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Energy Conservation for Potential Dependent special Relativity and string Quantum Energy with Imaginary Energy

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Abstract
The conservation of energy of the potential dependent special relativity has been studied. It was found that the conservation could be secured when using vector four-dimensional representation with the fourth time component is imaginary and related to the momentum. The energy expression has been used to derive new quantum equation for weak field. Treating particles as strings an imaginary energy has been found .the imaginary part is quantized and proportional to the liberated photon energy .This resembles the imaginary wave number reflecting the energy liberated by electromagnetic waves upon interaction with medium.

Key Words: *energy, conservation, potential dependent special relativity, string, imaginary energy.*

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I. Introduction:

Physical system can be well described by the equation of motion and conserved quantities. The equation of motion describes the time evolution and the dynamic of the system. Conserved quantities describe those constant physical quantities that does not change with space or time [1].

One of the most popular one is the concept and notion of energy. In Newtonian mechanics, the energy is the equal to sum of kinetic and potential energy [2]. In Special Relativity (SR) the total energy is related to the rest mass energy and momentum, without recognized the potential energy. This situation is puzzling , and indirect conflict with observations , beside Newton's laws and quantum laws .According to (SR) version tow charge identical particle moving with same velocity ,one in vacuum and the other revolves around a nucleus have the name total energy . This is in direct conflict with observed atomic spectra of different atoms; the electron total energy is directly affected by its potential energy. This result does not conform with SR energy from whish has no room for potential energy [3, 4]. This bizarre situation encourages some scientists to remedy this effect by constructing new models [5, 6, 7]. One of the most popular one is the one proposed by Mubarak Dirar, which recognize the effect of the potential. His original work is based on the energy expression in a curved space [8]. Later on the utilized Lorentz transformation to incorporate potential energy in the total energy from [9, 10]. Those models succeeded in bridging the gap between relativity and quantum beside Newtonian mechanics. Even same models tries to study conservation laws for this potential dependent special relativity and quantum beside Newtonian mechanics. Even some models tries to study conservation laws for this potential dependent special relativity (psr) [11]. However, no ultimate regions version is been obtained. This encourage trying to construct a new regiours model for conservation of energy. This has done in section (2). Section (3) has devoted for conclusion.

II. Relation of Newtonian Lagrangian and Energy to the energy Conservation in Generalized potential Dependent Space

It is well known that the suitable quantity, which simplify the equation of motion, is the lagrangian L, which is defined by:

$$L = T - V = \frac{1}{2}mv^2 - V \qquad (1)$$

With T, V, m, v stands for kinetic energy, potential energy mass and velocity. The lagrangian and the equation of motion are related to the action I, which is extremum, reflecting the tendency of the system to select a path that enable it to give minimum or extract maximum energy from the surrounding via changing the generalized coordinates due to the change of trajectories at affixed space – time point. The state of physical systems can also be simplified by extracting from the action integrand quantities that are invariant under space and time transition like momentum and energy E which is given by:

$$E = T + V \tag{2}$$

Which the Newtonian expression for energy using GSR energy mass relation.

$$E = mc^2 = \frac{m_o c^2}{\sqrt{1+\frac{2\emptyset}{c^2}-\frac{v^2}{c^2}}} = m_o c^2 \left(1 + \frac{2\emptyset}{c^2} - \frac{v^2}{c^2}\right)^{-1/2} \tag{3}$$

For a weak field and macroscopic world:

$$\frac{2\emptyset}{c^2} < 1 \quad , \frac{v^2}{c^2} < 1 \tag{4}$$

Using the identity:

$$(1+x)^n=1+nx \tag{5}$$

For small x.

Setting:

$$x = \frac{2\emptyset - v^2}{c} \tag{6}$$

In equation (3) then using (5) yields:

$$E = m_o c^2 \left(1 - \frac{\emptyset}{c^2} + \frac{1}{2}\frac{v^2}{c^2}\right) = -m_o\emptyset + \frac{1}{2}m_o v^2 + m_o c^2$$

$$E = T - V + m_o c^2 \tag{7}$$

With kinetic energy T and potential energy V given by:

$$T = \frac{1}{2}m_o v^2 \,, \qquad V = m_o\emptyset \tag{8}$$

But according to Newtonian and classical mechanics, that energy in equation (7) is the lagrangian:

$$L= T-V \tag{9}$$

Hence:

$$E=L \tag{10}$$

$$E = \frac{m_o c^2}{\sqrt{g_{\infty} - \frac{v^2}{c^2}}} = \frac{m_o c^2}{\sqrt{\frac{g_{\infty} m^2 c^4 - m^2 v^2 c^2}{m^2 c^4}}} \tag{11}$$

$$E = \frac{m_o c^2}{\sqrt{\frac{g_{\infty} E^2 - P^2 c^2}{E^2}}} = \frac{m_o c^2 E}{\sqrt{g_{\infty} E^2 - P^2 C^2}}$$

$$\left(\sqrt{g_{\infty} E^2 - P^2 c^2}\right)^2 = m_o^2 c^4$$

$$E^{\sim 2} = g_{\infty} E^2 = P^2 c^2 + m_o^2 c^4 \tag{12}$$

$$E^2 = g_{oo}^{-1} = (P^2 c^2 + m_o^2 c^4) \tag{13}$$

Thus, the energy conservation requires:

$$E= \text{constant} \tag{14}$$

$$g_{oo}^{-1}(P^2 c^2 + m_o^2 c^4)= \text{constant} \tag{15}$$

One can also look at the conservation 0f energy using space-time language. In view of equation (12) for a particle at rest in free space:

$$v = 0 \,, \qquad p = 0, g_{\infty} = 1, \quad \tilde{E} = \widetilde{E_o} = E_o \tag{16}$$

In this case, equation (12) reduces to:

$$\tilde{E} = \widetilde{E_o} = E = E_o = m_o c^2$$

$$\widetilde{E_o} = E_o = m_o c^2 \tag{17}$$

The energy can be written using complex notation as [see equ-12]

$$m_o^2 c^4 = \widetilde{E_o^2} = \widetilde{E^2} - P^2 c^2 = \tilde{E}^2 + (ipc)^2 \tag{18}$$

$$m_o^2 c^4 = \widetilde{E^2} + E_4^2 \tag{19}$$

Where:

$$E_4 = ipc \tag{20}$$

One can write (18) in the form:

$$\widetilde{\ E} = \widetilde{E_o} + i\tilde{E}_4 \tag{21}$$

Where the subscript stands for a complex quantity, thus a cording to the vector notion, one can written equation (21) in the form:

$$E^{\sim} = \widetilde{E_1}\widehat{e_1} + \widetilde{E_2}\widehat{e_2} + \widetilde{E_3}\widehat{e_3} + \widetilde{E_4}\widehat{e_4} \tag{22}$$

$$\widetilde{E_1} = E_x,\ \widetilde{E_2} = E_y,\ \widetilde{E_3} = E_z,\ \widetilde{E_4} = ipc \tag{23}$$

$$\widetilde{E^2} = m_o^2 c^4 = \widetilde{E_1} + \widetilde{E_2} + \widetilde{E_3} + \widetilde{E_4}$$

$$E_x^2 + \ E_y^2 + \ E_z^2 + i^2 p^2 c^2 = E^2 - p^2 c^2 \tag{24}$$

This means that the square of the components of the energy in the four-space time dimension is conserved provided that:

$$\tilde{E} = (\widetilde{E_1}, \widetilde{E_2}, \widetilde{E_3}, \widetilde{E_4}) = (\widetilde{E_\iota}) = \widetilde{E_\iota} \quad (25)$$

One can re write equation (18)in a complex form [see equation (12)].

$$\tilde{E} = g_{\circ\circ}E = p^2c^2 + m_o^2c^4 \quad (26)$$

By suggesting:

$$E_c = \tilde{E} = g_{\circ\circ}E \quad (27)$$

$$E_r = pc, \quad E_i = m_o c^2 \quad (28)$$

Thus, the complex energy representation is given by:

$$E_c = E_r + iE_i \quad (29)$$

Thus:

$$|E_c|^2 = \widetilde{E^2} = E_r^2 + E_i^2 = p^2c^2 + m_o^2c^4 \quad (30)$$

Comparing the vector and the complex representation looks more convenient with additional time component.

$$\widetilde{E_4} = ipc$$

Compared to the convention three-dimensional one. Generalized Potential dependent Weak Field Quantum Klein Gordon equation:

The g p s r energy momentum relation in a weak field is given by equation (7).

$$E = \frac{1}{2}m_o v^2 - V + m_\circ c^2 = \frac{p^2}{2m} - V + +m_\circ c^2 \quad (31)$$

Using the wave function:

$$\psi = Ae^{\frac{i}{\hbar}(px - Et)} \quad (32)$$

Clearly:

$$\frac{\partial\psi}{\partial t} = \frac{-E}{i\hbar}\psi, \quad \frac{\partial^2\psi}{\partial t^2} = -\frac{E}{\hbar}\psi$$

$$\nabla\psi = \frac{i}{\hbar}p\psi \ , \quad \nabla^2\psi = -\frac{p^2}{\hbar^2}\psi$$

$$i\hbar\frac{\partial\psi}{\partial t} = E\psi, \quad -\hbar^2 \ \frac{\partial^2\psi}{\partial t^2} = E^2\psi$$

$$\frac{\hbar}{i}\nabla\psi = p\psi, \quad -\hbar^2 \ \nabla^2\psi = p^2\psi \quad (33)$$

Multiplying both sides of (31) by ψ and using equation (33) given:

$$E\psi = \frac{p^2}{2m}\psi - V\psi + m_o c^2\psi$$

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi - V\psi + m_o c^2\psi \quad (34)$$

This equation resembles ordinary Schrodinger equation with V replaced by $m_o c^2 - V$

$$V \to m_o c^2 - V \quad (35)$$

The Schrodinger equation, r dependent part, for spherically symmetric potential is given by:

$$\frac{\hbar^2}{2m}\ddot{u} - V(r)u + Eu = c\frac{u}{r^2} \quad (36)$$

To study behavior of the nucleons, one consider nucleons as oscillators. Where for harmonic oscillator, the potential taken the form:

$$V(r) = \frac{1}{2}k_o r^2 \quad (37)$$

Rearranging equation (36) and setting for Schrodinger equation

$$k^2 = \frac{2mE}{\hbar^2}$$

$$c_2 = \frac{m}{\hbar^2}k_{o,} \quad c_1 = \frac{2mc}{\hbar^2} = \quad \frac{2ml(l+1)\hbar^2}{2m\hbar^2} = l(l+1) \quad (38)$$

Or using the gsr potential lagrangian Schrodinger equation (34), and in view of equation (35).equation (36) become

$$\frac{\hbar^2}{2m}\ddot{u} + V(r)u + (E - m_o c^2)u = \frac{\mathrm{cu}}{r^2} \quad (39)$$

Using equation (37) equation (39) becomes:

$$\ddot{u} + \frac{m}{\hbar^2}k_o r^2 + \frac{2m}{\hbar^2}(E - m_o c^2)u = \frac{2m}{\hbar^2}\frac{c}{r^2}u \quad (40)$$

Taking and setting:

$$c_2 = \frac{m}{\hbar^2}k_{o,} \quad k^2 = \frac{2mE}{\hbar^2}(E - m_o c^2), c_1 = l(l+1) \quad (41)$$

Equation (36) becomes:

$$\ddot{u} - c_2 r^2 u + k^2 u - \frac{c_1}{r^2}u = 0 \quad (42)$$

This equation can be simplified by defining the variable y to satisfy:

$$y = \propto r$$

$$u^{"} = \frac{d^2u}{dy^2} = \frac{\ddot{u}}{\alpha^2} \quad (43)$$

Inserting (43) in (42) given:

$$u^{"} - \frac{c_2}{\alpha^4} y^2 u + \frac{k^2}{\alpha^2} u - \frac{c_1}{y^2} u = 0 \quad (44)$$

The simplification can be a achieved by choosing $\propto$ to satisfy:

$\propto^4 = c_2$,

$$\lambda = \frac{k^2}{\alpha^2} = \frac{2m}{\hbar^2\alpha^2}(E - m_o c^2) \quad (45)$$

Thus inserting equation (45) in equation (44) given:

$$u^{"} + (\lambda - y^2)u - \frac{c_1}{y^2} u = 0 \quad (46)$$

This equation can be solved by taking u to be in the form:

$$u = He^{-\frac{1}{2}y^2} \quad (47)$$

$$\bar{u} = (\bar{H} - yH)e^{-\frac{1}{2}y^2}$$

$$u^{"} = \left(H^{"} - H - yH^{-}\right)e^{-\frac{y^2}{2}} - y(H^{-} - yH)e^{\frac{y^2}{2}}$$

$$u^{"} = \left(H^{"} - 2yH^{/} + y^2H - H\right)e^{-\frac{y^2}{2}} \quad (48)$$

Inserting equation (48) in equation (46) given:

$$H^{"} - 2yH^{/} + y^2H - H + \lambda H - y^2H - \frac{c_1}{y^2}H = 0 \quad (49)$$

If one consider (L=0), thus according to equation (38):

$$c_1 = 0 \quad (50)$$

Thus, equation (49) becomes:

$$H^{"} - 2yH^{/} + (\lambda - 1)H - \frac{c_1}{y^2}H = 0 \quad (51)$$

Let:

$$H = \sum_s a_s\, y^s \ \ ,\ H^{/} = \sum_s a_s y^{s-1}$$

$$H^{"} = \sum_s s(s-1)a_s\, y^{s-2} \quad (52)$$

Substituting equation (52) in equation (51) given:

$$\sum_s s(s-1)a_s\, y^{s-1} - 2\sum s a_s\, y^s + (\lambda - 1)\sum a_s\, y^s - c_1 \sum_s a_s y^{s-2} = 0 \quad (53)$$

Replacing s-2 by s in the first term gives:

$$\sum (s+2)(s+1)a_{s+2}y^s \sum_s [\lambda - 1 - 2s]a_s y^s = 0 \quad (54)$$

Equation the powers of y^s on both sides:

$$[(s+1)(s+2)]a_{s+2} - [\lambda - 1 - 2s]a_s = 0 \quad (55)$$

$$a_{s+2} = \frac{[2s+1-\lambda]}{(s+1)(s+2)} a_s \quad (56)$$

Since the wave function is finite, there for H should be a finite series with finite terms. This requires to terminate H, such that the last term is (s=n). This means that:

$$a_n \neq 0 \quad , \ a_n + 1 = 0 \quad , \ a_{n+2} = 0, a_{n+3} = 0 \ ... \quad (57)$$

$taking$ (s=n) in equation (56) yields

$$0 = a_{n+2} = \frac{[2n+1-\lambda]a_n}{(n+1)(n+2)} \quad (58)$$

This requires:

$$\lambda = 2n + 1 \quad (59)$$

To find the energy E, one uses equations (45) and (41) to get:

$$\propto^2 = (c_2)^{\frac{1}{2}} = \left(\frac{-mk_o}{\hbar^2}\right)^{\frac{1}{2}} - = \left(\frac{-m^2\omega^2}{\hbar^2}\right)^{\frac{1}{2}} = \frac{m\omega}{\hbar} i \quad (60)$$

There fore

$$\lambda = \frac{2m}{\hbar^2 \propto^2}(E - m_o c^2) = \frac{2}{\hbar\omega i} = (E - m_o c^2) \quad (61)$$

$$\frac{2(E - m_o c^2)}{\hbar\omega i} = (2n + 1)$$

$$E - m_o c^2 = (n + \frac{1}{2})\hbar\omega i$$

$$E = m_o c^2 + (n + \frac{1}{2})\hbar\omega i \quad (62)$$

$$m_o c^2 = E - i(n + \frac{1}{2})\hbar\omega \quad (63)$$

This resembles the representation of four-space time relativistic energy in equation (22) and (23) with:

$$\widetilde{E_1} = E_x = E,\ \widetilde{E_4} = ipc = -i(n + \frac{1}{2})\hbar\omega \quad (64)$$

$$\tilde{E} = \widetilde{E_1}\widehat{e_1} + \widetilde{E_4}\widehat{e_4} = E\widehat{e_1} - \left(n + \frac{1}{2}\hbar\omega\right) i\widehat{e_4} \quad (65)$$

Thus, the real energy has given according to equation (24) to be

$$\widetilde{E^2} = E_1^2 + i^2(-\left(n+\frac{1}{2}\hbar\omega\right)^2 \quad (66)$$

$$\widetilde{E^2} = \left|\widetilde{E^2}\right|^2 = E_1^2 + \left(n+\frac{1}{2}\right)^2 \hbar^2\omega^2$$

$$\widetilde{E^2} = \left|\widetilde{E^2}\right|^2 = E^2 + \left(n+\frac{1}{2}\right)^2 \hbar^2\omega^2 \quad (67)$$

Where

$$\left|\widetilde{E}\right| = m_o c^2 \quad (68)$$

Thus

$$m_o^2 c^4 = E^2 + \left(n+\frac{1}{2}\right)^2 \hbar^2\omega^2 \quad (69)$$

Here $m_o c^2$ is real

Since at rest p=0, v=0, $E = mc^2 = \frac{m_o c^2}{\sqrt{1-\frac{v^2}{c^2}}} = m_o c^2$

$$\widetilde{E_4} = ipc = 0 = -i((n+\frac{1}{2})\hbar\omega \quad (70)$$

Which requires

$$\omega = 0 \quad (71)$$

Thus a direct substitution of (70) and (71) in (63) gives

$$m_o c^2 = m_o c^2 - 0 \quad (72)$$

Thus according to equation (69):

$$E^2 = [m_o^2 c^4 - \left(n+\frac{1}{2}\right)^2 \hbar^2\omega^2 \quad (73)$$

For small rest mass

$$m_o = 0 \quad (74)$$

$$E = i(n+\frac{1}{2})\hbar\omega \quad (75)$$

Thus according to equation (75)

And according to the representation (23) the energy results mainly from the kinetic momentum quantized part. It also shows that the momentum is the relativistic energy momentum relation:

$$E^2 = m_o^2 c^4 + p^2 c^2 \quad (76)$$

Can be written in the complex form:

$$E = m_o c^2 + ipc \quad (77)$$

Such that

$$E^2 = |E|^2 = m_o^2 c^4 + p^2 c^2 \quad (78)$$

In view of equation (62)

$$E = m_o c^2 + (n+\frac{1}{2})\hbar\omega i \quad (79)$$

$$E^2 = |E|^2 = m_o^2 c^4 + \left(n+\frac{1}{2}\right)^2 \hbar^2\omega^2 \quad (80)$$

For small rest mass ($m_o = 0$), equation (77) gives :

$$E = ipc \quad (81)$$

Comparing equations (75) and (81)

$$E_4 = pc = (n+\frac{1}{2})\hbar\omega \quad (82)$$

Which stands for the time energy component [see equation (64)], while E represents the spatial components ($E_x, E_y E_z$) as equation (64)indicates , where [see equation(69)]

$$E^2 = E_x^2 + E_y^2 + E_z^2 \quad (83)$$

With

$$\underline{E} = E_x e_1^\wedge, + E_y e_2^\wedge + E_z e_3^\wedge) \quad (84)$$

Thus according to equation (69), as far as $m_o c^2$ is a constant , the fourth dimensional energy including time coordinate is conserved according to equations (62),(63),(69) and (82).

It is also very important to note that when the rest mass part is very small equation (81) gives

$$E = (n+\frac{1}{2})\hbar\omega \quad (85)$$

Which is the ordinary expression for the harmonic oscillator.

III. Conclusion:

The conservation of energy in potential dependent special relativity can be achieved by using 4-dimensional representation with the time part recognizing the momentum. In version, the square of momentum multiplied by the square of the free speed of light subtracted from the curved space energy is invariant and constant everywhere. The quantum equation derived from this equation in a weak filed limit shows that the momentum is quantized and the energy reduced to that of Schrodinger harmonic oscillator where neglecting the rest mass term.

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