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Doppler Effect without Light Waves

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Abstract: *The Doppler effect or sound and light is a well-known phenomenon: the wave emitted by a source undergoes a frequency change due to the velocity of the source and the observer relative to the medium.For sound, the medium is known: air, fluid, etc. For light, the medium is called the "luminoferous ether". However, according to Special Relativity of Einstein, (SRT) ether is not necessary to be includedin the theory.Therefore, relativistic formulas have been created, which apparently eliminate the carrier medium. The derivation of these formulas is based on a travelling electromagnetic wave which aberrates because of the move of the source or the observer. Thereafter the time dilation factor is applied.*
In this article we derive the relativistic Doppler formulas (both longitudinal and transverse) without referring to EM wave, frequency, wavelength, time dilation or flying photon.
Keywords: *Doppler effect, missing lightwave*

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I. Introduction

For the definition of the Doppler effect it is the simplest way to cite the appropriate entry in Wikipedia:[1]

The Doppler effect or Doppler shift (or simply Doppler, when in context) is the change in frequency of a wave in relation to an observer who is moving relative to the wave source.] It is named after the Austrian physicist Christian Doppler, who described the phenomenon in 1842. A common example of Doppler shift is the change of pitch heard when a vehicle sounding a horn approaches and recedes from an observer. Compared to the emitted frequency, the received frequency is higher during the approach, identical at the instant of passing by, and lower during the recession. The reason for the Doppler effect is that when the source of the waves is moving towards the observer, each successive wave crest is emitted from a position closer to the observer than the crest of the previous wave. Therefore, each wave takes slightly less time to reach the observer than the previous wave. Hence, the time between the arrivals of successive wave crests at the observer is reduced, causing an increase in the frequency. While they are traveling, the distance between successive wave fronts is reduced, so the waves "bunch together". Conversely, if the source of waves is moving away from the observer, each wave is emitted from a position farther from the observer than the previous wave, so the arrival time between successive waves is increased, reducing the frequency. The distance between successive wave fronts is then increased, so the waves "spread out".

The apparent similarity between sound and light waves attracted scientists to apply the for sound developed formulas also for light. [2] However, there is a significant difference: in the case of sound, there is a visible or detectable carrier medium: air, water, etc. For light, no such medium is seen, butsome of us could not imagine the propagation of light withouta light carrying (lumiferous) medium, the "ether". A. Einstein was the first in his seminal paper to detect that the ether was superfluous in compiling the laws of Physics:[3]

"The introduction of a ``luminiferous ether'' will prove to be superfluous inasmuch as the view here to be developed will not require an ``absolutely stationary space'' provided with special properties, nor assign a velocity-vector to a point of the empty space in which electromagnetic processes take place."

Since then, new, ether-centered theories appeared with a "preferred frame" (Mansouri-Sexl, Selleri and others) notion. We do not want to intervene in this debate, we solely ascertain that in order to analyze the Doppler effect no "ether" is needed. (See below, detailed)

II. The Doppler Effect For Sound

In this paper we do not want to describe the whole process of the acoustic Doppler effect, it can be found in any good College Textbook (e.g. [4])Notions used:

$\mathbf{v}$ velocity of sound in the medium

$\mathbf{v_S}$ velocity of the source (also referred to the medium)

$\mathbf{v_O}$ velocity of the observer

$\mathbf{f_S}, \mathbf{f_O}$ sound frequency at the source resp. the observer

It is worthwhile to introduce relative quantities: $\beta_S = \dfrac{\mathbf{v_S}}{\mathbf{v}}$, $\beta_O = \dfrac{\mathbf{v_O}}{\mathbf{v}}$

Remark: in this article we define all velocities in the same direction, as usual in Physics. So, the relative velocity between the source and the observer will be $\mathbf{v_O} - \mathbf{v_S}$, unlike in most textbooks describing the Doppler effect.Of course, this designation does not affect the physical laws.

The usual derivation goes as follows: We imagine a travelling wave emitted by the source and count the wave crests arriving to the observer. (For details see the textbooks.)

Here we just present the derived formulas:

Case 1. Stationary source, moving observer

$$\mathbf{f_O} = \mathbf{f_S}\left(1 - \frac{\mathbf{v_O}}{\mathbf{v}}\right) = \mathbf{f_S}\left(1 - \beta_O\right) \tag{1}$$

As Eq. (1) shows, the frequency shift depends linearly on the observer velocity. If this is negative, the frequency increases.

Case 2. Moving source, stationary observer:

$$\mathbf{f_O} = \mathbf{f_S}\left(\frac{1}{1 - \mathbf{v_S}/\mathbf{v}}\right) = \mathbf{f_S}\left(\frac{1}{1 - \beta_S}\right) \tag{2}$$

As can be seen, the velocity dependence is inverse.Remark: Here we use the velocity directions mentioned above.

For the sake of completeness, let us cite the General Case: both the source and the observer are moving:

$$\mathbf{f_O} = \mathbf{f_S}\left(\frac{1 - \beta_O}{1 - \beta_S}\right) \tag{3}$$

The velocity of sound is not very high, so it is not useful to expand these formulas into series. However, if we take the velocity of light, in SRT we usually round-up the formulas up to second order. (time dilation, length contraction, etc.) Let us rewrite the formulas (1..3) and neglect the third and higher order terms: (ready-to-use examples are found in math tables, e.g.[5])

Case 1: (Eq. 1, moving observer)

$$\frac{\mathbf{f_O}}{\mathbf{f_S}} = \left(1 - \beta_O\right) = 1 - \underline{1} \times \beta_O + \underline{0} \times \beta_O^{\,2} \tag{4}$$

(we intentionally write the coefficients, for easy comparison.)

Case 2: (Eq 2, moving source)

$$\frac{\mathbf{f_O}}{\mathbf{f_S}} = \frac{1}{1 - \beta_S} = 1 + \underline{1} \times \beta_S + \underline{1} \times \beta_S^{\,2} \tag{5}$$

Case 1 and 2 differ in the coefficient of the second order term: it is zero in Case 1, and 1 (one) in Case 2. As we will see, it is 1/2 in the relativistic formula.

General Case (Eq 3)

$$\frac{\mathbf{f_O}}{\mathbf{f_S}} = \left(1 - \beta_O\right)\left(1 + \beta_S + \beta_S^{\,2}\right) = 1 + \beta_S + \beta_S^{\,2} - \beta_O - \beta_O\beta_S - \cancel{\beta_O\beta_S^{\,2}} = 1 + \left(\beta_S - \beta_O\right) + \beta_S\left(\beta_S - \beta_O\right) \tag{6}$$

Eq. (6) shows that the Doppler effect depends on the relative velocity $\left(\beta_S - \beta_O\right) \equiv \left(\mathbf{v_S} - \mathbf{v_O}\right)/\mathbf{v}$ in first order, but the second-order terms contain absolute velocities. In other words, it is not ether-free. No wonder, because these formulas have been developed by assuming a wave-carrying medium.

Remark: the concept "ether-free" means that a quantity is a function of the velocity differences only, not separatedly.

III. Doppler Effect For Light (Usual Approach)

If the existence of the ether was reality, the formulas (1..6) could automatically be applied for light, too, only the velocity in medium (**v**) should be replaced by the usual **c**.

However, SRT requires a formula, where only relative velocities appear. We do not show here the derivation of the "relativistic" Doppler formula, it can be found in textbooks (see for example D. Morin [6]) We only mention the method of the derivation: one of the participants is treated as "stationary", then we calculate the time dilation at the "moving" one.

But one may ask: "stationary in what?" The answer is: " In an inertial frame, where the source or observer is in rest".But the "frame" is only a mathematical tool, without physical meaning, as it was pointed out by the present author.[7] So, the relativistic Doppler effect is usually derived by the following way:

- We temporarilybelieve in ether, and appoint "stationary" and "moving" objects
- We derive the necessary equations, using the acoustic formulas
- Thereafter, we clear away the traces of the ether, by applying time dilation. By this way, the absolute, ether-bound velocities become relative velocities.

The accepted formula for the relativistic Doppler effect is the following: [6]

$$\frac{\mathbf{f_O}}{\mathbf{f_S}} = \sqrt{\frac{1+\mathbf{v_r}/\mathbf{c}}{1-\mathbf{v_r}/\mathbf{c}}} \tag{7}$$

where $\mathbf{v_r}$ is the relative velocity between the source and the observer. It is positive, if the source and the observer are approaching. We introduce a new variable, on example of (1..6): $\beta_r = \mathbf{v_r}/\mathbf{c}$ Now we rewrite Eq. (7)

$$\frac{\mathbf{f_O}}{\mathbf{f_S}} = \sqrt{\frac{1+\beta_r}{1-\beta_r}} \tag{8}$$

Similarly to the equations above, we expand Eq. (8) into McLaurin series: (see Appendix A)

$$\frac{\mathbf{f_O}}{\mathbf{f_S}} = \sqrt{\frac{1+\beta_r}{1-\beta_r}} = 1+\beta_r+\frac{1}{2}\beta_r^{\,2} \tag{9}$$

As mentioned, the starting step in the derivation is to assign a "stationary" and a "moving" component, as found in SRT textbooks.However, as we derive in Appendix C, the (9) formula is valid for two arbitrary velocities, none of the participants needs to be stationary.This fact may be new for some readers.

IV. Doppler Effect For Light Without Waves (Our Approach)

At this point, we have repeated all the important knowledge about sound and light Doppler effect. We saw that the derivation of the Doppler formula is based on the investigation of wave propagation. In case of light, no "flying" photon or "traveling wave" can be observed, because if we try to observe them, we destroy them at the same time. This fact suggests that the phenomena of light, Doppler effect included, could be treated without wavelength, frequency or "photon" considering the processes at the source and the observer only. This is the reason that some scientists deny the photon's existence. Their question: "Who has seen a flying photon?"

In the followings, we develop the relativistic Doppler effect using momentum and energy conservation. This method, i.e. developing the equations for the Doppler effect with momentum and energy conservation can already be found in the scientific literature, [8,9,10] However, in this paper we develop the relativistic formulas too, omitting waves, frequency, wavelength etc. We use the equations of Classical (i.e. Nonrelativistic, Nonquantum) Mechanics, with two exceptions:

- We accept the $\mathbf{E}=\mathbf{mc}^2$ relation, which is proven in neutron capture experiments
- We consider that during the light emission/detection the ratio of the transmitted energy-momentum is **c**, the light velocity.

The processes:

a. Emission:

Variables used:

- $\mathbf{M_S}$ mass of the source.
- $\mathbf{v_S}$ velocity of the source prior to emission
- $\mathbf{w_S}$ change in the source velocity due emission
- $\mathbf{m_S}$ mass defect of the source due to atomic energy level change
- $\mathbf{h}\nu/\mathbf{c}$ momentum transferred to the observer (ν is only a parameter, not necessary a frequency!)
- $\mathbf{h}\nu$ energy transferred to the observer

Momentum conservation

$$\left(\mathbf{M_S}-\mathbf{m_S}\right)\left(\mathbf{v_S}+\mathbf{w_S}\right)+\frac{\mathbf{h}\nu}{\mathbf{c}}=\mathbf{M_S v_S}\Rightarrow \mathbf{M_S w_S}-\mathbf{m_S v_S}\cancel{-\mathbf{m_S w_S}}+\frac{\mathbf{h}\nu}{\mathbf{c}}=0 \tag{10}$$

We neglected the $\mathbf{m_S w_S}$ term, because it disappears when $\frac{\mathbf{m}}{\mathbf{M_S}}\to 0$.

<u>Remark</u>: It is sure that Eq (10) would be interesting if we do not neglect this term. However, we want to develop a formula for the Doppler effect, where the change in source kinetic energy due to emission is negligible. (Large atom or Mössbauer effect)

Rewriting (10)

$$\mathbf{M_S w_S}-\mathbf{m_S v_S}+\frac{\mathbf{h}\nu}{\mathbf{c}}=0 \tag{11}$$

Eq. (11) is the main equation for momentum conservation at the source.

Energy conservation

$$\begin{gathered}\frac{1}{2}\left(\mathbf{M_S}-\mathbf{m_S}\right)\left(\mathbf{v_S}+\mathbf{w_S}\right)^2+\mathbf{h}\nu=\mathbf{m_S c}^2+\frac{1}{2}\mathbf{M_S v_S}^2\Rightarrow\\ \frac{1}{2}\left(2\mathbf{M_S v_S w_S}+\cancel{\mathbf{M_S w_S}^2}-\mathbf{m_S v_S}^2-\cancel{2\mathbf{m_S v_S w_S}}-\cancel{\mathbf{m_S v_S}^2}\right)+\mathbf{h}\nu=\mathbf{m_S c}^2\end{gathered} \tag{12}$$

Cleared, rearranged

$$\mathbf{m_S c}^2=\left(\mathbf{M_S w_S}\right)\mathbf{v_S}-\frac{1}{2}\mathbf{m_S v_S}^2+\mathbf{h}\nu \tag{13}$$

Eq. (11) repeated, rearranged

$$\mathbf{M_S w_S}=\mathbf{m_S v_S}-\frac{\mathbf{h}\nu}{\mathbf{c}} \tag{14}$$

Inserting Eq. (14) into Eq. (13), we get

$$\mathbf{m_S c}^2=\left(\mathbf{m_S v_S}-\frac{\mathbf{h}\nu}{\mathbf{c}}\right)\mathbf{v_S}-\frac{1}{2}\mathbf{m_S v_S}^2+\mathbf{h}\nu\Rightarrow\mathbf{m_S}\left(\mathbf{c}^2-\frac{1}{2}\mathbf{v_S}^2\right)=\mathbf{h}\nu\left(1-\frac{\mathbf{v_S}}{\mathbf{c}}\right) \tag{15}$$

Introducing the following notations: $\frac{\mathbf{v_S}}{\mathbf{c}}\equiv\beta_S$, $\quad \mathbf{E_S}=\mathbf{m_S c}^2$

With these, rewriting (15)

$$\mathbf{E_S}\left(1-\frac{1}{2}\beta_S^2\right)=\mathbf{h}\nu\left(1-\beta_S\right)\Rightarrow\mathbf{h}\nu=\mathbf{E_S}\frac{1-\frac{1}{2}\beta_S^2}{1-\beta_S} \tag{16}$$

b. Now we write up the similar equations for the <u>observer.</u>Occasionally,we do not write all the neglectable terms.

Momentum conservation

$$\mathbf{M_O v_O}+\frac{\mathbf{h}\nu}{\mathbf{c}}=\left(\mathbf{M_O}+\mathbf{m_O}\right)\left(\mathbf{v_O}+\mathbf{w_O}\right)\Rightarrow\mathbf{M_O w_O}+\mathbf{m_O v_O}=\frac{\mathbf{h}\nu}{\mathbf{c}} \tag{17}$$

Rewriting:

$$\mathbf{M_O w_O}=\frac{\mathbf{h}\nu}{\mathbf{c}}-\mathbf{m_O v_O} \tag{18}$$

Energy conservation

$$\frac{1}{2}\mathbf{M_O v_O}^2+\mathbf{h}\nu=\mathbf{m_O c}^2+\frac{1}{2}\left[\left(\mathbf{M_O}+\mathbf{m_O}\right)\left(\mathbf{v_O}+\mathbf{w_O}\right)^2\right] \tag{19}$$

Unfolded

$$\mathbf{h}\nu==\mathbf{m_O c}^2+\frac{1}{2}\left[2\mathbf{M_O v_O w_O}+\cancel{\mathbf{M_O w_O}^2}+\mathbf{m_O v_O}^2+\cancel{2\mathbf{m_O v_O w_O}}+\cancel{\mathbf{m_O w_O}^2}\right] \tag{20}$$

Cleared, omitting the neglectable terms

$$\mathbf{h}\nu = \mathbf{m_O c}^2 + \left(\mathbf{M_O w_O}\right)\mathbf{v_O} + \frac{1}{2}\mathbf{m_O v_O}^2 \tag{21}$$

Inserting Eq. (18) into (21)

$$\mathbf{h}\nu = \mathbf{m_O c}^2 + \left(\frac{\mathbf{h}\nu}{\mathbf{c}} - \mathbf{m_O v_O}\right)\mathbf{v_O} + \frac{1}{2}\mathbf{m_O v_O}^2 \Rightarrow \mathbf{m_O}\left(\mathbf{c}^2 - \frac{1}{2}\mathbf{v_O}^2\right) = \mathbf{h}\nu\left(1 - \frac{\mathbf{v_O}}{\mathbf{c}}\right) \tag{22}$$

Introducing the following notations: $\frac{\mathbf{v_O}}{\mathbf{c}} \equiv \beta_\mathbf{O}$, $\mathbf{E_O} = \mathbf{m_O c}^2$

$$\mathbf{E_O}\left(1 - \frac{1}{2}\beta_\mathbf{O}^2\right) = \mathbf{h}\nu\left(1 - \beta_\mathbf{O}\right) \Rightarrow \mathbf{h}\nu = \mathbf{E_O}\frac{1 - \frac{1}{2}\beta_\mathbf{O}^2}{1 - \beta_\mathbf{O}} \tag{23}$$

Let us compare Eqs. (16) and (23) utilizing the fact that $\mathbf{h}\nu$ is the same in both equations.

$$\frac{\mathbf{E_O}}{\mathbf{E_S}} = \frac{\dfrac{1 - \frac{1}{2}\beta_\mathbf{S}^2}{1 - \beta_\mathbf{S}}}{\dfrac{1 - \frac{1}{2}\beta_\mathbf{O}^2}{1 - \beta_\mathbf{O}}} = \frac{\left(1 - \beta_\mathbf{O}\right)\left(1 - \frac{1}{2}\beta_\mathbf{S}^2\right)}{\left(1 - \beta_\mathbf{S}\right)\left(1 - \frac{1}{2}\beta_\mathbf{O}^2\right)} \tag{24}$$

Expanding Eq. (24) up to 2[nd] order:

For better readability, we temporarily rename the variables: $\mathbf{x} \to \beta_\mathbf{S}$, $\mathbf{y} \to \beta_\mathbf{O}$

$$\frac{\mathbf{E_O}}{\mathbf{E_S}} = \frac{(1-\mathbf{y})}{(1-\mathbf{x})}\frac{\left(1 - \frac{1}{2}\mathbf{x}^2\right)}{\left(1 - \frac{1}{2}\mathbf{y}^2\right)} \approx \left(1 - \mathbf{y} - \frac{1}{2}\mathbf{x}^2 + \cancel{\frac{1}{2}\mathbf{x}^2\mathbf{y}}\right)\left(1 + \mathbf{x} + \mathbf{x}^2\right)\left(1 + \frac{1}{2}\mathbf{y}^2\right) =$$

$$\left(1 - \mathbf{y} - \frac{1}{2}\mathbf{x}^2 + \mathbf{x} - \mathbf{xy} + \mathbf{x}^2\right)\left(1 + \frac{1}{2}\mathbf{y}^2\right) = \left(1 - \mathbf{y} - \frac{1}{2}\mathbf{x}^2 + \mathbf{x} - \mathbf{xy} + \mathbf{x}^2 + \frac{1}{2}\mathbf{y}^2\right) = \tag{25}$$

$$1 - (\mathbf{y} - \mathbf{x}) + \frac{1}{2}\mathbf{x}^2 + \frac{1}{2}\mathbf{y}^2 - \mathbf{xy} = 1 + \left(\mathbf{x} - \mathbf{y}\right) + \frac{1}{2}\left(\mathbf{x} - \mathbf{y}\right)^2$$

Replacing the variables

$$\frac{\mathbf{E_O}}{\mathbf{E_S}} = 1 + \left(\beta_\mathbf{S} - \beta_\mathbf{O}\right) + \frac{1}{2}\left(\beta_\mathbf{S} - \beta_\mathbf{O}\right)^2 \tag{26}$$

We notice that the expression $\beta_\mathbf{S} - \beta_\mathbf{O}$ is proportional to the relative velocity between the source and the observer:

$$\beta_\mathbf{S} - \beta_\mathbf{O} = \frac{\mathbf{v_S} - \mathbf{v_O}}{\mathbf{c}} = \frac{\mathbf{v_r}}{\mathbf{c}} = \beta_\mathbf{r} \tag{27}$$

Inserting (27) into (26), we get, with help of Appendix A

$$\frac{\mathbf{E_O}}{\mathbf{E_S}} = 1 + \beta_\mathbf{r} + \frac{1}{2}\left(\beta_\mathbf{r}\right)^2 = \sqrt{\frac{1 + \beta_\mathbf{r}}{1 - \beta_\mathbf{r}}} \tag{28}$$

As can be see, the energy ratio is ether-free up to second order and equals to the traditional relativistic formula, Eq (9)

If we observe a “light wave”, we never perceive the frequency, but only the atom energy level change. Therefore, in the case of the Doppler effect for light, the ratio of the atomic level change observer/source is the real indicator of Doppler aberration. Therefore, Eq. (9) and (28) are equivalent.

The main result: we derived the formula for the relativistic Doppler effect without*light waves, flying photons and time dilation*. As seen, the formula contains the relative, “ether-free” velocities up to second order.

An interesting question arises: if we neglect the terms emerging from the mass defect or gain, $\mathbf{m_S v_S}^2$ in Eq (12) and $\mathbf{m_O v_O}^2$ in Eq (20) we would obtain the nonrelativistic Doppler equation. This fact shows that a strong correspondence exists between the mass-energy equivalence and time dilation in the case of light emission/detection,

V. Transverse Doppler Effect

Unlike longitudinal Doppler effect, with transverse Doppler effect the source and the observer approaching each other in uniform inertial motion along paths that do not collide. According to the opinion of most scientist, Doppler effect in classical physics disappears at perpendicular light transmission. It is only in SRT, according to them. The derivation is made by supposing one of the participants to be stationary, then apply time dilation for the moving part.

Here we prove that Doppler effect occurs also in perpendicular transmission,according to SRT. However, similarly to the above derivations, we do not involve light waves.

Let us suppose, that source and observer are moving in the x direction, but in parallel lines. Let the light transmission occur at angle of θ.The parameters:

- $\mathbf{M_S}$ mass of the source
- $\mathbf{m_S}$ mass loss due to emission
- $\mathbf{v_{Sx}}$ its velocity before emission (no **y** component)
- $\mathbf{w_{Sx}}$, $\mathbf{w_{Sy}}$ velocity change in **x** resp.**y** direction
- $\mathbf{h}\nu$ energy transferred to the observer

For the Observer, we use similar notations.

The derivation is very similar to the longitudinal case. Therefore, we advise he hurried reader to continue at Eq (46)

a. <u>Source</u>

Momentum Conservation. x direction

$$\mathbf{Mv_{Sx}} = (\mathbf{M_S} - \mathbf{m_S})(\mathbf{v_{Sx}} + \mathbf{w_{Sx}}) + \frac{\mathbf{h}\nu}{\mathbf{c}}\cos\theta \tag{29}$$

Unfolded

$$\cancel{\mathbf{Mv_{Sx}}} = \cancel{\mathbf{M_S v_{Sx}}} + \mathbf{M_S w_{Sx}} - \mathbf{m_S v_{Sx}} - \mathbf{m_S w_{Sx}} + \frac{\mathbf{h}\nu}{\mathbf{c}}\cos\theta \tag{30}$$

Expressed for $\mathbf{M_S w_{Sx}}$

$$\mathbf{M_S w_{Sx}} = \mathbf{m_S v_{Sx}} - \frac{\mathbf{h}\nu}{\mathbf{c}}\cos\theta \tag{31}$$

Momentum conservation in y direction

$$0 = (\mathbf{M_S} - \mathbf{m_S})\mathbf{w_{Sy}} + \frac{\mathbf{h}\nu}{\mathbf{c}}\sin\theta \tag{32}$$

Energy conservation

$$\mathbf{m_S c}^2 + \frac{1}{2}\mathbf{M_S v_{Sx}}^2 = \frac{1}{2}(\mathbf{M_S} - \mathbf{m_S})\left[(\mathbf{v_{Sx}} + \mathbf{w_{Sx}})^2 + \mathbf{w_{Sy}}^2\right] + \mathbf{h}\nu \tag{33}$$

Unfolded

$$\begin{aligned} &\mathbf{m_S c}^2 + \frac{1}{2}\mathbf{M_S v_{Sx}}^2 = \frac{1}{2}(\mathbf{M_S} - \mathbf{m_S})\left[(\mathbf{v_{Sx}} + \mathbf{w_{Sx}})^2 + \mathbf{w_{Sy}}^2\right] + \mathbf{h}\nu = \\ &\frac{1}{2}\mathbf{M_S}\left(\mathbf{v_{Sx}}^2 + 2\mathbf{v_{Sx}w_{Sx}} + \mathbf{w_{Sx}}^2 + \mathbf{w_{Sy}}^2\right) - \frac{1}{2}\mathbf{m_S}\left(\mathbf{v_{Sx}}^2 + 2\mathbf{v_{Sx}w_{Sx}} + \mathbf{w_{Sx}}^2 + \mathbf{w_{Sy}}^2\right) = \\ &\frac{1}{2}\mathbf{M_S v_{Sx}}^2 + \mathbf{Mv_{Sx}w_{Sx}} - \frac{1}{2}\mathbf{m_S v_{Sx}}^2 \end{aligned} \tag{34}$$

Get $\mathbf{M_S w_{Sx}}$ from (31)

$$\mathbf{m_S c}^2 = \mathbf{v_{Sx}}\left(\mathbf{m_S v_{Sx}} - \frac{\mathbf{h}\nu}{\mathbf{c}}\cos\theta\right) - \frac{1}{2}\mathbf{m_S v_{Sx}}^2 + \mathbf{h}\nu \Rightarrow$$
$$\mathbf{m_S c}^2 = -\mathbf{h}\nu\frac{\mathbf{v_{Sx}}}{\mathbf{c}}\cos\theta + \frac{1}{2}\mathbf{m_S v_{Sx}}^2 + \mathbf{h}\nu \tag{35}$$

Rearranged

$$\mathbf{m_S}\left(\mathbf{c}^2 - \frac{1}{2}\mathbf{v_{Sx}}^2\right) = \mathbf{h}\nu\left(1 - \frac{\mathbf{v_{Sx}}}{\mathbf{c}}\cos\theta\right) \Rightarrow \mathbf{h}\nu = \frac{\mathbf{m_S}\left(\mathbf{c}^2 - \frac{1}{2}\mathbf{v_{Sx}}^2\right)}{1 - \frac{\mathbf{v_{Sx}}}{\mathbf{c}}\cos\theta} = \mathbf{m_S c}^2\frac{\left(1 - \frac{1}{2}\beta_S^2\right)}{1 - \beta_S\cos\theta} \tag{36}$$

b. Equations for the <u>Observer</u>

Momentum Conservation. x direction

$$\mathbf{M v_{Ox}} + \frac{\mathbf{h}\nu}{\mathbf{c}}\cos\theta = \left(\mathbf{M_O} + \mathbf{m_O}\right)\left(\mathbf{v_{Ox}} + \mathbf{w_{Ox}}\right) \tag{37}$$

Unfolded

$$\cancel{\mathbf{M v_{Ox}}} + \frac{\mathbf{h}\nu}{\mathbf{c}}\cos\theta = \cancel{\mathbf{M_O v_{Ox}}} + \mathbf{M_O w_{Ox}} + \mathbf{m_O v_{Ox}} \tag{38}$$

Expressed to $\mathbf{M_O w_{Ox}}$

$$\mathbf{M_O w_{Ox}} = \frac{\mathbf{h}\nu}{\mathbf{c}}\cos\theta - \mathbf{m_O v_{Ox}} \tag{39}$$

Momentum conservation in y direction

$$\frac{\mathbf{h}\nu}{\mathbf{c}}\sin\theta = \left(\mathbf{M_O} + \mathbf{m_O}\right)\mathbf{w_{Oy}} \tag{40}$$

Energy Conservation

$$\mathbf{h}\nu + \frac{1}{2}\mathbf{M_O v_{Ox}}^2 = \mathbf{m_O c}^2 + \frac{1}{2}\left(\mathbf{M_O} + \mathbf{m_O}\right)\left[\left(\mathbf{v_{Ox}} + \mathbf{w_{Ox}}\right)^2 + \mathbf{w_{Oy}}^2\right] \tag{41}$$

Unfolded

$$\mathbf{h}\nu + \frac{1}{2}\mathbf{M_O v_{Ox}}^2 = \mathbf{m_O c}^2 + \frac{1}{2}\mathbf{M_O}\left[\left(\mathbf{v_{Ox}} + \mathbf{w_{Ox}}\right)^2 + \mathbf{w_{Oy}}^2\right] + \frac{1}{2}\mathbf{m_O}\left[\left(\mathbf{v_{Ox}} + \mathbf{w_{Ox}}\right)^2 + \mathbf{w_{Oy}}^2\right] =$$
$$\frac{1}{2}\mathbf{M_O}\left(\mathbf{v_{Ox}}^2 + 2\mathbf{v_{Ox}w_{Ox}} + \mathbf{w_{Ox}}^2\right) + \frac{1}{2}\mathbf{m_O}\left(\mathbf{v_{Ox}}^2 + 2\mathbf{v_{Ox}w_{Ox}} + \mathbf{w_{Ox}}^2\right) \Rightarrow \tag{42}$$
$$\mathbf{h}\nu = \mathbf{m_O c}^2 + \mathbf{M_O v_{Ox} w_{Ox}} + \frac{1}{2}\mathbf{m_O v_{Ox}}^2$$

Insert $\mathbf{M_O w_{Ox}}$ from (39)

$$\mathbf{h}\nu = \mathbf{m_O c}^2 + \left(\frac{\mathbf{h}\nu}{\mathbf{c}}\cos\theta - \mathbf{m_O v_{Ox}}\right)\mathbf{v_{Ox}} + \frac{1}{2}\mathbf{m_O v_{Ox}}^2 \Rightarrow \mathbf{h}\nu = \mathbf{m_O c}^2 + \frac{\mathbf{v_{Ox} h}\nu}{\mathbf{c}}\cos\theta - \frac{1}{2}\mathbf{m_O v_{Ox}}^2 \tag{43}$$

Expressed for $\mathbf{h}\nu$

$$\mathbf{h}\nu\left(1 - \frac{\mathbf{v_{Ox}}}{\mathbf{c}}\cos\theta\right) = \frac{1}{2}\mathbf{m_O v_{Ox}}^2 \Rightarrow \mathbf{h}\nu = \frac{\mathbf{m_O c}^2 - \frac{1}{2}\mathbf{m_O v_{Ox}}^2}{1 - \frac{\mathbf{v_{Ox}}}{\mathbf{c}}\cos\theta} = \mathbf{E_O}\frac{1 - \frac{1}{2}\beta_O^2}{1 - \beta_O\cos\theta} \tag{44}$$

Rewriting

$$\mathbf{h}\nu = \mathbf{E}_s \frac{\left(1-\frac{1}{2}\beta_s{}^2\right)}{1-\beta_s\cos\theta} = \mathbf{E}_o \frac{1-\frac{1}{2}\beta_o{}^2}{1-\beta_o\cos\theta} \tag{45}$$

Remember: $\mathbf{E}_s = \mathbf{m}_s\mathbf{c}^2$, $\mathbf{E}_o = \mathbf{m}_o\mathbf{c}^2$

Rearranging, using the $\mathbf{h}\nu$ equality

$$\mathbf{h}\nu = \mathbf{E}_s \frac{\left(1-\frac{1}{2}\beta_s{}^2\right)}{1-\beta_s\cos\theta} = \mathbf{E}_o \frac{1-\frac{1}{2}\beta_o{}^2}{1-\beta_o\cos\theta} \Rightarrow \frac{\mathbf{E}_o}{\mathbf{E}_s} = \frac{\frac{\left(1-\frac{1}{2}\beta_s{}^2\right)}{1-\beta_s\cos\theta}}{\frac{1-\frac{1}{2}\beta_o{}^2}{1-\beta_o\cos\theta}} = \frac{\left(1-\frac{1}{2}\beta_s{}^2\right)\left(1-\beta_o\cos\theta\right)}{\left(1-\frac{1}{2}\beta_o{}^2\right)\left(1-\beta_s\cos\theta\right)} \tag{46}$$

Expanding into series

$$\frac{\mathbf{E}_o}{\mathbf{E}_s} = \frac{\left(1-\beta_o\cos\theta\right)\left(1-\frac{1}{2}\beta_s{}^2\right)}{\left(1-\beta_s\cos\theta\right)\left(1-\frac{1}{2}\beta_o{}^2\right)} = 1-\beta_o\cos\theta+\beta_s\cos\theta+\beta_s{}^2\left(\cos^2\theta-\frac{1}{2}\right)-\beta_s\beta_o\cos^2\theta+\frac{1}{2}\beta_o{}^2 \tag{47}$$

for the general case. (β_s and β_o are independent, the MacLaurin expansion is in Appendix B)

Different special cases

Eq (47) contains the velocities in a very asymmetric form. it is clearly not ether-free.This fact needs further investigation (not here). However, it may be interesting to simplify the initial condition by assuming one of the participants to be stationary, as can be found in textbooks. (e. g. [11])

Stationary observer, θ arbitrary

$$\frac{\mathbf{E}_o}{\mathbf{E}_s} = \frac{\left(1-\frac{1}{2}\beta_s{}^2\right)}{\left(1-\beta_s\cos\theta\right)} = \frac{\sqrt{1-\beta_s{}^2}}{\left(1-\beta_s\cos\theta\right)} = \frac{1}{\gamma_s\left(1-\beta_s\cos\theta\right)} \tag{48}$$

Here we used expansion to second order.

We can further simplify the equation by taking perpendicular light transmission $\left(\cos\beta = 0\right)$

Stationary observer, $\beta_o = 0,\ \cos\theta = 0$ “visually closest approach”, redshift.

$$\frac{\mathbf{E}_o}{\mathbf{E}_s} = \frac{1-\frac{1}{2}\beta_s{}^2}{} = \sqrt{1-\beta_s{}^2} = \frac{1}{\gamma_s} \tag{49}$$

Stationary source $\beta_s = 0$

$$\frac{\mathbf{E}_o}{\mathbf{E}_s} = \frac{\left(1-\beta_o\cos\theta\right)}{\left(1-\frac{1}{2}\beta_o{}^2\right)} = \left(1-\beta_o\cos\theta\right)\frac{1}{\sqrt{1-\beta_o{}^2}} = \gamma_o\left(1-\beta_o\cos\theta\right) \tag{50}$$

This is the usual formula found in SRT textbooks.

With perpendicular transmission:

Stationary observer $\beta_s = 0,\ \cos\theta = 0$, „geometrically closest point”, blueshift.

$$\frac{\mathbf{E}_o}{\mathbf{E}_s} = \frac{1}{\left(1-\frac{1}{2}\beta_o{}^2\right)} = \frac{1}{\sqrt{1-\beta_o{}^2}} = \gamma_o \tag{51}$$

Zero Doppler effect.

It is usual to mention the case when no Doppler effect occurs. It is somewhere between the geometrically and visually closest approach. It is easy from Eq (46) that this case occurs when $\mathbf{v_S}^2 = \mathbf{v_O}^2$, with perpendicular transmission.

Remark: When we say that the velocity of the source or the observer is zero, it does not mean that we apply its frame.

VI. Conclusions

- We proved that the relativistic Doppler effect for light can be derived without waves and spacetime by using momentum and energy conservation and mass-energy equivalence.
- Our result is ether-free up to second order for the longitudinal Doppler effect
- It is shown that the mass-energy equivalence causes similar effect as the time dilation for light
- The transverse Doppler effect can be derived, too. However, one of the transmission participants must be stationary, as in textbooks
- The article raises the probability for the nonexistence of light waves and free photon, and the impossibility of detectionless light emission.

Appendix A

MacLaurin expansion of the $\sqrt{\frac{1+\mathbf{x}}{1-\mathbf{x}}}$ function up to second order

$$\sqrt{1+\mathbf{x}} = 1+\frac{1}{2}\mathbf{x}-\frac{1}{8}\mathbf{x}^2 \quad [5]$$

$$\frac{1}{\sqrt{1-\mathbf{x}}} = 1+\frac{1}{2}\mathbf{x}+\frac{3}{8}\mathbf{x}^2$$

$$\sqrt{\frac{1+\mathbf{x}}{1-\mathbf{x}}} = \left(1+\frac{1}{2}\mathbf{x}-\frac{1}{8}\mathbf{x}^2\right)\left(1+\frac{1}{2}\mathbf{x}+\frac{3}{8}\mathbf{x}^2\right) = 1+\frac{1}{2}\mathbf{x}-\frac{1}{8}\mathbf{x}^2+\frac{1}{2}\mathbf{x}\left(1+\frac{1}{2}\mathbf{x}\right)+\frac{3}{8}\mathbf{x}^2 =$$

$$1+\mathbf{x}+\mathbf{x}^2\left(-\frac{1}{8}+\frac{1}{4}+\frac{3}{8}\right) = \underline{\underline{1+\mathbf{x}+\frac{1}{2}\mathbf{x}^2}}$$

Appendix B.

Transverse Doppler Effect derivation. Expanding up to second order.

$$\frac{\mathbf{E_O}}{\mathbf{E_S}} = \frac{(1-\beta_O\cos\theta)\left(1-\frac{1}{2}\beta_S^2\right)}{(1-\beta_S\cos\theta)\left(1-\frac{1}{2}\beta_O^2\right)} =$$

$$(1-\beta_O\cos\theta)\left(1-\frac{1}{2}\beta_S^2\right)\left(1+\frac{1}{2}\beta_O^2\right)\left(1+\beta_S\cos\theta+\beta_S^2\cos^2\theta\right) =$$

$$\left(1-\beta_O\cos\theta-\frac{1}{2}\beta_S^2\right)\left(1+\frac{1}{2}\beta_O^2\right)\left(1+\beta_S\cos\theta+\beta_S^2\cos^2\theta\right) =$$

$$= \left(1-\beta_O\cos\theta-\frac{1}{2}\beta_S^2+\frac{1}{2}\beta_O^2\right)\left(1+\beta_S\cos\theta+\beta_S^2\cos^2\theta\right) =$$

$$1-\beta_O\cos\theta-\frac{1}{2}\beta_S^2+\frac{1}{2}\beta_O^2+\beta_S\cos\theta-\beta_S\beta_O\cos^2\theta+\beta_S^2\cos^2\theta =$$

$$1-\beta_O\cos\theta+\beta_S\cos\theta+\beta_S^2\left(\cos^2\theta-\frac{1}{2}\right)-\beta_S\beta_O\cos^2\theta+\frac{1}{2}\beta_O^2$$

Appendix C.

Relativistic Doppler effect. Traditional derivation with two independent velocities.

a. moving source

$$\frac{1}{1-\beta_s}\times\sqrt{1-\beta_s^{\,2}}=\frac{\sqrt{1-\beta_s}\sqrt{1+\beta_s}}{1-\beta_s}=\sqrt{\frac{1+\beta_s}{1-\beta_s}}$$

b. moving observer

$$\left(1+\beta_o/\mathbf{c}\right)\times\frac{1}{\sqrt{1-\beta_o^{\,2}}}=\frac{1+\beta_o}{\sqrt{1-\beta_o}\sqrt{1+\beta_o}}=\sqrt{\frac{1+\beta_o}{1-\beta_o}}$$

c. Combined: both are moving

$$\frac{\left(1-\beta_o\right)}{\sqrt{1-\beta_o^{\,2}}}\frac{\sqrt{1-\beta_s^{\,2}}}{1-\beta_s}=\frac{\left(1-\beta_o\right)}{\sqrt{1-\beta_o}\sqrt{1+\beta_o}}\frac{\sqrt{1-\beta_s}\sqrt{1+\beta_s}}{1-\beta_s}=\frac{\sqrt{1-\beta_o}}{\sqrt{1+\beta_o}}\cdot\frac{\sqrt{1+\beta_s}}{\sqrt{1-\beta_s}}=$$

$$\left(1-\beta_o+\frac{1}{2}\beta_o^{\,2}\right)\left(1+\beta_s+\frac{1}{2}\beta_s^{\,2}\right)=1-\beta_o+\frac{1}{2}\beta_o^{\,2}+\beta_s-\beta_s\beta_o+\frac{1}{2}\beta_s^{\,2}=1-\left(\beta_s-\beta_o\right)+\frac{1}{2}\left(\beta_s-\beta_o\right)^2$$

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