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Correlation of Proton- Neutron Separation Energy and Energy Gap for Some Exotic Rich Nuclei

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Abstract
Exotic nuclei are nuclei with ratios of neutron number N to proton number Z much larger or much smaller than those of nuclei found in nature. In exotic nuclei the binding or interaction is due to the nucleon-nucleon interaction without the exchange of any third particle between the two interacting nucleons. In the study of such systems, the qualitative behavior of the energy gap has been studied in terms of the nucleon-nucleon interactions, called the pairing correlations that lead to the finite pairing energy, energy gap and the importance of such a relation has been studied, especially in neutron-rich (N>Z) nuclear systems. The dependence of pairing gap on the asymmetry parameter (η) whether small or large has been studied. To calculate the numerical values for the energy gap for nuclei, nuclear potentials have been used to ascertain the effect of the nuclear potentials and the physical quantities involved. A formula has been derived which correlates nucleon separation energy and energy gap using some known nucleon-nucleon interactions. Pairing in infinite neutron matter and nuclear matter show that in the lowest order approximation, where the pairing interaction is taken to be the bare nucleon-nucleon (NN) and (PN) interaction, the pairing interaction and the energy gap can be determined directly. This is due to the almost separable character of the nucleon-nucleon interaction. Since the most recent NN interactions are charge-dependent, we have solved gap equations for nucleon-nucleon interaction (for proton-proton, neutron-neutron, and neutron-proton) pairing in nuclear matter. The results are, however, found to be close to those obtained with charge-independent potentials.
Key Words: Exotic nuclei, Pairing energy, Isotopes, Binding energy
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I. Introduction

Nuclei with ratios of neutron number N to proton number Z much larger or much smaller than those of nuclei found in nature are called exotic nuclei (Pastore *et al.*, 2013). Studies of nuclear matter under extreme conditions have been revisited in which the nuclei are quite different in some way from those found in nature, are at the forefront of nuclear research (Bohr, *et al.*, 1958). Such extreme conditions include nuclei at high temperature and at high density (several times normal nuclear density), as well as those with larger or smaller *N/Z* ratios. The *N/Z* ratio depends on the nature of the attractive nuclear force that binds the protons and neutrons in the nucleus and its competition and complex interplay with the disruptive Coulomb or electrical force that pushes the positively charged protons apart (Meng, et al., 2017).

Stable nuclei lie in the so-called valley of beta-stability. As the *N/Z* ratio decreases (proton-rich nuclei) or increases (neutron-rich nuclei) compared to that of the stable isotopes, there is, respectively, energy for a proton or neutron in the nucleus to undergo beta ($ß^+$, $ß^-$) decay to move the nucleus back toward stability (Kobyakov & Pethick, 2013). Most knowledge of nuclear structure and decay has been gained from nuclei in or near the valley of beta-stability. Exotic nuclei exhibit decay modes not seen near stability, such as proton radioactivity and beta-delayed particle emission. (After beta decay, the highly excited nucleus can emit one or more particles, such as one or two protons, an alpha particle, or one or two neutrons.)

In nuclear physics, a measure of the pairing energy (also called pairing gap) is the systematic staggering of the ground state energies between nuclei with even numbers of neutrons (or protons) and those with odd numbers (Dong *et al.*, 2013). Explicitly for the case of (fermions) neutrons, in systems with even neutron number, N, all neutrons are paired, while for odd N, one nucleon is not paired. This pairing gap, Δ , can be expressed in terms of the second difference, i.e, $\Delta = (-1)N + 1\left\{ E(N) - \frac{1}{2}\left[E(N+1) + E(N-1) \right] \right\}$

(1)

where $E(N)$ is the ground state energy of the system with N neutrons. Equation (1) can be written in terms of the neutron separation energy $S(N)$;

$$S(N) = E(N) - E(N-1) \tag{2}$$

Equation (1) can now be expressed in terms of the neutron separation energy, i.e,

$$\Delta = \frac{(-1)^N(-1)}{2}\{2E(N) - E(N+1) - E(N-1)\}$$

$$= \frac{(-1)^N}{2}(-1)\{[-E(N+1) + E(N)] + [E(N) - E(N-1)]\}$$

$$= \frac{(-1)^N}{2}\{[E(N+1) - E(N)] - [E(N) - E(N-1)]\} = \frac{(-1)^N}{2}\{S(N+1) - S(N)\} \tag{3}$$

To calculate Δ from equation (3), we have to know the separation energy S_n of a neutron, and for a nucleus of mass $A = N + Z$, it is given by (Gezerlis *et al.*, 2015)

$$S_n(A) = f + (A-1)\frac{df}{dA} \tag{4}$$

where f is the binding energy per nucleon, i.e,

$$f = \frac{(B) Binding\ energy\ of\ the\ nucleus}{(A) Mass\ number\ of\ the\ nuclus} \tag{5}$$

The pairing energy of a pair of nucleons (neutrons or protons) is a consequence of the pairing interaction between the nucleons. A finite-range interaction in the pairing channel is essential for the existence of the pairing energy due to pairing correlations (Baldo & Schulze, 2007).

PAIRING IN UNIFORM NEUTRON SYSTEM

The pairing properties of neutron supper-fluid according to the BCS (Floerchinger *et al.*, 2008) model are characterized by the pairing energy gap which satisfy the integral equation (Dean & Hjorth-Jensen, 2001)

$$\Delta_{(p)} = -\frac{1}{(2\pi)^3}\int d^3\kappa \upsilon(p-\kappa)\frac{\Delta(\kappa)}{2\sqrt{(\varepsilon(\kappa) - \varepsilon_F)^2 + \Delta(\kappa)^2}} \tag{6}$$

where $\upsilon(p-\kappa)$ is the matrix element of the interaction in the superfluid channel calculated between waves, $\varepsilon(\kappa)$ is the single-particle-energy as a function of the momentum while ε_F is the neutron Fermi energy (Pastore, Margueron, Schuck, & Vi˜nas, 2013).

NON-UNIFORM NEUTRON MATTER

Single particles energy can be calculated in H-F approximation and the Fermi energy proximity of the neutron Fermi wavenumber κ_N is (Carlson, Chang, Pandharipande, & Schmidt, 2003)

$$\varepsilon(\kappa) - \varepsilon_F \approx \hbar^2\frac{(\kappa^2 - \kappa_N^2)}{2m^*(\kappa_N)} \tag{7}$$

where the effective mass at the Fermi surface is defined as (Gezerlis & Carlson, 2010)

$$2m^*(\kappa_N) = \frac{m\hbar\kappa_N}{(d\varepsilon(\kappa)/d\kappa)_{\kappa=\kappa_N}} \tag{8}$$

The ratio between the effective and bare mass at the Fermi surface is (Floerchinger *et al.*, 2008)

$$\frac{2m^*(\kappa_N)}{m} = \left(1 + \frac{m\kappa_N}{4\pi^2\hbar^2}\sum y_i^N v_i^N c_i + \frac{m}{\hbar^2\kappa_N}\sum y_i^p v_i^p c_i\right)^{-1} \tag{9}$$

where

$$v_i^N = \left(\frac{2}{r_i\kappa_N}\right)^4\left[\frac{r_i^2\kappa_N^2}{2} - 1 + \left(1 + \frac{r_i^2\kappa_N^2}{2}\right)\exp(-r_i\kappa_N)\right] \tag{10}$$

And (Gubbels & Stoof, 2008)

$$v_i^p = \exp\left[-\frac{(\kappa_N+\kappa_P)^2 r_i^2}{4}\right]\left[\frac{1}{2\pi^2 r_i^2}-\frac{1}{\pi^2 r_i^2}\left(\frac{1}{\kappa_N^2 r_i^2}+\frac{1}{2}+\frac{\kappa_P}{2\kappa_N}\right)\right]$$
$$+\exp\left[-\frac{(\kappa_N-\kappa_P)^2 r_i^2}{4}\right]\left[-\frac{1}{2\pi^2 r_i^2}+\frac{1}{\pi^2 r_i^2}\left(\frac{1}{\kappa_N^2 r_i^2}+\frac{1}{2}-\frac{\kappa_P}{2\kappa_N}\right)\right] \quad (11)$$

Then some of the quantities can be calculated as $c_i = \frac{\pi^{3/2} r_i^3}{4}$, while $y_i^N = v_3^i + 2v_4^i + v_1^i + 2v_2^i$ and $y_i^p = v_3^i - 2v_4^i$. The values of k_N and k_p are obtained using the relations $k_N = (3\pi^2 \rho N)^{\frac{1}{3}}$ and $k_p = (3\pi^2 \rho P)^{\frac{1}{3}}$ (Aissaoui *et al*., 2009).

II. Results And Discussions

Thus, we calculate the energy gaps for isotopic chains of Ca, Ni, Mo and Ytt. Using equation (4), where we can write $A' = (N+1+Z)$;

$$S(N+1) = f(N+1,Z) + (N+1+Z-1)\frac{df\,(N+1,Z)}{dA} \quad (12)$$

$$S(N) = f(N,Z) + (N+Z-1)\frac{df\,(N+Z)}{dA} \quad (13)$$

Substituting for $S(N+1)$ from equation (12) and $S(N)$ from equation (13) in equation (3), we can calculate the values of Δ for different chains of isotopes, like (Cherop *et al*., 2019)$^{40}_{20}Ca$, $^{42}_{20}Ca$, $^{43}_{20}Ca$, $^{44}_{20}Ca$, $^{46}_{20}Ca$, $^{58}_{28}Ni$, $^{60}_{28}Ni$, $^{61}_{28}Ni$, $^{62}_{28}Ni$, $^{64}_{28}Ni$, $^{58}_{28}Ni$, $^{92}_{42}Mo$, $^{94}_{42}Mo$, $^{95}_{42}Mo$, $^{96}_{42}Mo$, $^{98}_{42}Mo$, $^{100}_{42}Mo$, $^{106}_{56}Sn - {}^{180}_{56}Sn$

The equation $\Delta = \eta e^{-\frac{1}{G\rho(\epsilon_F,k)}}$ (14)
is used to determine the pairing gap. The following data from table 1 is used in calculating the pairing gap. If the energy window at the Fermi surface is 3 MeV (Draayer, et al., 2018), and the number of single particle states is 14, then the density of states at the Fermi surface is

$$\rho(\in_F, \mathrm{K}) = \frac{14}{3MeV} \cong 3.7(MeV)^{-1} \quad (15)$$

The value of G=0.260MeV. The pairing gap Δ=0.8MeV. Thus

$$0.8MeV = \eta e^{-\frac{1}{3.7(MeV)^{-1}\times 0.260\ MeV}} = \eta e^{-\frac{1}{0.962}}$$

$$0.8MeV = \frac{\eta}{e^{1.04}}$$

$$\eta = 0.8\times e^{-1.04} MeV = 2.2634MeV \quad (16)$$

The number of particle states and number of neutrons that are active for ^{161}Dy, ^{162}Dy and ^{171}Yb, ^{172}Yb nuclei are in table 1.

Table 1: Table of Yb and Dy particle states

	Yb		Dy	
N_{sp} (Number of particle states)	14	11	10	10
G(MeV)	0.26	0.27	0.32	0.284
n_a (Number 0f neutrons that are active)	16	12	10	10

If the number of single-particle states or the energy window at the Fermi surface is 14, then

$$\rho(\in_F, \mathrm{K}) = \frac{14}{14\ MeV} = 1(MeV)^{-1} \tag{17}$$

Which means that there is one energy state in an energy gap of 1 MeV and

$$\rho(\in_F, \mathrm{K})G = 0.260 \tag{18}$$

Hence using Eq. (17) and Eq. (18) we get

$$0.8\,MeV = \frac{\eta}{e^{\frac{1}{0.26}}} \quad \text{or } \eta = 0.8\,MeV \times e^{3.9} = 39.52\ MeV \tag{19}$$

which is quite large as compared to the value obtained in Equation (16).

There are also exotic nuclei with very large η that exist in the inner crust of the neutron stars, such as $^{250}_{40}Zr$, $^{500}_{40}Zr$, $^{110}_{50}Sn$ and $^{1800}_{50}Sn$, i.e $\eta = 0.68, 0.84, 0.090909$, *and* 0.944444 respectively.

PAIRING ENERGY OF THE NEUTRON PAIR P_N

Pairing energy of the neutron pair is defined as the binding energy of the second neutron minus the binding energy of the first neutron when the neutrons are added one after the other.The calculation of proton pairing gaps is a challenging problem due to the coupling of protons to the denser neutron background (Haensel *et al*., 2017). An important effect is that the proton effective mass, and therefore the density of states that enters the pairing strength in the exponent in the expression for the gap, is smaller than the neutron effective mass in neutron rich matter, $m_p^* < m_n^*$ (Chen *et al*., 1993). This decreases the proton $^1S_0$ gap significantly for a particular proton density, n_p compared to the neutron gap for a neutron density equal to n_p (Pastore *et al.*, 2011). In addition to effective-mass effects, the coupling to the denser neutron background amplifies the repulsive contributions to the pairing interaction from nucleon-nucleon forces and decreases the proton $^1S_0$ gap further (Clark *et al.*, 1993).

The situation regarding induced interactions is less clear. (Zhou *et al.*, 2004) found out that the induced interactions are repulsive with a stronger reduction of the proton $^1S_0$ gap compared to the neutron one, whereas (Baldo & Schulze, 2007) found attractive induced interactions. A systematic study of proton pairing in neutron-rich matter incorporating the discussed many-body effects consistently remains an important outstanding problem (Hammer *et al*., 2013).

Let us take the nucleus $X(N, Z)$ where binding energy is $B(N, Z)$. Then by definition

$$B(N+2, Z) - B(N+1, Z) \tag{20}$$

The binding energy of the second neutron is the difference

$$B(N+2, Z) - B(N+1, Z) \tag{21}$$

The binding energy of the first neutron is the difference

$$B(N+1, Z) - B(N, Z) \tag{22}$$

Now the pairing energy P_N of the neutron pair is the difference of equation (21) and equation (22), i.e

$$P_N = B(N+2, Z) - B(N+1, Z) - B(N+1, Z) + B(N, Z)$$

$$= B(N+2, Z) - 2B(N+1, Z) + B(N, Z) \tag{23}$$

In terms of separation energy, if a neutron is removed from the nucleus $X(N+2, Z)$ to give $X(N+1, Z)$, then the separation energy is say S_N^1, then

$$S_N^1 = B(N+2, Z) - B(N+1, Z) \tag{24}$$

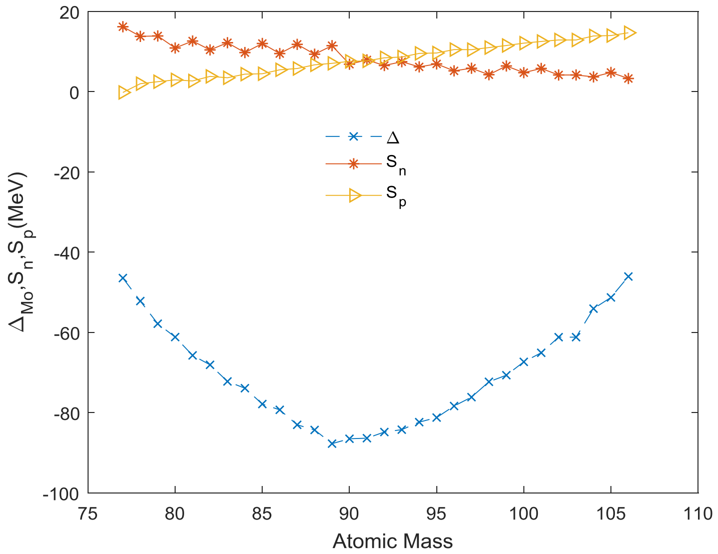

Figure 1: Energy gap, neutron-proton separation energy versus atomic mass for Molybdenum Isotopes

Similarly, when a second neutron is removed from the nucleus $X(N+1, Z)$, then the separation energy for the second neutron say S_N^2, it will be,

$$S_N^2 = B(N+1, Z) - B(N, Z) \tag{25}$$

The pairing energy of the neutron pair is P_N is the difference of equation (24) from equation (25), such that

$$P_N = B(N+2, Z) - 2B(N+1, Z) + B(N, Z) \tag{26}$$

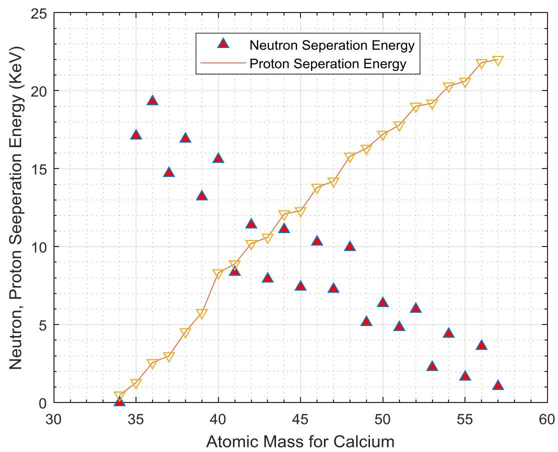

Figure 2: Proton, Neutron separation energy versus atomic mass for Calcium Isotopes.

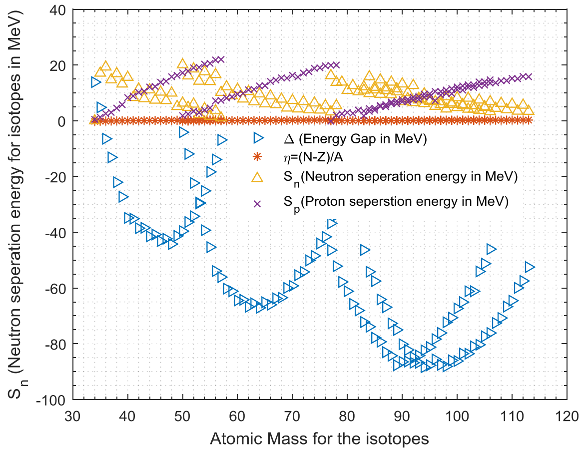

Figure 3: Neutron separation energy versus atomic mass for $^{40}_{20}Ca - ^{46}_{20}Ca, ^{58}_{28}Ni - ^{58}_{28}Ni, ^{92}_{42}Mo - ^{100}_{42}Mo$ isotopes

The energy gap Δ, separation energy calculated has been fitted to nucleon-nucleon scattering data the potentials are specified according to the legend at the middle panel. Energy gaps in the exotic nuclei, obtained by solving the derived gap equation with a free-particle spectrum for the for the normal states.

III. Conclusion

In exotic nuclei the binding or interaction is due to the nucleon-nucleon interaction without the exchange of any third particle between the two interacting nucleons. The qualitative behavior of the energy gap has been studied in terms of the nucleon-nucleon interactions, called the pairing correlations that lead to the finite pairing energy, energy gap and the importance of such a relation has been studied, especially in neutron-rich (N>Z) nuclear systems. To calculate the numerical values for the energy gap for nuclei, nuclear potentials have been used to ascertain the effect of the nuclear potentials and the physical quantities involved. The derived formula correlates well with nucleon separation energy and energy gap using some known nucleon-nucleon interactions. Pairing in infinite neutron matter and nuclear matter showed that in the lowest order approximation, where the pairing interaction is taken to be the bare nucleon-nucleon (NN) and (PN) interaction, the pairing interaction and the energy gap can be determined directly. This is due to the almost separable character of the nucleon-nucleon interaction. Since the most recent NN interactions are charge-dependent, we have solved gap equations for nucleon-nucleon interaction (for proton-proton, neutron-neutron, and neutron-proton) pairing in nuclear matter. It is well known that the separation energy and the energy gap, Δ , also depends on the nucleon-nucleon interaction for the isotopes studied. Thus, we established that there exists a definite relationship between $V(r)$ and Δ. The experimental and the theoretically calculated results are, however, found to be close to those obtained with charge-independent potentials, however, more experimental studies should be done on other isotopes whose data are not available. It is difficult to predict pairing gaps for the protons because it is bedeviled by the fact that the protons are the minority in the exotic nuclei, consequently induced interactions due to the surrounding neutrons play a large part, but its role is poorly understood.

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References

[1]. Aissaoui, L., Benrachi, F., and Boumala, D. (2009). Pairing energy correction in shell model forthe neutron rich-tin isotope. Brazilian Journal of Physics, 39, 333-339.
[2]. Baldo, M., and Schulze, H.-J. (2007). Proton pairing in neutron stars. Physics Review, C75, 025802.
[3]. Bohr, A., Mottelson, B. R., and Pines, D. (1958). Possible analogy between the excitation spectra of nuclei and those of the superconducting metallic state. Physics Review, 75, 936.
[4]. Carlson, J., Chang, S.-Y., Pandharipande, V. R., and Schmidt, K. E. (2003). Superfluid Fermi gases with large scattering lenght. Physics Review Letters, 91, 050401.
[5]. Cherop, H., Muguro, K., & Khanna, K. (2019). The role of Shell model in Determing Pairing interactions in Nuclei. National Conference on Technological Innovations in Management, Engeering and Science, 26-27.
[6]. Chen, J. M., Clark, J. W., Dave, R. D., and Khodel, V. (1993). Pairing gaps in nucleonic superfluids. Nuclear Physics, A 555, 59.

[7]. Dean, D. J., and Hjorth-Jensen, M. (2001). Pairing in nuclear systems: From nuclear stars to finite nuclei. Reviews in Modern Physics, 75, 607.
[8]. Dong, J. M., Lombardo, U., and Zuo, W. (2013). 3P-F2 pairing in high density neutron matter. Physics Review, C 87, 062801.
[9]. Draayer, J. P., Gueorguiev, V. G., Sviratcheva, K. D., Bahri, C., Feng, P., and Geoorgieva, A. I. (2018). Exactly sovable pairing models. Institute of Nuclear Research and Nuclear Energy. doi:arXiv.org/abs/1802.06508vl
[10]. Floerchinger, S., Schere, M., Diehl, S., & Wetterich, C. (2008). Particle-hole fluctuations in the BCS-BEC crossover. Physics Review, B 78, 174528.
[11]. Gerzerlis, A., and Carlson, J. (2008). Strongly paired fermions: Cold atoms and neutron matter. Physics Review, C 77, 032801.
[12]. Gezerlis, A., and Carlson, J. (2010). Low-density neutron matter. Physics Review, C 81, 025803.
[13]. Gezerlis, A., Pethick, C. J., & Schwenk, A. (2015). Pairing and superfluidity of nucleons in neutron stars. Nuclear Theory. doi:arXiv.1406.6109v2
[14]. Gubbels, K., and Stoof, H. T. (2008). Renorlization group theory for imbalanced Fermi gas. Physics Review Letters, 100, 140407.
[15]. Haensel, P., Potekhin, A., andYakovlev, D. (2017). Neutron Stars I Equation of State and Structure. (Springer Publisher, New York)
[16]. Hammer, H.-W., Nogga, A., and Schwenk, A. (2013). Three-body forces: From cold atoms to nuclei. Review of Modern Physics, 85, 197.
[17]. Kobyakov, D., and Pethick, C. J. (2013). Dynamics of the inner crust of neutron stars: Hydrodynamics, elasticity, and collective modes. Physics Review C 87, 055803.
[18]. Meng, W., Audi, G., Kondev, F., Huang, W. J., Naimi, S., and Xing, X. (2017). The AMEA2016 Atomic mass evaluation. Chinese Physics C 41 (3), 030003.
[19]. Pastore, A., Baroni, S., & Losa, C. (2011). Superfluid properties of the inner crust of neutron stars. Physics Review, C 43, 065807.
[20]. Pastore, A., Margueron, J., Schuck, P., and Vi˜nas, X. (2013). Pairing in exotic neutron rich nuclei around the drip line and in the crust of neutron stars. Physics Review C 88, 034314.
[21]. Zhou, X.-R., Schulze, H.-J., Zhao, E.-J., Pan, F., and Draayer, J. (2004). Pairing gaps in neutron stars. Physics Review, C 70, 048802.

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