed) of an object or event B can satisfy the following:

$$Q(A) \times P(B) = \underline{Q\Phi P}(A\Phi B) = R(M)$$

Then Division is defined as:

$$R(M)/Q(A) = \underline{Q\Phi P}(A\Phi B)/Q(A) \equiv P(B)$$

2. Definition of Inverse Physical Quantity

If there is an object or event R can satisfy:

$$Q(A) \times P(R) = 1$$

Then the inverse physical quantity 1/Q(A) is defined as:

$$1/Q(A) \equiv P(R)$$

Also,

$$Q(A) \times 1/Q(A) = 1$$

3. Theory

If

$A\Phi B = M$

$Q(A) \times P(B) = \underline{Q\Phi P}(A\Phi B) = R(M)$

Then

$R(M)/Q(A) = P(B)$

$R(M) \times 1/Q(A) = (Q(A) \times P(B)) \times 1/Q(A) = P(B)$

Therefore,

$$R(M)/Q(A) = R(M) \times 1/Q(A)$$

V. Arithmetic Operations and Measured Quantities

Measured Quantities mU(S) are composed of two components, m is a real number and U(S) is a Unit Quantity. Unit Quantity is an Associated Quantity of a specific property of a standard object or event. For examples, "meter" (m) – the quantity of dimension (property U_1) of a standard ruler (object S_1) can be represented as $U_1(S_1)$, "Kilogram" (Kg) – the quantity of mass (property U_2) of a standard weight (object S_2) can be represented as $U_2(S_2)$ and "second"(s) – the quantity of duration (property U_3) of a standard atomic clock (event S_3) can be represented as $U_3(S_3)$.

1. Addition

mU(S) and nU(S) are two Measured Quantities, where U(S) is the Unit Quantity (Associate Quantity) of a property U of a standard object or event S, m and n are real numbers.

Because

mU(S) = U(mS)

nU(S) = U(nS)

Also,

mU(S) + nU(S) = U(mS) + U(nS) = U(mS ψ nS) = U((m+n)S) = (m+n)U(S)

Therefore,

$$mU(S) + nU(S) = (m+n)U(S)$$

2. Subtraction

If

gU(S) +nU(S) = (g+n)U(S) = mU(S)

g + n = m

Then

mU(S) – nU(S) = gU(S) = (m-n)U(S)

Therefore,

$$mU(S) - nU(S) = (m-n)U(S)$$

3. Multiplication

$U_1(mS_1)$ is the Measured Quantity of property U_1 of an object or event mS_1. $U_2(nS_2)$ is the Measured Quantity of property U_2 of an object or event nS_2. $\underline{U_1}\Phi\underline{U_2}(S_1\Phi S_2)$ is the Associated Quantity of an induced property $\underline{U_1}\Phi\underline{U_2}$ of an interactive object or event $S_1\Phi S_2$ between S_1 and S_2. m and n are real numbers.

Because

$mU_1(S_1) = U_1(mS_1)$

$nU_2(S_2) = U_2(nS_2)$

Also,

$mU_1(S_1) \times nU_2(S_2) = U_1(mS_1) \times U_2(nS_2) = \underline{U_1}\Phi\underline{U_2}(mS_1\Phi nS_2) = m\underline{U_1}\Phi\underline{U_2}(S_1\Phi nS_2) = mn\ \underline{U_1}\Phi\underline{U_2}(S_1\Phi S_2) = mn\ U_1(S_1) \times U_2(S_2)$

Therefore,

$$mU_1(S_1) \times nU_2(S_2) = mn\ U_1(S_1) \times U_2(S_2)$$

4. Division

If

$hU_3(S_3) \times nU_2(S_2) = hn\ U_3(S_3) \times U_2(S_2) = hn\ \underline{U_3}\Phi\underline{U_2}(S_3\Phi S_2) = mU_1(S_1)$

Then

$mU_1(S_1)/nU_2(S_2) = hn\ \underline{U_3}\Phi\underline{U_2}(S_3\Phi S_2)/nU_2(S_2) = hU_3(S_3)$

$(m/n)\ ((U_1(S_1)/U_2(S_2)) = (m/n)(\underline{U_3}\Phi\underline{U_2}(S_3\Phi S_2)/U_2(S_2)) = hU_3(S_3)$

Therefore,

$$mU_1(S_1)/nU_2(S_2) = (m/n)\ (U_1(S_1)/U_2(S_2))$$

VI. Conclusion

Three different physical quantities: Nature Quantities (such as a piece of space or time); Associated Quantities (such as the dimension and duration of an object or event); and Measured Quantities (such as X number of minutes and Y number of meters) are clearly defined and compared. Also, arithmetic operations including addition, subtraction, multiplication and division applied on Associated Quantities (such as M/S and M_1M_2/R^2) and Measured Quantities (such as m/s and Kgm/s^2) are interpreted. Furthermore, commutativity, associativity and distributivity of Associated Quantities and Measured Quantities are derived.

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