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Entropy and phase transitions in Calabi-Yau space

Tetiana Obikhod, Ievgenii Petrenko[1]
[1]*Institute for Nuclear Research NAS of Ukraine, Kyiv 03028, Ukraine*

Abstract:
Background: *The evolution of the concept of black hole entropy from the classical expression for a Schwarzschild black hole to Bekenstein-Hawking entropy for a black hole at Planck scales is considered. Phase transitions of the multidimensional space of Calabi-Yau and its identification with a microscopic black hole are considered in the aspect of string and D-brane theory.* ***Results***: *Using a K-functor, we calculated the topological invariant of a black hole, compressed into a point-like photon-type particle and obtained an equidistant set of energy levels of the final state.*
Key Word: *Calabi-Yau, K-functor, black hole*

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I. Introduction

The connection between black holes and elementary particles through the discovery of string theory and possible unification of the general theory of relativity and quantum theory, made it possible to investigate the question of properties. The most important of these properties is the entropy of the black hole. The study of BPS black hole solutions has become interesting in the context of Bekenstein-Hawking entropy [1] through the counting of microscopic degrees of freedom of D-branes [2]. Moreover BPS solutions account for the light states of moduli space in order to describe phase transitions between different branches of the moduli space. So, Dp-branes to be of great importance in studies of the physics of black holes in the context of string theory. Here we will study black holes and their p-brane cousins, which are the best candidate for a unified quantum theory of all interactions including gravity.

II. History of entropy determination

a) Schwarzschild black holes

The Schwarzschild metric is a solution of the d = 4 action, [3]

$$S = \frac{1}{16\pi G_4}\int d^4x\sqrt{-g}R[g].$$

The metric takes the form

$$ds^2 = -\left(1-\frac{r_H}{r}\right)dt^2 + \left(1-\frac{r_H}{r}\right)^{-1}dr^2 + r^2 d\Omega_2^2$$

The radius is in Schwarzschild coordinates $rH = 2G4M$ is expressed by the Newton constant and black hole mass, M. The black hole temperature called Hawking temperature of the black hole to be $T_H = \frac{1}{8\pi G_4 M}$.

Bekenstein came to the conclusion that the second law of thermodynamics will not be violated only if the black hole also has entropy, and this entropy is constantly growing as it is sucked in into a black hole of matter, compensating observed decrease in entropy outside black hole.

$$S_{BH} = \frac{A}{4\hbar}$$

b) Problem of 'lost' information

Taking Hawking radiation into account, a black hole after gravitational collapse will evaporate, and the space-time has no event horizon. This is presented by the following diagram, Fig.1

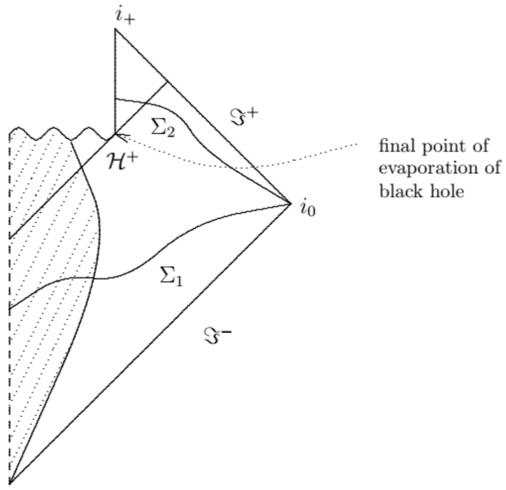

Figure 1: Diagram of black hole evolution.

$\Sigma 1$ is a Cauchy surface for this space-time, $\Sigma 2$ is not, its past does not include the black hole region. Information from $\Sigma 1$ propagates into the black hole region instead of $\Sigma 2$. It appears that information is 'lost' into the black hole. But from the point of view of a static external observer the information is not really lost and nothing passes through $\mathcal{H}^+$.

The black hole information puzzle resolved within string and D-brane theory. In Einstein's theory there are no restrictions on the minimum mass of a black hole. According to the theory of relativity, the compression of matter of any mass leads to black hole with small size. However, there is a small catch connected with the black holes in the Universe and black holes of small size when the mass of the black hole becomes of the order of the Planck mass. An understanding of the Planck scale physics requires the inclusion of Quantum Gravity effects and the temperature in string theory is then of the order maximum (Hagedorn) temperature.

c) Collapse of black hole to small size and D-brane theory

Studying the equations of string theory, physicists realized the possibility that in the process of evolution in time, three-dimensional spheres can contract, collapse to vanishingly small sizes. Such compression of space is represented as a string surrounding the singularity or, in general, as a D-brane wrapped around the singularity. Strominger reasoned [3] that one-dimensional string, i.e. 1-brane in the new language of theorists can completely surround a one-dimensional spatial object, similarly, a two-dimensional membrane, i.e. a 2-brane, can wrap and completely cover a two-dimensional sphere, 3-branes can envelop and completely cover three-dimensional spheres, although this is difficult to visualize. The evolution of a black hole is represented through a space of extra dimension or a Calabi-Yau manifold. During the evolution of the Calabi-Yau manifold, accompanied by a conifold transition with a gap in space, the initially nonzero mass of the black hole decreases to zero, after which the black hole turns into a massless particle (like photon), which in the language of string theory is described by a certain vibrational mode of the string.

In the first phase - the presence of black holes will be detected through the original Calabi-Yau manifold. In the second phase - black holes underwent a phase transition, "melted", and passed into fundamental vibrational modes of the string connected with another Calabi-Yau manifold. So black holes and elementary particles are two sides of the same coin. Any Calabi-Yau manifold can transform into any other by physically valid discontinuous conifold transitions. All string models with different coupling constants and geometry of the Calabi-Yau space are different phases of a unified theory. Such black holes related to BPS states are endowed with a charge and have the smallest mass compatible with charge.

III. New concept of entropy

In the framework of superstring and D-brane theory, the concept of entropy has changed due to the presence of the extra-dimensional Calabi-Yau space, which is folded at each point of the usual Minkowski space.

Up to constants, the black hole entropy is just the area of the horizon in Planck units. The area scaling is the evidence for "holography". There are several versions of holography, but the idea is that since the entropy scales like the area rather than the volume, the degrees of freedom of the system are characterized by a quantum field theory with one fewer space dimensions. This idea called "*AdS/CFT* correspondence" was elevated to a principle by't Hooft and Susskind [4].

Since the area of the horizon is proportional to the black hole entropy, it might appear that this area decrease signals a violation of the second law. On the other hand, the entropy in the Hawking radiation increases, providing a possible way out. Defining a generalised entropy, which includes the entropy of the black hole plus the other stuff such as Hawking radiation, $S_{tot} = S_{BH} + S_{other} \geq 0$, was argued by Bekenstein to fix up the second law. Therefore, the Bekenstein-Hawking or Black Hole entropy is in any space-time dimension d

$$S_{BH} = \frac{A_d}{4\hbar G_d},$$

where A_d is the area of the event horizon, and G_d is the d-dimensional Newton constant, which in units $\hbar = c = 1$ has dimensions of (length)$^{d-2}$.

Identifying the Bekenstein-Hawking entropy as the physical black hole entropy gives rise to puzzles:

1. the microscopic quantum degrees of freedom and thermodynamic entropy;
2. the information problem which requires a quantum gravity theory with non-locality.

The *AdS/CFT* correspondence connected with the holographic description in string theory appears to possess an explicit realization the information retention scenario, [4].

Through the holographic description of $AdS(5) \otimes S(5)$ near horizon geometry of a stack of D3-branes a solution of ten dimensional super-gravity was presented. The new metric of the boundary $AdS(5) \otimes S(5)$ is (8 + 1) dimensional

$$dS^2 = \left\{ \left(1+r^2\right)^2 dt^2 - 4dr^2 - 4r^2 d\Omega^2 \right\} + \left\{ \left(1-r^2\right)^2 d\omega_5^2 \right\}$$

The size of the 5-sphere shrinks to zero as the boundary at $r = 1$ is approached. The boundary of the geometry is therefore 3 + 1 dimensional. The number of quantum degrees of freedom in the gauge theory description satisfies the holographic behavior

$$\frac{N_{dof}}{A} \sim \frac{1}{G}$$

with

$$A = \frac{R^3}{\delta^3} \times R^5 = \frac{R^8}{\delta^3}$$

and G_d - is d-dimensional Newton constant which in units $\hbar = c = 1$ has dimension of (length)$^{d-2}$. Therefore, as $S \sim A/G$, we have the resulting formula for the entropy required by the holographic principle,

$$S_{max} \sim \frac{1}{\delta^3} \frac{R^8}{l_p^8}.$$

So, as black hole can be represented as a submanifold of such a Calabi-Yau like a pea in a shell. String theory space-times with conserved quantum numbers can be black holes, but more commonly they are black p-branes.

$$\frac{S_{array}}{S_{string}} \sim \left(\frac{R}{r_H}\right)^{1/(d-3)}.$$

So, the array dominates for small horizon radii, and the black string dominates for large horizon radii. The entropy of a BPS black hole in the context of string theory is written in [5].

IV. The calculation of topological invariant of vector bundle

Since D-branes are multidimensional objects, represented as vector bundles with Calabi-Yau manifolds as the base of such bundles, phase transitions of D-branes are associated with phase transitions between Calabi-Yau manifolds, [6]. Using BPS-states of D-branes represented by vector bundles of the type

$$\begin{array}{ccc} Spin(k) & \longrightarrow & Spin(k+1) \\ & & \downarrow \\ & & S^k \end{array}$$

it can be shown that for $k = 6$, $Spin(6)$ group is isomorphic to the $SU(4)$ group. Since the group describing black holes is $SU(2,2|4) \sim SU(2,2) \times SU(4)$ ($SU(2,2)$ describes the external degrees of freedom, and $SU(4)$ - the internal ones), the greatest interest is of group $SU(4)$. Then we can work with *Spin* vector bundles, which present D-branes with the phase transitions between them classified with Grothendieck K-group in the framework of the Clifford algebra formalism. As a result, we obtain a chain of phase transitions of a black hole represented by transitions between topological invariants of vector bundles described by K-groups

$$K(S^6) \to K(S^4) \to K(S^2) \to K(S^0) = Z.$$

The value $K(S^0)=Z$ signals about an equidistant set of energy levels of a point-like particle into which the black hole has passed during the phase transition [7].

V. Conclusions

We have considered the evolution of the black hole concept of entropy. The problem of 'lost' information as well as the problem of collapse of black hole to small size at Plank energies leads the scientists to the necessity of using of D-brane and string theory. The ability to describe the phase transitions of a black hole using the conifold transitions of the Calabi-Yau manifold with a gap in space, leads to the fact that initially nonzero mass of the black hole decreases to zero, after which the black hole turns into a massless particle (like photon). Due to AdS/CFT correspondence connected with holographic description of $AdS(5) \otimes S(5)$ near horizon geometry of a stack of D3-branes we can say about the system characteristic by a quantum field theory with one fewer space dimensions. This fact was demonstrated by number of quantum degrees of freedom in the gauge theory description which satisfies the holographic behavior. So, string theory of space-times with conserved quantum numbers can be black holes, but more commonly they are black p-branes. Using the presentation of D-branes through vector bundles we presented chain of phase transitions of black hole in words of transitions of topological invariants of vector bundles and calculated K-group of the resulting point-like black hole, which signals about the equidistant energy levels of soliton-like black hole.

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