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The spiral Hall's effect

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Abstract: *In this paper the Hall's effect in spiral geometry is studied, i.e. the formula of the voltage produced by a magnetic field perpendicular to a spiral metal sheet in which an electric current flows is determined. The mathematical relationship between the potential and the electric field is discussed by solving the Laplace equation in spiral coordinates together with the boundary conditions.*
Since both rectangular and circular geometries are limiting cases of spiral geometry, the spiral Hall's effect is a generalization of the homonymous effect discovered by Hall in 1879 and of the circular Corbino's effect.
At the end of the paper it will be clear that spiral geometry is one of the best choices for designing a long stripe Hall's effect device in a small space.

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I. Background:

The discovery of the Hall's effect dates back to the second half of the nineteenth century when classical electromagnetism was in full development. The theory arises from a critique of the theories supported by Clerk Maxwell on the interaction between magnets and charges [1] contained in conductors defended by Edwin Herbert Hall in his research under the supervision of Prof. Henry Rowland at Johns Hopkins University in Baltimore [2].

In reality the effect named after Hall had already been the subject of discussion in the scientific circles of the time and Prof. Wiedemann published a paper in 1872 stating that it did not exist [3].

However, Hall with great intuition was able to demonstrate the effect that now bears his name through a modification to an experiment carried out by Prof. Rowland, simply using very thin gold leaf instead of thick brass and copper plates [2]. Later Hall carried out the same experiment with long thin leaves of gold, silver, platinum, nickel, tin, reporting the results in 1880 [4].

A few years later, Prof. O.M.Corbino [5],[6] applied a magnetic field on a disk with two circular electrodes, obtaining results similar to those obtained by Hall but in circular rather than rectangular geometry.

Since both rectangular and circular geometry are limiting cases of spiral geometry, we can say that the spiral Hall's effect is a generalization of the original Hall and Corbino effects.

Hall effect devices nowadays have myriad of applications (see [7] for a complete list) among which we mention those that can be classified as "applications for digital output sensors" such as RPM/speed detectors (engine control), position sensors, valve position sensors, Relays, Proximity detectors, Banking machines, Pressure sensors, etc. and what can be classified as "linear output sensor applications" such as the disk drives, motor control protection/indicators, flow meters, rotary encoders, vibration sensors, tachometers, etc.

Conceiving new Hall effect devices in spiral geometries means expanding the range of solutions available to designers for different applications and expanding the geometric concept underlying these devices which often limits their use. The question of how to cope with Hall effects in different geometries has been debated for more than a century [5],[7], but so far no one has discussed the case of spiral orthogonal manifolds (i.e. spiral conducting sheets). Although galvanomagnetic effects such as current detection and magnetoresistance effect have the same physical origin as the conventional Hall effect (see [5]), their treatment in spiral geometry is beyond the scope of this paper.

II. Materials and Methods:

Let's start analyzing the Hall effect in a comparative way in the two cases (R) rectangular / Cartesian and (S) spiral, starting from the properties of the conductors.

Let's restrict our analysis to the case of materials that obey a linear relationship between the current density J and the applied electric field E as described by Ohm's law,

$$\vec{J} = \sigma\left(\vec{r}\right)\vec{E}. \qquad (1)$$

In general the electrical properties of materials are a function of the position r but in this context it is assumed that the conductivity σ is uniform. Moreover, suppose we are in stationary conditions, i.e. the excitations are substantially constant over time, so the continuity equation for the current (see for example [10] or chap. 7 [9]) becomes

$$\vec{\nabla} \cdot \vec{J} = \sigma \vec{\nabla} \cdot \vec{E} = 0. \qquad (2)$$

and therefore the current density and the electric field are solenoidal.
Moreover, under the same steady state conditions, the Maxwell- Faraday (see chap. 7 [9] and [10]) law becomes

$$\vec{\nabla} \times \vec{E} = 0. \qquad (3)$$

That is, the electric field E is irrotational, and therefore there is a potential function for which

$$\vec{E} = -\vec{\nabla} V. \qquad (4)$$

Now, combining the equations (4),(1),(2) with the condition of uniform conductivity is obtained

$$\vec{\nabla}^2 V = 0. \qquad (5)$$

That is, in a conductor with uniform conductivity, in steady state, the electric potential satisfies the Laplace equation, the solution of which depends on the coordinate system used and on the boundary conditions.
Then, the link between the electric field and the electric potential depends on the gradient mathematical operator whose form changes according to the coordinates used (for a visual introduction of differential geometry and manifolds see for example [11]) . Therefore to determine the electric field it will be necessary to solve the Laplace equation in the reference coordinates, that is Cartesian or polar or spiral etc., with the appropriate boundary conditions (for an in-depth analysis in Cartesian coordinates see [12]).
Spiral coordinates (see [13],[14],[15],[16]),

$$\begin{cases} x = e^{\frac{\delta}{g} - g\theta} \cos(\delta + \theta + \psi), \\ y = e^{\frac{\delta}{g} - g\theta} \sin(\delta + \theta + \psi), \quad \psi, g \in \Re. \end{cases} \qquad (6)$$

allow us to represent curves that can approximate both a short straight line and a circle. Observing eq. (6) it can be seen that the circle is obtained in the particular condition

$$\frac{\delta}{g} - g\theta = k. \qquad (7)$$

Now, if we fix δ we must make sure that in the interval $\theta \in [\theta_i, \theta_f]$ we have

$$\delta - g^2\theta = kg^2 \Rightarrow |\theta| << \frac{1}{g^2}. \qquad (8)$$

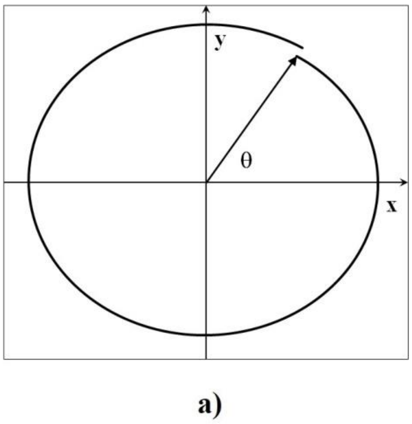
that is, the parameter g must be very small and consequently the equation of the spiral lines becomes

$$\begin{cases} x(\theta) \approx e^{\frac{\delta}{g}} \cos(\delta + \theta + \psi), \\ y(\theta) \approx e^{\frac{\delta}{g}} \sin(\delta + \theta + \psi). \end{cases} \qquad (9)$$

So if we want to represent a circle of radius R with spirals we will have to choose δ in such a way that

$$\begin{cases} \delta \approx g\ln(R), \\ g^2 \ll \left|\frac{1}{\theta_i}\right|, \left|\frac{1}{\theta_i + 2\pi}\right|. \end{cases} \qquad (10)$$

Figure 1 shows two cases that approach the circle with radius R=3. As we can see from the graphs, in case (b) the spiral curve approximates the circle much better than in case (a).



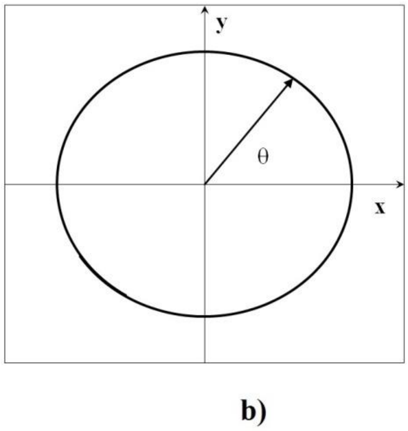


Figure 1. (a) θi = 10, g= 0.01, δ = 0.011 (b) θi = 10, g= 0.001, δ = 0.0011 (a little closer to the shape of the circular curve than the spiral (a)).

If we use a very large g on the other hand, the spiral tends to become a straight line.
Let us now consider the Drude's [17] model underlying the explanation of the Hall's effect. The assumptions (see 1-4 in chap. 1 [18] p. 4-6 and [19] p. 36) concerning the mean velocity and free path of electrons in the metal do not depend on the differential geometry we use.
If n electrons per unit of volume all move with velocity v , then the current density they give rise to will be parallel to v . Since each electron carries a charge -e, the current density is [18]

$$\vec{J} = -en\,\vec{v}. \qquad (11)$$

In the presence of a field E, there will be a mean electronic velocity v_{avg} directed opposite to the field (the electronic charge being negative), which satisfies

$$\begin{cases} \vec{v}_{avg} = -\dfrac{e\vec{E}\tau}{m}, \\ \vec{J} = \left(\dfrac{ne^2\tau}{m}\right)\vec{E}. \end{cases} \qquad (12)$$

This relationship establishes a linear dependence between the current J and the electric field E through the relaxation time τ which is independent of the chosen geometry.
However, it should be noted that the electric field E depends on the geometry and must be treated mathematically differently if the metal leaf has a rectangular or spiral shape.
This result is usually expressed in terms of the inverse of the resistivity, i.e. the conductivity σ

$$\begin{cases} \vec{J} = \sigma\,\vec{E}, \\ \sigma = \left(\dfrac{ne^2\tau}{m}\right). \end{cases} \qquad (13)$$

Now, while in the case of a rectangular sheet the relationship between the potential drop and the electric field is linear and the latter is uniform, in the spiral case the electric field depends on the coordinates and is not uniform.

III. Results:

Let us compare the Hall's effect in Cartesian coordinates with the same effect in spiral coordinates.
A simple scheme of a long and thin sheet suitable for the study of the spiral Hall's effect is shown in Figure 2(a) together with a rectangular thin sheet in Figure 2(b)[8],[6]. A sample for a Hall's effect experiment is said to be long if its dimension in the direction of the current is much longer than that in the direction of the magnetic force acting on the carriers. In such a sample the influence of the current supply contacts on the phenomena can certainly be neglect.

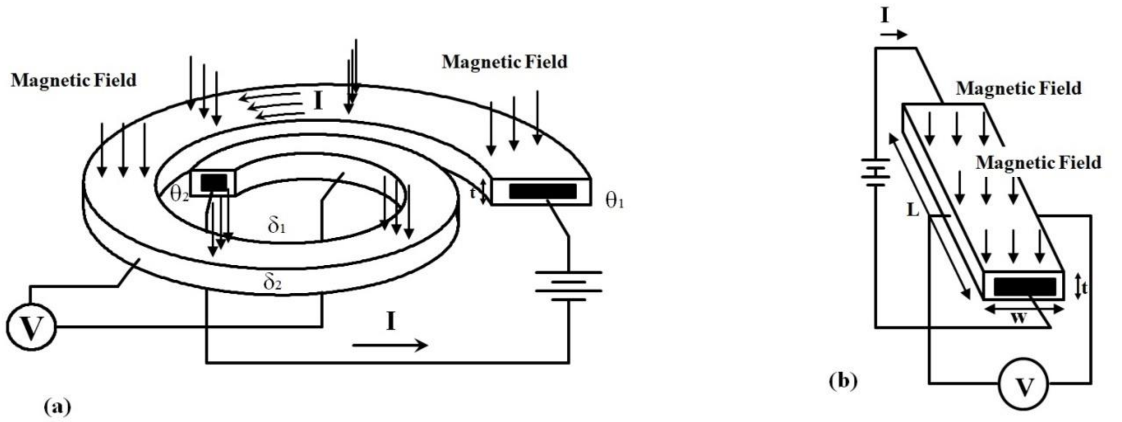


Figure 2. (a) The long spiral sheet and (b) the long rectangular sheet Hall concepts.

It is worth highlighting that the two edges of the Hall device in Figure 2 are not two logarithmic spirals shifted in Cartesian space but are two parallel spiral coordinate lines (in the spiral manifold) corresponding to two different δ, i.e.δ_1; δ_2.
We can imagine that a bias current I is supplied through the two current contacts and that two other voltage or sensing contacts are placed at the edges of the thin spiral foil of Figure 2(a).
If a perpendicular magnetic field $B_\perp$ is applied to the device, a voltage will appear across the two sensing contacts whose amplitude depends on the geometry adopted, i.e. in this case the spiral one.
In both Cartesian and spiral cases, the electron moving in the plates is subject to the Lorentz force and collisions with other electrons in the sample, [20],[18][1].

$$\begin{cases} \frac{d\vec{p}}{dt} = -\frac{\vec{p}}{\tau} + \vec{F}_L, \\ \vec{F}_L = -e\left(\vec{E} + \frac{\vec{p}}{m}\times\vec{B}\right), \\ \vec{p} = m\,\vec{v}. \end{cases} \tag{14}$$

In steady state the current is independent of time, dp/dt = 0 (see p. 13 [20] and therefore in Cartesian coordinates the eq. (14) becomes,

$$(R)\begin{cases} \frac{\vec{p}}{m}\times\vec{B} = \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ \frac{p_x}{m} & \frac{p_y}{m} & 0 \\ 0 & 0 & B_\perp \end{vmatrix} = B_\perp\frac{p_y}{m}\hat{e}_x - B_\perp\frac{p_x}{m}\hat{e}_y, \\ \Rightarrow \begin{cases} 0 = -\frac{p_x}{\tau} - eE_x - \omega_c p_y, \\ 0 = -\frac{p_y}{\tau} - eE_y + \omega_c p_x, \end{cases} \\ \omega_c = \frac{eB_\perp}{m}. \end{cases} \tag{15}$$

and in spiral coordinates

$$(S)\begin{cases} \frac{\vec{p}}{m}\times\vec{B} = \begin{vmatrix} \hat{e}_\delta & \hat{e}_\theta & \hat{e}_z \\ \frac{p_\delta}{m} & \frac{p_\theta}{m} & 0 \\ 0 & 0 & B_\perp \end{vmatrix} = B_\perp\frac{p_\theta}{m}\hat{e}_\delta - B_\perp\frac{p_\delta}{m}\hat{e}_\theta, \\ \Rightarrow \begin{cases} 0 = -\frac{p_\delta}{\tau} - eE_\delta - \omega_c p_\theta, \\ 0 = -\frac{p_\theta}{\tau} - eE_\theta + \omega_c p_\delta, \end{cases} \end{cases} \tag{16}$$

[1] Ashcroft uses the magnetic vector H instead of the magnetic vector B, noting (see p.12 [18] that for non-magnetic materials the difference between the two is extremely small.

Multiply equations (15) and (16) by $-ne\tau/m$ and introduce the current density components through eq. (11) to find

$$\text{(R)}\ \begin{cases} 0 = -J_x + \sigma_0 E_x - \tau\omega_c J_y, \\ 0 = -J_y + \sigma_0 E_y + \tau\omega_c J_x, \end{cases} \tag{17}$$

And

$$\text{(S)}\ \begin{cases} 0 = -J_\delta + \sigma_0 E_\delta - \tau\omega_c J_\theta, \\ 0 = -J_\theta + \sigma_0 E_\theta + \tau\omega_c J_\delta, \end{cases} \tag{18}$$

where ωc is the well-known Larmor cyclotron frequency [26] and $J\delta$; $J\theta$, Jx; Jy are the longitudinal and transverse electric currents respectively in spiral and Cartesian coordinates.
It is worth noting that when $\tau\omega c$ is small, i.e. mainly when the magnetic field $B\perp$ is not very intense, then the current density vector J is almost parallel to the electric field vector E .
In general this is not true and the current density vector forms an angle ϕ_H with the electric field vector (known as Hall angle, see [18] p. 14), which is identical for the rectangular and spiral case, such that tan (ϕ_H) = $\tau\omega_c$.
The Hall fields EH_y and EH_θ in Cartesian and spiral coordinates (see Figure 2(a) and Figure 2(b)) are determined by the requirement that there is no transverse current. In reality this approximation reported in many textbooks [21], [22], [23] is the subject of discussion by many researchers and is strictly valid only at the edges of the plates (see [12]). Away from the edges the transverse current is not really zero and the magnetic force is not entirely in the transverse direction. Although the precise mathematical treatment in spiral coordinates requires the solution of the Laplace equation with the boundary conditions at the edges (see [12]) and a precise analysis of the Hall angle, this is far beyond the introductory purposes of this paper.
Setting the transverse current densities J_y or J_δ to zero everywhere (not just at the edges as in [12]) in eqs. (17) and (18), we find the Hall fields EH_y ; EH_θ (see [18], [20])

$$\text{(R)}\begin{cases} E_{H_y} = -\dfrac{\tau\omega_c}{\sigma_0} J_x\,, \\ J_x = \sigma_0 E_x\,, \end{cases} \Rightarrow E_{H_y} = -\tau\omega_c E_x\,,$$

$$\text{(S)}\begin{cases} E_{H_\delta} = -\frac{\tau\omega_c}{\sigma_0} J_\theta\,, \\ J_\theta = \sigma_0 E_\theta\,, \end{cases} \Rightarrow E_{H_\delta} = -\tau\omega_c E_\theta\,. \tag{19}$$

The Hall coefficient RH (see for example [18]) even in the spiral case does not depend on any parameter of the metal except on the density of the carriers.

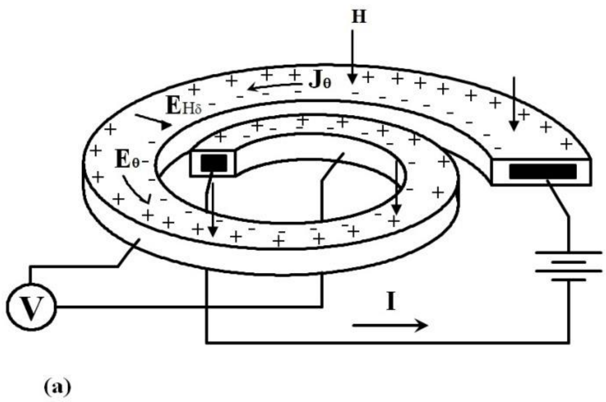
$$\text{(R)}\Rightarrow E_{H_y} = R_H B_\perp J_x\,,$$
$$\text{(S)}\Rightarrow E_{H_\delta} = R_H B_\perp J_\theta\,, \tag{20}$$

where $R_H = -1/ne$.
The relations of equations (19) can be rewritten in terms of electric potential using the definition of gradient in Cartesian and spiral coordinates[2],

$$\text{(R)}\,\frac{\partial V}{\partial y} = \tau\omega_c \frac{\partial V}{\partial x},$$

$$\text{(S)}\,\frac{\partial V}{\partial \delta} = g\tau\omega_c \frac{\partial V}{\partial \theta}, \tag{21}$$

[2] (R) $\vec{E} = \frac{\partial V}{\partial x}\hat{e}_x + \frac{\partial V}{\partial y}\hat{e}_y$, (S) $\vec{E} = \frac{1}{h_\delta}\frac{\partial V}{\partial \delta}\hat{e}_\delta + \frac{1}{h_\theta}\frac{\partial V}{\partial \theta}\hat{e}_\theta$.



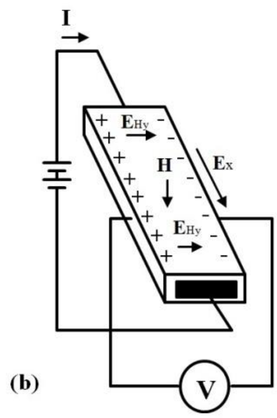


Figure 3: (a) The long spiral plate and (b) the long rectangular plate Hall voltage.

where we have used the Lamè coefficients h_δ , h_θ of the metric tensor in the orthogonal spiral coordinates[3].
If we set J_x or J_δ (transverse currents) null only to the edges, then the boundary conditions change and we need to make a series of mathematical considerations on the Laplace equation (see [12]). Despite the great importance of these considerations for the real cases of galvanomagnetic effects, their treatment in spiral coordinates requires a specific dedicated and in-depth study that goes beyond the objectives of this paper.
Once the problem is posed, it is necessary to solve the Laplace equation (5) in the spiral coordinates system and then compare the solution with the well-known case of the rectangular conductor. Laplace's equation in the two coordinate systems is given by [10], [13][4]

$$(\mathrm{R})\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}=0\,,$$

$$(\mathrm{S})\frac{\partial^2 V}{\partial \delta^2}+\frac{\partial^2 V}{\partial \theta^2}=0\ . \tag{22}$$

The two dimensional equations (22) become one-dimensional equations by imposing that the transverse current is zero (see eq. (21) , restricted solution) and therefore to the Cauchy-Laplace problems with boundary conditions defined by the voltage applied to the ends of the thin sheet.

$$(\mathrm{R})\begin{cases}\dfrac{\partial^2 V}{\partial x^2}=0\,,\\ V(x=0,y,z)=V_0\,,\\ V(x=L,y,z)=0,\forall\ \ x,z\in \text{plate}\end{cases}$$

$$(\mathrm{S})\begin{cases}\dfrac{\partial^2 V}{\partial \theta^2}=0,\\ V(\delta,\theta=\theta_1,z)=V_0\,,\\ V(\delta,\theta=\theta_2,z)=0,\forall\ \ \delta,z\in \text{plate}\end{cases} \tag{23}$$

The solutions of equations (23) are respectively $V(x) = V_0\left(1-\frac{x}{L}\right)$ and $V(\theta)=\frac{V_0}{\theta_1-\theta_2}(\theta-\theta_2)$.
Using the expression of the gradient in the two coordinate systems we can at this point determine the longitudinal electric fields in the case of a rectangular and spiral conductor.

$$(\mathrm{R})\ E_x=\frac{V_0}{L},$$

[3] $h_\delta = e^{\frac{\delta}{g}-g\theta}\frac{\sqrt{1+g^2}}{g}, h_\theta = e^{\frac{\delta}{g}-g\theta}\sqrt{1+g^2}.$

[4] The z component is not considered because the lamina is thin and therefore the potential is almost constant along it.

$$(S)\ E_\theta = \frac{V_0}{\sqrt{1+g^2(\theta_1-\theta_2)}}\ e^{-\frac{\delta}{g}+g\theta}. \tag{24}$$

As can be seen from eq. (24) the electric field in the rectangular case is uniform while in the spiral case it is different in each point of the sample.
Let's determine the relationship between the total current $I = \int \vec{J} \cdot \vec{dA}$ and the voltage applied $V_0 = \int \vec{E} \cdot \vec{dl}$ in both cases.
While the expressions of the total longitudinal currents I in the two cases are given by

$$(R)\ \ I = \int_0^w \int_0^t \sigma_0 \frac{V_0}{L} dydz = wt\sigma_0 \frac{V_0}{L},$$

$$(S)\ \ I = \int_{\delta_1}^{\delta_2} \int_0^t J_\theta\, h_\delta\, d\delta dz = (\delta_2 - \delta_1)t\sigma_0 \frac{V_0}{g(\theta_1-\theta_2)}. \tag{25}$$

the expressions of the Hall transverse voltages are given by

$$(R)\ \ V_H = \int_0^w E_{H_y} dy = -\frac{e\tau}{m}\frac{w}{L} V_0 B_\perp,$$

$$(S)\ \ V_H = \int_{\delta_1}^{\delta_2} E_{H_\delta} h_\delta d\delta = -\frac{e\tau}{m}\frac{(\delta_2-\delta_1)}{g(\theta_1-\theta_2)} V_0 B_\perp\ . \tag{26}$$

The relationship between the Hall tensions, the total longitudinal currents and the magnetic fields perpendicular to the conductive strips in the two cases is identical (see [6] eq.(3.47))

$$V_H = \frac{R_H}{t} IB_\perp\ . \tag{27}$$

We observe that thin and long spiral (i.e. $|\delta_2 - \delta_1| \ll |\theta_1 - \theta_2|$) conductive strips can be housed in much smaller containers than rectangular ones (i.e. w << L).

Conclusion: In this paper the Hall's voltage for long strips of thin spiral conductor was determined. A comparison was made between the classical case of long rectangular conductor strips and spiral ones. From the formulas it is clear that it is possible to conceive Hall devices with very long spiral strips that occupy very small spaces. The spiral ones is therefore a very suitable geometry for the construction of compact Hall devices. The analysis was carried out through a comparison with the rectangular case by using the approximation of the zero transverse current (classical restricted case). This paper is a pioneering introduction to the spiral Hall's effect. Further investigations into spiral galvanomagnetic effects and the use of other spiral differential varieties are possible.

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