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Approximation of the Inverse Fine-Structure Constant UsingOther Mathematical Constants

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Abstract:*The fine-structure constantα is a dimensionless number and very nearly equal to 1 / (137.036). For reasons of convenience,the reciprocalvalue of the fine-structure constant*$\left(\frac{1}{\alpha} = 10^2 + \frac{10^3 - 10^{-3}}{3^3} - 10^{-3}\right)$*is often specified. The 2018 CODATA recommended value of* α^{-1} *= 137.035999084 [1]. In this paper value of* α^{-1}*wasestimated using the equation:*$\boxed{\alpha^{-1} = \sqrt{e^{\pi + \pi\Phi + \Phi}} = \frac{\Phi}{2} + e\pi^3\Phi = \frac{\pi^{12}}{5^3 . e^4} + \Phi = (e + \pi + \Phi)^{\sqrt{6}} - 1 =}$ $\boxed{\left(\frac{\Phi\pi}{\log(e)}\right)^2}$.

Keywords: *Fine-Structure Constant α, Golden Ratio (Φ = 1.618), Euler's number(e = 2.7182),and π = 3.1416.*

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I. Introduction

The fine structure constantgot its name from Arnold Sommerfield, who introduced it in 1916 [2].It is noted that when an electron orbits the nuclei in different energy shells, the energy levels of each individual shell split into much finer ones. And the gaps between the fine layer of these energy levelsare directly proportional to the square of number of protons in the nucleus multiplied by α[3].And thus it got its name. The value of fine structure constantcan be derived from other constants like:G (Newton's constant), c (Einstein's constant), ħ (reduced Planck's constant),K_B (Boltzmann's constant),K_E (Coulomb's constant),&e (Charge of an electron).

Sign	Name	Formula	Value	Dimension
G	The universal gravitational constant	$G = gR_E^2/M_E$	6.67408×10^{-11} m^3 kg^{-1} s^{-2}	$M^{-1}L^3T^{-2}$
c	The speed of light in a free space	$c = 1/\sqrt{\mu_o \varepsilon_o}$	$299792458 \approx 3\times 10^8$m s^{-1}	LT^{-1}
ħ	The reduced Planck's constant	ħ = h/2π	$1.0545718 \times 10^{-34}$m^{2}kg s^{-1}(J.s)	ML^2T^{-1}
K_B	The Boltzmann's constant	$K_B = R/N_A$	1.3806×10^{-23} m^2 kg s^{-2} K^{-1} (J/K)	$ML^2T^{-2}\theta^{-1}$
K_E	The Coulomb's constant	$K_E = 1/4\pi\varepsilon_0$	8.9875517923×10^9 m^3·kg.s^{-2}·C^{-2}	$ML^3T^{-2}Q^{-2}$
e	Charge of an electron	$96500/N_A$	$1.60217662 \times 10^{-19}$ C	Q

N_A = Avogadro's number = 6.023×10^{23} mole^{-1}. Named after the Italian scientist Amedeo Avogadro.

Fundamental Entity	**Planck's Expression**	**Value in SI unit**	**Stoney's Expression**	**Value in SI unit**
Length	$\sqrt{\frac{\hbar G}{c^3}}$	1.616255×10^{-35} m	$\sqrt{\frac{GK_e e^2}{c^4}}$	1.3807×10^{-34} m
Mass	$\sqrt{\frac{\hbar c}{G}}$	2.176434×10^{-8}kg	$\sqrt{\frac{k_e e^2}{G}}$	1.8592×10^{-9} kg
Time	$\sqrt{\frac{\hbar G}{c^5}}$	5.391247×10^{-44} s	$\sqrt{\frac{GK_e e^2}{c^6}}$	4.6054×10^{-45} s
Temperature (Absolute Hot)	$\sqrt{\frac{\hbar c^5}{GK_B{}^2}}$	1.416784×10^{32} K	$\sqrt{\frac{K_e e^2 c^4}{GK_B{}^2}}$	1.2119522×10^{31} K
Charge	$\sqrt{\frac{\hbar c}{K_e}}$	1.875546×10^{-18} C	.e	1.6021766×10^{-19} C

Now from the table above it can be seen that the ratio between the corresponding values of Planck's units & Stoney's units are also a constant, which is$11.706237481 = \sqrt{137.036}$. Means, the ratio has the value of $\alpha^{-1/2}$.It can be observed that if we take square of each expressions and after that divide the Stoney's unit with corresponding Planck's unit then the ratio will always be$\frac{K_e e^2}{\hbar c}$. Hence, this is the fundamental expression for α.

	Dimensional Analysis of Stoney's Units	Dimensional Analysis of Planck's Units
L	$\sqrt{\frac{GK_e e^2}{c^4}}$ = $[M^{-1}L^3T^{-2}.\ ML^3T^{-2}Q^{-2}.\ Q^2.\ L^{-4}T^4]^{(0.5)} = \sqrt{L^2}$	$\sqrt{\frac{\hbar G}{c^3}}$ = $[ML^2T^{-1}.\ M^{-1}L^3T^{-2}.\ L^{-3}T^3]^{(0.5)} = \sqrt{L^2}$
M	$\sqrt{\frac{k_e e^2}{G}}$ = $[ML^3T^{-2}Q^{-2}.\ Q^2.\ ML^{-3}T^2]^{(0.5)} = \sqrt{M^2}$	$\sqrt{\frac{\hbar c}{G}}$ = $[ML^2T^{-1}.\ LT^{-1}.\ ML^{-3}T^2]^{(0.5)} = \sqrt{M^2}$
T	$\sqrt{\frac{GK_e e^2}{c^6}}$ = $[M^{-1}L^3T^{-2}.\ ML^3T^{-2}Q^{-2}.\ Q^2.\ L^{-6}T^6]^{(0.5)} = \sqrt{T^2}$	$\sqrt{\frac{\hbar G}{c^5}}$ = $[ML^2T^{-1}.\ M^{-1}L^3T^{-2}.\ L^{-5}T^5]^{(0.5)} = \sqrt{T^2}$
θ	$\sqrt{\frac{K_e e^2 c^4}{GK_B{}^2}}$ = $[ML^3T^{-2}Q^{-2}.\ Q^2.\ L^4T^{-4}.\ ML^{-3}T^2.\ M^{-2}L^{-4}T^4\theta^2]^{(0.5)}$	$\sqrt{\frac{\hbar c^5}{GK_B{}^2}}$ = $[ML^2T^{-1}.\ L^5T^{-5}.\ ML^{-3}T^2.\ M^{-2}L^{-4}T^4\theta^2]^{(0.5)}$
Q	e = Charge of a single electron or a single proton	$\sqrt{\frac{\hbar c}{K_e}}$ = $[ML^2T^{-1}.\ LT^{-1}.\ M^{-1}L^{-3}T^2Q^2]^{(0.5)} = \sqrt{Q^2}$

(Stoney Length ÷ Planck Length)

$= \sqrt{\frac{GK_e e^2}{c^4}} \div \sqrt{\frac{\hbar G}{c^3}} = \sqrt{\frac{K_e e^2}{\hbar c}} = \sqrt{\alpha}$

$= 1.3807\times10^{-34} \div 1.61625\times10^{-35}$

$= 1/\sqrt{137.036}$

□

(Stoney Mass ÷ Planck Mass)

$= \sqrt{\frac{k_e e^2}{G}} \div \sqrt{\frac{\hbar c}{G}} = \sqrt{\frac{K_e e^2}{\hbar c}} = \sqrt{\alpha}$

$= 1.8592\times10^{-9} \div 2.176434\times10^{-8}$

$= 1/\sqrt{137.036}$

□

(Stoney Time ÷ Planck Time)

$= \sqrt{\frac{GK_e e^2}{c^6}} \div \sqrt{\frac{\hbar G}{c^5}} = \sqrt{\frac{K_e e^2}{\hbar c}} = \sqrt{\alpha}$

$= 4.6054\times10^{-45} \div 5.39124\times10^{-44}$

$= 1/\sqrt{137.036}$

□

(Stoney Temp. ÷ Planck Temp.)

$= \sqrt{\frac{K_e e^2 c^4}{GK_B{}^2}} \div \sqrt{\frac{\hbar c^5}{GK_B{}^2}} = \sqrt{\frac{K_e e^2}{\hbar c}} = \sqrt{\alpha}$

$= 1.211952\times10^{31} \div 1.41678\times10^{32}$

$= 1/\sqrt{137.036}$

□

(Stoney charge ÷ Planck charge)

$= e \div \sqrt{\frac{\hbar c}{K_e}} = \sqrt{\frac{K_e e^2}{\hbar c}} = \sqrt{\alpha}$

$= 1.60217\times10^{-19} \div 1.8755\times10^{-18}$

$= 1/\sqrt{137.036}$

So (1) $E = mc^2$ (2) $E = hf$ (3) $c = f\lambda$(4) $\hbar = \frac{h}{2\pi}$ (5) $2\pi r = n\lambda$(6) $c^2 = \frac{1}{\square_o\varepsilon_o}$

$E = mc^2 = hf$
$\Rightarrow mc.c = hf$
$\Rightarrow p.c = hf$
$\Rightarrow p.f\lambda = hf$
$\Rightarrow p.\lambda = h$
$\Rightarrow (mv).\lambda = h$
$\Rightarrow h = (mv).\lambda$
$\Rightarrow h = (mv).(2\pi r)$
Considering n = 1
$\Rightarrow h/2\pi = (mv).(r)$
$\Rightarrow \hbar = mvr$
$\therefore \hbar = L$

Energy:
$E = mc^2$
$\Rightarrow E = mc.c$
$\Rightarrow E = pc$
$\Rightarrow E = \frac{p}{\sqrt{\square_0\varepsilon_0}}$
$\circlearrowleft E = hf$
$\Rightarrow E = h(c/\lambda)$
$\Rightarrow E = h/t$
$\Rightarrow E = \frac{h}{2\pi}.\frac{2\pi}{t}$
$\Rightarrow E = \hbar\omega$
$\therefore E = L\omega$

Attractive force between electron & proton of a hydrogen atom n = 1,⇒ $F_1 = K_e.e^2/r_e^2$, P.E. = $(F_1 \times r_e)$. Centripetal force perceived by the same electron in the Bohr's model, ⇒ $F_2 = m_e v_e^2/r_e$,K.E. = $(½F_2 \times r_e)$.
Now, $F_1 = F_2$
$\Rightarrow K_e.e^2/r_e^2 = m_e v_e^2/r_e$
$\Rightarrow K_e.e^2 = m_e v_e^2.r_e$
$\Rightarrow K_e.e^2 = (m_e.v_e.r_e).v_e$
$\Rightarrow K_e.e^2 = L.v_e$
$\Rightarrow K_e.e^2 = \hbar.v_e$
$\Rightarrow v_e = K_e.e^2/\hbar$
$\Rightarrow v_e/c = K_e.e^2/\hbar c$
$\therefore v_e/c = \alpha$

For an electron of first orbital–
Let's say, r_e = Bohr's radius n = 1
$\lambda_e = (h/mc)$ = Compton wavelength
So, $2\pi r_e = n\lambda$
$\Rightarrow 2\pi r_e = \lambda$ (For n = 1)
$\Rightarrow 2\pi r_e = h/p$
$\Rightarrow 2\pi r_e = h/mv_e$
$\Rightarrow 2\pi r_e = h/mc\alpha$
$\Rightarrow 2\pi r_e = \lambda_e/\alpha$
$\therefore \alpha = \lambda_e/2\pi r_e$

Velocity of the electron:
$v_e = c\alpha = 3\times10^8/137.036$ m/s
$\therefore v_e = 2.2\times10^6$m/s

Charge of the electron:
e = Faraday Const./Avogadro N_A.
$\Rightarrow e = [(96500)/(6.023\times10^{23})]$ C
$\therefore e = 1.875546\times10^{-18}$ C

Mass of the electron:
$\frac{The\ Mass\ of\ a\ Proton\ (H^+)}{The\ Mass\ of\ an\ Electron\ (m_e)} = 1836$
m_e = gm atomic mass of $H_2/1837N_A$
$= [1.00784/(1837\times6.023\times10^{23})]$ gm
$= 9.11\times10^{-28}$ gm $= 9.11\times10^{-31}$ kg

Radius of the orbital:
Now, $2\pi r_e = h/mc\alpha = 2\pi\hbar/mc\alpha$
$\therefore r_e = \hbar/mc\alpha = 5.3\times10^{-11}$m

The Lagrange Equation:
$\mathcal{L}$ = KE– PE = $(½)m_e.v_e^2 - K_e e^2/r_e$
$\therefore \mathcal{L} = (½)m_e.(c\alpha)^2 - K_e e^2/(\hbar/mc\alpha)$
$= (½)m_e c^2\alpha^2 - m_e c^2\alpha^2 = (½)m_e c^2\alpha^2$

$$\alpha = \frac{K_e e^2}{\hbar c} = \frac{\mu_o}{4\pi}\frac{ce^2}{\hbar} = \frac{e^2}{2\varepsilon_o hc} = \frac{\mu_o ce^2}{2h} = \frac{\mu_o c}{2R_K} = \frac{e^2 Z_o}{2h} = \frac{e^2 Z_o}{4\pi\hbar} = \frac{v_e}{c} = \left(\frac{e}{Q_p}\right)^2 = \frac{\lambda_e}{2\pi r_e} = \frac{\lambda_e}{\lambda} = \frac{2^{256}}{N_{Edd}} = \sqrt{\frac{r_0}{r_e}}$$

ε_0	is the electric constant or permittivity in vacuum or free space	$8.854\ 187\ 8128\times10^{-12}$ F/m
μ_0	is the magnetic constant or permeability in vacuum or free space	$1.25663706212\times10^{-6}$ N/A^2
Z_0	is the vacuum impedance or impedance in free space E/H = $\mu_o c = 1/c\varepsilon_0$	376.730313668… Ω.
R_K	is the Von Klitzing constant = h/e^2	25812.80745… Ω
λ	is the De Broglie's wavelength = h/mv_e	3.33×10^{-10} m
λ_e	is the Compton's wavelength = h/mc	2.43×10^{-12} m
Q_p	is the Planck charge (Ref: first page)	1.87555×10^{-18} C
N_{Edd}	Is the Eddington's Number,Total number of protons in the universe	1.57×10^{79}

r_0	Classical Electron Radius (Not same as Bohr's radius as discussed later)	2.818×10^{-15} m

Again, P.E. = $\frac{K_e.e^2}{r_0^2}\times r_0 = mc^2 \Rightarrow r_0 = \frac{K_e.e^2}{mc^2} = \frac{\hbar.K_e.e^2}{\hbar.mc^2} = \frac{K_e.e^2}{\hbar c}\times\frac{\hbar}{mc} = \frac{\hbar\alpha}{mc}$. Now, $\frac{r_0}{r_e} = \frac{\hbar\alpha}{mc}\div\frac{\hbar}{mc\alpha} = \alpha^2$.

One of the prominent issue with the expression of Bohr's radius of orbital $r_e = \hbar/mc\alpha$ is, its discrepancy with the expression of the classical radius of orbital r_0. Although the size of the electron is beyond the scope of ordinary quantum mechanics, one can think of its size as something the electron would need to have if its rest energy were only due to its electrostatic potential energy P.E. = ($F_1 \times r_0$) instead of ($F_1 \times r_e$).Also, $F_1 = K_e.e^2/r_0^2$instead of $K_e.e^2/r_e^2$. But, P.E. = mc^2. So, P.E. = $\frac{K_e.e^2}{r_0^2}\times r_0 = mc^2 \Rightarrow r_0 = \frac{K_e.e^2}{mc^2} = \frac{\hbar.K_e.e^2}{\hbar.mc^2} = \frac{K_e.e^2}{\hbar c}\times\frac{\hbar}{mc} = \frac{\hbar\alpha}{mc}$. Now, $\frac{r_0}{r_e} = \frac{\hbar\alpha}{mc}\div\frac{\hbar}{mc\alpha} = \alpha^2$. Thus, the ratio of the classical radius of electron to the Bohr's radius isα^2.

The Bohr radius uses the center of the proton as center, while the classical radius includes the fact that both the electron and the proton have mass, putting the center little away from the geometric center of the proton. If the electron clouds observed around the nuclei of atoms are purely statistical phenomena, then there should be no need for a radius. On the other hand, if the electron is moving in an orbit, like a moon around a planet, then the radius should be used on its own. The electron orbit is in other words neither a purely statistical phenomenon nor a conventional orbit. This is exactly what we should expect if the electron is bouncing on the atomic nucleus. Hence, electron would neither orbit, nor be entirely random it would be something in between.

The α^2 is also the ratio between the Harte energy (27.2 eV = 2×Rydberg energy =2×its ionization energy) and the electron rest energy (511 keV).α is also the ratio of other two energies: (i) the energy needed to overcome the electrostatic repulsion between two electrons at a distance d, &(ii) the energy of a single photon of a wavelength 2πd. if λ = 2πd, then$\left[\frac{e^2}{4\pi\epsilon_0 d}\div\frac{hc}{\lambda} = \frac{e^2}{4\pi\epsilon_0\, d}\times\frac{2\pi d}{hc} = \frac{e^2}{4\pi\epsilon_0\, d}\times\frac{d}{\hbar c} = \frac{e^2}{4\pi\epsilon_0\,\hbar c} = \frac{K_e e^2}{\hbar c} = \alpha\right]$. Thus, the fine structure constant is not only the square root of the ratio of the classical radius of electron to the Bohr's radius, but also the ratio of the velocity of the electron in the first circular orbit of the Bohr model of the atom, to the speed of light in vacuum (v_e/c). This was the Summerfield's original physical interpretation. Therefore, α can similarlybeexpressed, as the ratio between the Compton's wavelength (h/mc) to the De Broglie's wavelength (h/mv_e) at ground state [4].Enos Øye made the discovery that the Fine-structure constant is equal to the wavelength of the electron of a hydrogen atom, divided by half the wavelength of the photon required to kick it out of orbit, thus ionizing the hydrogen atom. The fine structure constant relates the energy of an electron in orbit around a proton with the energy of the photon required to free it from its orbit. Hence, αactually represents the probability that an electron will emit or absorb a photon.

We have already seen that$\sqrt{\alpha}$ is the conversion factor of Stoney units to Planck units. In this context, it must be pointed out that more than 25 years before Planck introduced his quantities, the Irish physicist Johnston Stoney in 1881 introduced the quantities of mass, length and time [5].Thus, Stoney units came out in classical era while on the other hand Planck units introduced thequantum era.

II. Literature Review

Now question arises, what then determines the value of α,are there hidden dimensions in nature that somehow fix its value? Some scientists think so. But the enigma of α remains. As one of the students of Sommerfeld, Wolfgang Pauli wrote about α in 1948: "The theoretical interpretation of its numerical value is one of the most important unsolved problems of atomic physics". Scientists began to mystify the number 137:

137 is the 33rd prime number after 131 and before 139. It is also a Pythagorean prime: a prime number of the form 4n + 1, where n = 34 (137 = 4 × 34 + 1) or the sum of two squares $11^2 + 4^2$ = (121 + 16). Also, 137 is the only known primeval number whose sum of digits equals the number of primes "contained", it is the largest prime factor of 123456787654321 and also divides 11111111. It is the smallest prime with 3 distinct digits that remains prime if any one of its digits is removed. But we need to keep in mind the inverse of fine structure constant is almost 137.036, not 137 the full number. Again, α = $1/\sqrt{\pi^2 + 137^2}$ = $1/\sqrt{6\zeta(2) + 137^2}$. Which means, a triangle with base 137 and height π has the hypotenuse of a length that's very close to the measured inverse of the fine structure constant. Its close connection with π is uncanny as the sum of the squares of the first seven digits of π is also 137. As, (3^2 + 1^2 + 4^2 + 1^2 + 5^2 + 9^2 + 2^2 = 137) [6].

$$\alpha = \frac{1}{4\pi^3+\pi^2+\pi} = \left(\frac{9}{16\pi^4}\right)\left(\frac{\pi^5}{5!}\right)^{\frac{1}{4}} = \left[\frac{3^2}{(2\pi)^4}\right]\left(\sqrt[4]{\frac{\pi^5}{5!}}\right) \approx \frac{1}{\sqrt{2}\pi^4} \approx \frac{7\pi}{\pi^7} \approx \frac{5e\phi}{\pi^7} = \frac{1}{20\phi^4} = \frac{(5e)^4}{20(7\pi)^4} = \frac{36}{500\pi^2} = \frac{6}{500.\zeta(2)} \approx \frac{360}{\phi^2} = \alpha$$

$$ie, \alpha = \frac{(2\times8\times18\times32)(\pi-1)+8}{8^2[(2\times8\times18\times32)(\pi-1)^2+8(\pi-1)-8]} = \frac{1}{\frac{126-\frac{3}{200}}{1-\frac{5}{2\pi^3}} + \left[\frac{1}{10}\left\{\frac{1}{2}-\left(\frac{1}{5}\right)^2\right\}\right]^2 + \left[\frac{1}{2}\left(\frac{1}{5}\right)^4\right]^2} = \frac{-137+\sqrt{137^2+16}}{8}$$

Therefore, α is the positive root of the quadratic equation:$4x^2 + 137x - 1 = 0$ or,$x^2 + \left(2\sqrt{3}\pi^2 + \frac{1}{16}\right)x - \frac{1}{4} = 0$[7]. Again, $\sqrt{e} \approx \Phi \approx 2^{\ln 2}$, hence, $\sqrt{e}/\Phi \approx 1 + \alpha.\Phi^2$. Here, Φ is the golden ratio and e is the Euler's number [8, 9 & 10].

Although it was Arnold Sommerfeld who formally introduced the fine structure constant in 1916, its history can be traced back to Max Planck, as discussed previously in this article. Planck had noticed that the combination of $K_e.e^2/c$ has the same dimensions as the Planck constant h. He wondered if h was identical to $K_e.e^2/c$ and if this could somehow explain the value of the elementary charge. In 1909, while reviewing the status of the theory of blackbody radiation AlbertEinstein tried to predict the value of "hc"from the value of K_e and e^2, but few decimals were missing. Lorentz reacted to Einstein's notes saying that, three missing decimals were too much and concluded that h had nothing to do with e. However, this agreement ofprediction with the observed fine-structuresplitting was bit accidental and led to considerable confusion in the early days ofquantum theory. Althoughrelativistic mass and momentum were used, the computed energy using classical mechanics led to a correction much larger thanthat actually due only to relativistic effects. Since, the fine structureis associated with a completely nonclassical property of the electron called spin. As α is a dimensionless number formed of universal constants, all observers will measure the same value for it. Therefore, several numerological experiments continued for some time, and these attempts are probably a measure of how desperate physicists were in their pursuit of a fundamental reason for the value of α [11].

Even before Bohr formally announced his model of hydrogen atom 1913, an Austrian physicist Arthur Erich Haas in 1910, observed that the different spectral red lines was actually a doublet, which was termed the 'fine structure' of lines. It means, the size of a hydrogen atom is a factor $\alpha^{-2} \approx 20000$ times the size of an electron. Arnold Sommerfeld thought he could improve upon the Bohr model by assuming that the orbits can be elliptical. In addition, he considered the effect of variation of mass with speed. He presented his calculations at the Bavarian Academy of Sciences in December 1915 & January 1916. The spectroscopist Friedrich Paschen soon set to work on comparing the prediction with observations. By May 1916 he reported to Sommerfeld that "my measurements are now finished, and they agree everywhere most beautifully with your fine structures". One month later, Paschen determined the value of α^{-1} as 137.9. This was when α got its name 'fine structure constant'. Sommerfeld's model was praised as a great progress. Einstein wrote to him a year later that, "Your investigation of the spectra belongs among my most beautiful experiences in physics. Only through it do Bohr's ideas become completely convincing." Planck went to the extent of comparing this work with that of the prediction of Neptune's orbit in astronomy[12].

However, all this work was superseded by the advent of wave mechanics of Schrödinger when the classical picture of fixed orbits of electrons was abandoned in favor of a probabilistic wave function. The uncertainly principle pointed out that the classical way of calculating the electron orbit was wrong because the position and velocity could not be determined at any given time. These models could explain the fine structure and much more, without referring to elliptical orbits. For the fine structure of spectral lines, a new quantum number was invoked, that of the electron 'spin', which took the place of Sommerfeld's 'k' quantum number. But the role of the fine structure constant in the scheme of the subatomic world was already secured, and it keeps appearing in all expressions of energy levels in atoms. It is now viewed as one of the 'coupling constants' of Nature. The force of gravity couples all particles with the Newton's gravitational constant G. Similarly, one can think of the fine structure constant being a parameter that couples all charged particles[13].

Since the value of α is important for the electronic energy levels in atoms, scientists have wondered what would have happened if its value had been different. In the 1950s, astronomers Fred Hoyle and others worked out the detailed process with which stars produce heavy elements such as carbon, oxygen etc. They found that the abundance of carbon in the Universe could be explained only if the fine structure constant had this value. Hence, Richard Feynmanfamously quoted about α saying, "It's one of the greatest damn mysteries of physics: a magic number that comes to us with no understanding by man. You might say the hand of God wrote that number, and we don't know how He pushed his pencil. We know what kind of a dance to do experimentally to measure this number very accurately, but we don't know what kind of dance to do on the calculation to make this number come out"[14].

Arthur Stanley Eddington (1882–1944) argued that the value of the fine-structure constant, α, could be obtained by pure deduction. He related α to the Eddington number, which was his estimate of the number of protons in the observable universe. This led him in 1929 to conjecture that α was exactly 1/137. Other physicists did not adopt this conjecture and did not accept his argument [15]. In the late 1930s, the best experimental value of the fine-structure constant, α, was approximately 1/136. Eddington then argued, from aesthetic and numerological considerations, that α should be exactly 1/136. He devised a "proof" that $N_{Edd} = 136 \times 2^{256}$ or about 1.5747×10^{79}. Current estimates of N_{Edd} point to a value of about 10^{80} [16]. These estimates assume that all matter can be considered to be hydrogen and require assumed values for the number and size of galaxies and stars in the universe. During a course of lectures that he delivered in 1938 as Lecturer at Trinity College, Cambridge, Eddington averred that: I believe there are 1.5747×10^{79},protons in the observable universe and the same number of electrons [17]. This large number was soon named the "Eddington number". Shortly thereafter, improved measurements of α yielded values closer to 1/137, whereupon Eddington changed his proof to show that α had to be exactly 1/137 [18].

In 2000, Kosinov suggested the more complex but more accurate formula$\alpha^{20} = (\pi.\Phi^{14})^{1/13}.10^{-43}$. He followed the footsteps of two American electrochemists, Lewis and Adams, who proposed back in 1914 that "all of the universalconstants involve only integral numbers and π". After applying cube root to the solution of Stefan-Boltzmann law (as it involves a 3D volume), Lewis derived [19]:

$$\alpha^{-1} = 32\pi\left(\frac{\pi^5}{5!}\right)^{\frac{1}{3}} = 137.35.$$

The Lewis–Adam's conjecture was discussed among physicists. In 1935, Heisenberg wrote to Dirac: "I do not believe at all any more in your conjecture that the Sommerfeld fine-structure constant may have something to do with the concept of temperature; that is, neither do I any more believe in the Lewis value". Indeed, Lewis' value is wrong, but his idea led to another dimensionless constant, involving the continued spectra of blackbody radiation [20]. Heisenberg wrote to Bohr with a joke formula suggested by Lunn in 1922, $\alpha^{-1} = 2^4.3^3/\pi$. Bohr replied, $\alpha^{-1} = 360/\Phi^2$. So back in 1935, after Heisenberg's letter to Dirac came into Pauli's notice,he then suggested that the five-dimensional Kaluza–Klein theory might help to understand the problem[21]. Following Pauli, Wyler came up with another formula in 1969 exposing a similar pattern with the Lewis formula, but in 4th root and in the reciprocal way [22].

$$\alpha^{-1} = \sqrt[4]{\frac{1^0 4^0 5^1 8^0 9^0}{2^{19} 3^7 6^0 7^0 10^0}\pi^{11}}$$

In 1989, Bailey and Ferguson used a supercomputer to check Wyler's formula, and automatically produced several "other relations of comparable complexity with even better accuracy". One example is $\alpha^{-5} = 150\pi(6^5 5^2 \pi^3)^8$, ie, $\alpha^{-1} = 137.036048362143$ [23]. This clearly showed that a Wyler-type formula could not be the unique answer for the fine structure constant. Wyler's formula is later discussed in the E8 lie groups. In 2006, Castro reviewed the coupling constant with the Complex Domains [24]. However, Wyler's work made people devise simpler ways to obtain the magic number, with no more care given to physical dimensional analysis. In this article a similar approach has been followed. Aether Theory in 1972 [25], Stoyan 2004 [26], Heyrovska 2005 [27], Naschia 2006 [28] Gilson 2007 [29], Lestone 2008 [30], Markovich 2009 [31], Rhodes 2010 [32], Kirakosyan 2011 [33], Code 2012 [34], Schonfeld 2013 [35–36], suggests that the pursuit never ended. Nevertheless, among all the approximations, Michael J. Bucknum and Eduardo A. Castro came up with the most elegant solution in last year 2020 [37] with a convergent series, within a few terms, to better than 99999 parts in 100,000 of the true value of α. They suggested:

$$\sqrt{\alpha} = \sum_{n=0}^{\infty}\left(\frac{2n+1}{2n+3}\right)^{2n}\frac{(e\pi)^{n+1}}{10^{4n+2}} = \left(\frac{1}{3}\right)^0 \cdot \frac{(e\pi)^1}{10^2} + \left(\frac{3}{5}\right)^2 \cdot \frac{(e\pi)^2}{10^6} + \left(\frac{5}{7}\right)^4 \cdot \frac{(e\pi)^3}{10^{10}} + \left(\frac{7}{9}\right)^6 \cdot \frac{(e\pi)^4}{10^{14}} + \cdots\infty$$

Even after these countless efforts, Pauli's simplest question still remains unanswered: "Why 137?" [38–42]. In his Nobel Lecture delivered in Stockholm on 13 December 1946, Pauli expressed his goal was to establish a theory, "which will determine the value of the fine-structure constant and will thus explain the atomistic structure of electricity, which is such an essential quality of all atomic sources of electric fields actually occurring in nature" [43]. As the initialization, "from a physical point of view, that the existence of atomicity, in itself so simple and basic, should also be interpreted in a simple and elementary manner by theory and should not, so to speak, appear as a trick in analysis" [43]. His lifelong search for 137, a millennium puzzle, ended in hospital room 137 [44]. The difficulty of finding the correct α formula is partly due to the uncertainty of the experimental values - approximately 137.036. Some experimental data of the inverse of the fine structure constant is listed in the Table below [45–53].

Year	**1/α**	**Source**	**Year**	**1/α**	**Source**
1916	137.360563948	A. Sommerfeld	2000	137.03599976(50)	CODATA 1998
1929	137.29 ± 0.11	R. Birge	2002	137.03599911(46)	CODATA 2002
1930	136.94 ± 0.15	W. Bond	2007	137.035999070(98)	G. Gabrielse
1932	137.305 ± 0.005	R. Birge	2008	137.035999679(94)	CODATA 2006
1935	137.04 ± 0.02	F. Spedding et al.	2008	137.035999084(51)	G GabrielseD Hanneke
1941	137.030 ± 0.016	R. Birge	2010	137.035999037(91)	R. Bouchendira
1943	137.033 ± 0.092	U. Stille	2010	137.03599913296(33)	T. Kinoshita
1949	137.027 ± 0.007	J. DuMond, E. Cohen	2011	137.035999074(44)	CODATA 2010
1949	137.041 ± 0.005	H Bethe, C Longmire	2015	137.035999139(31)	CODATA 2014
1957	137.0371 ± 0.0005	J.Bearden, J.Thomsen	2017	137.035999150(33)	Aoyama et al.

1969	137.03602(21)	CODATA 1969	2018	137.035999046(27)	Parker et al.
1973	137.03612(15)	CODATA 1973	2019	137.035999084(21)	CODATA 2018
1987	137.0359895(61)	CODATA 1986	2020	137.035999206(11)	Morel et al. 2020
1998	137.03599883(51)	T. Kinoshita			

III. Results and Discussion

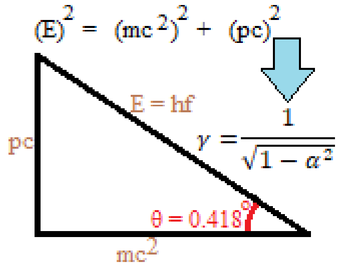
The theory does not predict its value. Therefore, α must be determined experimentally. In fact, α is one of the empirical parameters in the Standard Model of particle physics, whose value is not determined within the Standard Model. The true value of the fine structure constant can be approximatedusing the following mathematical equation:

$$\alpha^{-1} = \sqrt{e^{\pi+\pi\Phi+\Phi}} = \frac{\Phi}{2} + e\pi^3\Phi = \frac{\pi^{12}}{5^3.e^4} + \Phi = (e + \pi + \Phi)^{\sqrt{6}} - 1 = \left(\frac{\Phi\pi}{\log(e)}\right)^2$$

π = 3.14159265358979323846264338327950288419716939937510…

e = 2.71828182845904523536028747135266249775724709369995…

Φ = 1.61803398874989484820458683436563811772030917980 58…



$$tan\theta = \frac{pc}{mc^2} = \frac{p}{mc} = \frac{mv}{mc} = \frac{v}{c} = \alpha$$

The difference between theexperimental value andthe approximated value from the formula above is less than 0.00001.Means, the error between the actual &the calculated value is about + 0.11%. So, $\alpha_{actual} = \alpha_{approx.} \times$ P(– 3.25 <Z< + 3.25).Yet, the measurement of α has a relative standard uncertainty of 2.5×10^{-10}.Also, it is evident from the diagram, θ = $\tan^{-1}(\alpha)$ = 0.418° = 0.007297rad = (α) rad [54].

A simple VISUAL BASIC computer program was written to generate the factors for various particle pairs. The table below shows the largest common factors and multiples for electron-proton bonds that produce fine structure constants within a ±3 standard error window of the measured fine structure constant value. All of the components are considered to be integer. The factor analysis is periodic, with several α candidates appearing within the search window. The factor analysis also shows that the α candidates with the highest common factors, all exhibited the same multiple, 472. This means that at every 472nd electron wave period, the electron and proton total energy waves overlap.

Table: Largest common factors for electron-proton bond & Fine Structure Constant (Brian Dale Nelson) [55]

Proton Mass Component (Np)	Electron Mass Component (Ne)	Proton Electron Mass Ratio (Np/Ne = $6\pi^5$)	Kinetic Energy Component Nv = $1.45\times10^{-8}\times$Np	$\frac{1}{\alpha} = \sqrt{\frac{Ne}{2Nv}}$	Largest Common Factor	Multiple
553900587503	301663688	**1836.15267444121**	8032	137.035999065357	639135	472
434872647725	236839046	**1836.15267444119**	6306	137.035999080312	501791	472
810231325561	441265771	**1836.15267444118**	11749	137.035999110981	934910	472
375358677836	204426725	**1836.15267444117**	5443	137.035999124446	433119	472
691203385783	376441129	**1836.15267444116**	10023	137.035999140230	797566	472
315844707947	172014404	**1836.15267444115**	4580	137.035999158988	364447	472
572175446005	311616487	**1836.15267444113**	8297	137.035999181648	660222	472
828506184063	451218570	**1836.15267444113**	12014	137.035999190287	955997	472
256330738058	139602083	**1836.15267444111**	3717	137.035999209569	295775	472
709478244285	386393928	**1836.15267444110**	10288	137.035999232087	818653	472
453147506227	246791845	**1836.15267444109**	6571	137.035999244824	522878	472
649964274396	353981607	**1836.15267444108**	9425	137.035999258728	749981	472
846781042565	461171369	**1836.15267444107**	12279	137.035999266169	977084	472

$$\alpha = \frac{K_e e^2}{\hbar c} = \frac{\mu_o}{4\pi}\frac{ce^2}{\hbar} = \frac{e^2}{2\varepsilon_o hc} = \frac{\mu_o ce^2}{2h} = \frac{\mu_o c}{2R_K} = \frac{e^2 Z_o}{2h} = \frac{e^2 Z_o}{4\pi\hbar} = \frac{v_e}{c} = \left(\frac{e}{Q_p}\right)^2 = \frac{\lambda_e}{2\pi r_e} = \frac{\lambda_e}{\lambda} = \frac{2^{256}}{N_{Edd}} = \sqrt{\frac{r_0}{r_e}}$$

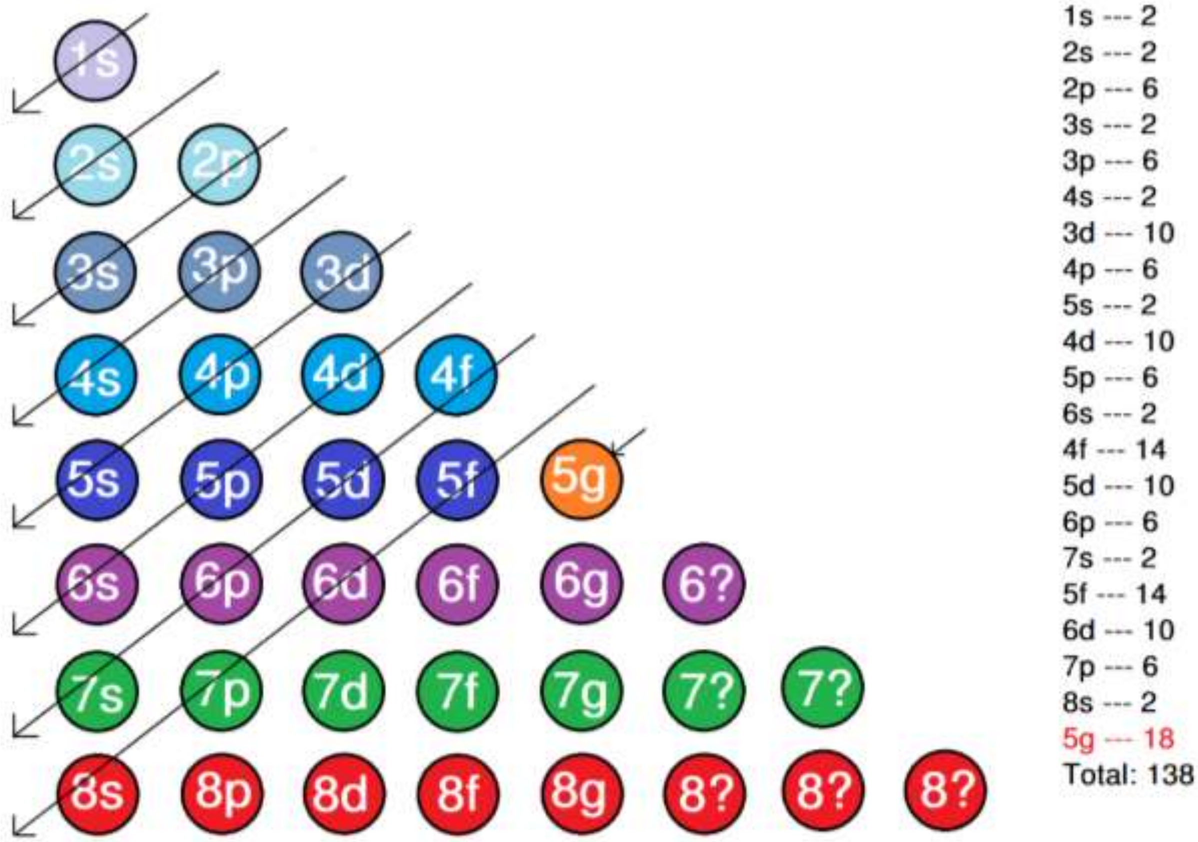
Hemce,

$$\alpha^{-1} = \sqrt{e^{\pi+\pi\Phi+\Phi}} = \frac{\Phi}{2} + e\pi^3\Phi = \frac{\pi^{12}}{5^3.e^4} + \Phi = (e + \pi + \Phi)^{\sqrt{6}} - 1 = \left(\frac{\Phi\pi}{\log(e)}\right)^2$$

IV. Conclusion

To realize the significance of the value of α we need to look into the 137th element of the periodic table. The element is Feynmanium, an undiscovered hypothetical element with the symbol Fy& atomic number 137.It is named in honor of Richard Feynman. The outer most electron of this element of the periodic table is supposed to move nearly at the speed of light. The idea is quite simple, as 1/137 is the odds that an electron will absorb a single photon. Protons and electrons are bound by interactions with photons. So when we get 137 protons, we get 137 photons, and we get (137/137.036)×100% chance of absorption and electron in the ground state is supposed to orbit at the speed of light. This is the electromagnetic equivalent of a black hole. But for the element number 138 the g orbital get fully occupied for the very first time. For this reason it is the most unstable and a temporarily observable hypothetical element. There is (138/137.036)×100% probability that an electron will absorb or emit a photon. As per the Aufbau principal when the g orbital gets fully occupied for the first time then it is supposed to get an atomic number of 138. The maximum occupancy level of s, p, d, f, & g orbitals are 2, 6, 10, 14, & 18 respectively, [2(2n + 1)]; where n = 0, 1, 2, 3, & 4.

According to the Aufbau principal till the element number 120 we do not observe the presence of g orbital. Unbinilium, is the hypothetical chemical element in the periodic table with atomic number of 120. After this hypothetical 120th element for the first time the g orbital comes into existence. Which means, even the g orbital itself is a hypothetical one. When it gets fully occupied with the allotted 18 electrons, then the total number of electrons in the element becomes (120 + 18) = 138. Hence, more than 100% probability that an electron will absorb or emit photon. So, α is directly related to the coupling constant determining the strength of the interaction between electrons–photons.



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