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Is the fine-structure constant actually a constant?

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Abstract:*The fine-structure constant denoted by α is a dimensionless number and very nearly equal to 1/137. This number actually represents the probability that an electron will absorb a photon. Also it is the ratio of the velocity of the electron in the first circular orbit of the Bohr model of the atom, to the speed of light in vacuum. This is Sommerfeld's original physical interpretation $\alpha = v_e/c$. For reasons of convenience, historically the value of the reciprocalof the fine-structure constant (1/α) is often specified. The 2018 CODATA recommended value is given byα^{-1} = 137.035999084[1]. Itis a fundamental physical constant which quantifies the strength of the electromagnetic interaction between elementary charged particles. It is a dimensionless quantity related to the elementary charge e, which denotes the strength of the coupling of an elementary charged particle with the electromagnetic field, by the formula $4\pi\varepsilon_0\hbar c\alpha = e^2$This paper addresses some issues regarding this constant.*

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I. Introduction

Mass, Length (which corresponds to area & volume), Time, charge, temperature are quite unique properties. These are so unique that classical physics used M, L and T for mass, Length and Time respectively for dimensions. Later, they build up on the idea of charge and temperature as well. But all of these quantities comes with units. Just for clarification, here we can see how classical physicist used to calculate some of these unique properties for electron by considering it as a particle but not a wave.

Mass:

Mass of an electron = (Mass of a proton / 1835) = (Mass of a H^+ Hydrogen ion / 1835). So, $M_{e-} = M_{p+} / 1835$.
Hence, $1835.M_{e-} = M_{p+}$. Or, $(1835.M_{e-} + M_{e-}) = 1836.M_{e-} = M_{p+} + M_{e-} = M_{H+} + M_{e-}$ = Mass of a Hydrogen atom.
∴ $1836.M_{e-}$ = [Hydrogen H_2 one mole (Gram atomic mass) / Number of atoms inside one mole of H_2Hydrogen]
∴$1836.M_{e-}$ = [1.00784 gm /Avogadro constant] = [1.00784 /6.02214076×10^{23}] = 1 /N = $1.6735576934\times10^{-24}$.
So, the mass of electron, M_{e-} = (Mass of a proton / 1836) = ($1.6735576934\times10^{-24}$/ 1836) = **$9.11\times10^{-28}$ gm.**[2]

Velocity:

Now, $2\pi r = n\lambda = n(h/p) \Rightarrow pr = (nh/2\pi) \Rightarrow mvr = (nh/2\pi) \Rightarrow$Angular Momentum, $L = (nh/2\pi)$. As per Coulomb's law, $F = [(e)\times(e)] / (4\pi.\varepsilon_o.r^2) = e^2/(4\pi.\varepsilon_o.r^2)$ = centripetal force = mv^2/r

$\Rightarrow mv^2/r = e^2/(4\pi.\varepsilon_o.r^2)$

$\Rightarrow mv^2 = e^2/(4\pi.\varepsilon_o.r)$

$\Rightarrow mvr = e^2/(4\pi.\varepsilon_o.v)$

$\Rightarrow (nh/2\pi) = e^2/(4\pi.\varepsilon_o.v)$

$\Rightarrow nh = e^2/(2.\varepsilon_o.v)$

$\Rightarrow v = e^2/(2nh\varepsilon_o)$

So, $\mathbf{v = e^2/(2h\varepsilon_o)}$ as for first orbital n = 1. [3]

Charge:

Now we need to figure out the charge of an electron to figure out the velocity. As in the above equation only the value of e is unknown. Now from electrochemistry, Faraday's first law of electrolysis says that, W = Zit = ZQ. Where, Z = M /nF, here, M = atomic weight, n = number of electrons, &, F = Faraday's constant = 96500 Coulomb. Z is widely known as electrochemical equivalent. Hence, **W = (MQ)/n.96500**. Now for hydrogen, let's consider after electrolysis one mole H_2 got liberated, hence, W = M/n. Or, Q = 96500 Coulomb. As one hydrogen atom consists only one electron, thus, charge of 6.02214076×10^{23}electrons are 96500 Coulomb. So, charge of only one electron is (96500 / 6.02214076×10^{23}) Coulomb = **1.60217662×10^{-19}** coulomb. [4]

Radius:

Now by putting this value of the charge in this equation, $\mathbf{v = e^2/(2h\varepsilon_o)}$ we can get the value of v. We know that, h = 6.62607004×10^{-27}cm^{2}gm/s, e = 1.60217662×10^{-19} coulomb, ε_o = $8.8541878128\times10^{-10}$ F/cm. Hence, the velocity we will get is 2.3×10^8 cm/s.Again, $2\pi r = n\lambda = n(h/p)$

$\Rightarrow r = nh/2\pi p = (nh)/(2\pi mv) = (nh\times2nh\varepsilon_o)/(2\pi m\times e^2) = (n^2h^2\varepsilon_o)/(\pi me^2) = (h^2\varepsilon_o)/(\pi me^2)$, for K orbital n = 1.
Now, for the first orbital r = h / 2πmv = [6.626×10^{-27}cm^{2}gm/s] / [2π (9.11×10^{-28} gm) × (2.3×10^8 cm/s)].
Hence, the radius r = 0.5×10^{-8}cm = 0.5 °A. The actual value is about 0.53 °A. [5]

Energy:
So, we will calculate the total energy of an electron to see whether it matches with the famous $E = mc^2$ equation. Kinetic Energy K.E. = $(1/2).mv^2 = (me^4) / (8n^2\varepsilon_o{}^2h^2)$, we know that, $v = e^2/ (2nh\varepsilon_o)$. Now, $r = (n^2h^2\varepsilon_o) / (\pi me^2)$. Potential Energy (potential difference) $FS\cos\Theta = [\{(e)\times(e)\} / (4\pi.\varepsilon_o.r^2)].r = [e^2/ (4\pi.\varepsilon_o.r)] = (me^4) / (4n^2\varepsilon_o{}^2h^2)$. Which means the potential energy is twice as much high as the kinetic energy. As the direction of these energies are opposite to each other, hence, total energy is: $(me^4) / (8n^2\varepsilon_o{}^2h^2) - (me^4) / (4n^2\varepsilon_o{}^2h^2) = - (me^4) / (8n^2\varepsilon_o{}^2h^2)$.
If an electron jumps from n_1 orbital to n_2 orbital, then the energy emitted: $\Delta E = E_1 - E_2 = hf = \hbar\omega$. Therefore,

$$f = \frac{me^4}{8\varepsilon o^2 h^3}\left(\frac{1}{{N_2}^2} - \frac{1}{{N_1}^2}\right)$$

II. Literature Review

While there are multiple physical interpretations for α, it received its name from Arnold Sommerfeld, who introduced it in 1916,[6] when extending the Bohr model of the atom. α quantifies the gap in the fine structure of the spectral lines of the hydrogen atom, which had been measured precisely by Michelson and Morley in 1887.[7]

The value of the fine structure constant comes into existence from the cosmological constants like: G (Newton's constant), c (Einstein's constant), ħ (reduced Planck's constant), K_B (Boltzmann's constant) and finally Ke (Coulomb's constant).

$$\mathbf{G} = (gR^2/M) = (gR^2/\rho V) = (3gR^2/4\pi\rho R^3) = (3g/4\pi\rho R) = 6.67408 \times 10^{-11}\ m^3kg^{-1}s^{-2} = \left(\frac{R_s c^2}{2m}\right)$$

$$\mathbf{c} = 1/\sqrt{Vacuum\ Permeability\ X\ VacuumPermittivity} = 1/\sqrt{\mu 0\ X\ \varepsilon 0} = 3\ X\ 10^8 m/s$$

$$\mathbf{\hbar} = E/\omega = mc^2t/2\pi = Et/2\pi = h/2\pi = (6.62607004 \times 10^{-34})/2\pi = 1.0545718 \times 10^{-34} m^2\ kg/s$$

$$\mathbf{K_B} = PV/N_AT = (10^5\times 22.4 \times 0.001)/(6.023 \times 10^{23}\times 273) = 1.38065 \times 10^{-23}\ m^2\ kg\ s^{-2}\ K^{-1}$$

$$\mathbf{K_e} = 1/4\pi\varepsilon_0 = 8.9875517923\times10^9\ kg\cdot m^3\cdot s^{-2}\cdot C^{-2}$$ (Note: the unit of ħ can be written as Js)

Using these values the fundamental values for Mass, Length, Time, Temperature and Charge can be counted. All of those corresponding values are shown in the table below.

Fundamental Entity	**Calculated Expression**	**Value in SI unit**
Unit Length	$\sqrt{\frac{\hbar G}{c^3}}$	1.616255×10^{-35} m
Unit Mass	$\sqrt{\frac{\hbar c}{G}}$	2.176434×10^{-8} kg
Unit Time	$\sqrt{\frac{\hbar G}{c^5}}$	5.391247×10^{-44} s
Unit Temperature (Absolute Hot)	$\sqrt{\frac{\hbar c^5}{G{K_B}^2}}$	1.416784×10^{32} K
Unit Charge	$\sqrt{\frac{\hbar c}{K_e}}$	1.875546×10^{-18} C

Using these fundamental values for Mass, Length, Time, Temperature and Charge all other values like area, volume, Force, Pressure, Density, Acceleration, Energy, Power and every other values can also be calculated.Now from the table above it can be understood that all other values in this table has a significance except the value of Mass and Charge. As we have calculated before, the mass of an electron is $9.10938356 \times 10^{-31}$ kg while the unit mass is 2.176434×10^{-8} kg, that is, 2.398×10^{22} times higher than the mass of an electron. This value 2.398 will later come on handy.Also the charge of an electron is 1.60217662×10^{-19}C while the unit charge is 1.875546×10^{-18}C that is,11.706237481 times higher than the charge of an electron. Now,11.706237481 = $\sqrt{137.035999084}$. Means,the ratio has the value of$\alpha^{-1/2}$.

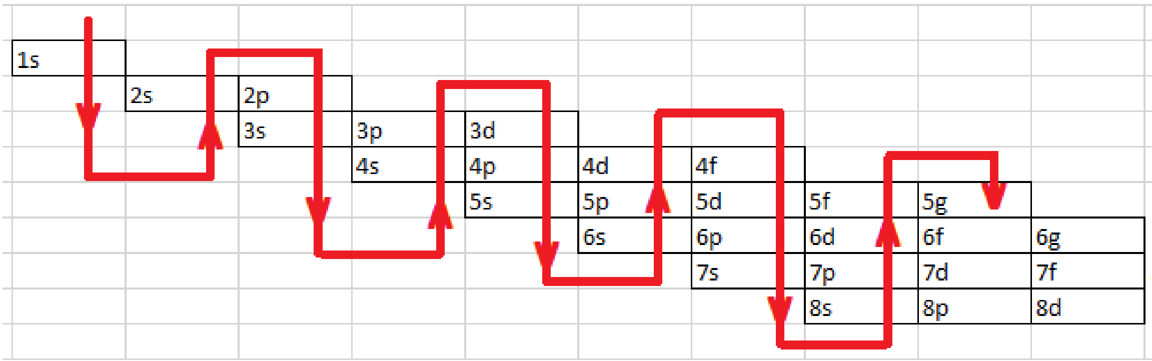
The major finding of this particular article is the expression of Planck entropy $S_p = K_B/16\pi$. It is derived from the following expressions: Schwarzschild radius of a black hole Rs = $\frac{2GM_B}{c^2}$ and the blackhole entropy S = $\frac{K_B A}{4{L_p}^2} = \frac{K_B A}{4G\hbar/c^3}$, [$L_p$ = Planck length and A = $4\pi R_s{}^2$]. Now, Hawking's equation $T_B = \frac{M_B C^2}{2S} = \frac{\hbar c^3}{8\pi K_B G M_B}$. If the blackhole temperature T_B equals the Planck temperature T_P, then, ratio of the mass of the blackhole and Planck mass $M_B/M_P = 1/(8\pi)$. Hence, $S = 4\pi GK_B(M_B{}^2/\hbar c) = 4\pi K_B(M_B/M_p)^2 = K_B/16\pi$

III. Result and Discussion!

137 is the 33rd prime number after 131 and before 139. It is also a Pythagorean prime: a prime number of the form 4n + 1, where n = 34 (137 = 4x34 + 1) or the sum of two squares $11^2 + 4^2$= (121 + 16). But we need to keep in mind the inverse of fine structure constant is almost 137.036, not 137 the full number. And if we multiply the almost precise value of this fine structure constant with the (almost precise) value of the ratio of mass of proton and electron then we get 13.4. Anyway here are some of the significance of this constant:

If we multiply the fine structure constant with π, e and Φ then the value we get is almost equal to 1/10. To make it precisely $\frac{1}{10}$we need to take the value 2.398 into consideration, as it was mentioned earlier that the value will come on handy. Here is the equation: e.Φ.α.π.P(z ≤ 2.398) = 1/10.

Classical physics tells us that electrons captured by element #137 (Feynmanium an undiscovered element with the symbol Fy and atomic number 137, named in honor of Richard Feynman) of the periodic table will move at the speed of light. The idea is quite simple, as1/137 is the odds that an electron will absorb a single photon. Protons and electrons are bound by interactions with photons. So when we get 137 protons, we get 137 photons, and we get a 100% chance of absorption. An electron in the ground state will orbit at the speed of light. This is the electromagnetic equivalent of a black hole. Which means, if the first g orbital gets fully occupied, then the element will instantly turn into energy by making its existence as temporary as possible.



Here in the Aufbau principal diagram we can see that, for the first time when any g orbital gets fully occupied then it is supposed to get an atomic number of 138. The maximum occupancy level of these s, p, d, f, & g orbitals are given as [2.(2n + 1)]; where n = 0, 1, 2, 3, & 4. Hence, it is 2, 6, 10, 14, & 18 for s, p, d ,f, &g respectively. The maximum occupancy for K, L, M, N etc. shell can be calculated using the formula $2n^2$; where n = 1, 2, 3, 4… etc. The detail calculation is given below when for the first time any g orbital gets fully occupied:

1s2 2s2 2p6 3s2 4s2 3p63d10 4p6 5s2 6s2 5p6 4d10 4f14 5d10 6p6 7s2 8s2 7p6 6d10 5f14 5g18

Here we can see up to the element number 120 we do notobserve the presence of g orbital. Even Unbinilium, also known as eka-radium or simply element 120, is the hypothetical chemical element in the periodic table with symbol Ubn and atomic number 120. The common error occurs if we use the shell formula instead of Aufbau principal. Cause with the formula $2n^2$shell K, L, M, N, O, & P has a maximum occupancy level of 2, 8, 18, 32, & 50 respectively. And the chemical is supposed to be (2 + 8 + 18 + 32 + 50) = 110, Darmstadtium (Ds), artificially produced highly radioactive element. Anyway, after the hypothetical 120th element for the first time g orbital comes into existence. And when it gets fully occupied with the allotted 18 electrons, then the total number of electrons in the element becomes (120 + 18) = 138. And there is a 100% probability that an electron will absorb a photon.

IV. Conclusion

Some equivalent definitions of α in terms of other fundamental physical constants are given below:

$$\alpha = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\hbar c} = \frac{\mu_0}{4\pi}\frac{e^2 c}{\hbar} = \frac{k_e e^2}{\hbar c} = \frac{e^2}{2\varepsilon_0 ch} = \frac{c\mu_0}{2R_K} = \frac{e^2 Z_0}{2h} = \frac{e^2 Z_0}{4\pi\hbar} = \frac{v_e}{c} = \left(\frac{e}{q_P}\right)^2 = \frac{2\pi r_e}{\lambda_e} = \frac{\lambda_e}{2\pi a_0}$$

Here,

- e is the elementary charge (= $1.602176634\times10^{-19}$ C).
- π is the mathematical constant pi.
- h is the Planck constant (= 6.62607015×10^{-34} J·s).
- $\hbar = h/2\pi$ is the reduced Planck constant (= 6.62607015×10^{-34} J·s/2π.
- c is the speed of light in vacuum (= 299792458 m/s).
- ε_0 is the electric constant or permittivity in vacuum (or free space).
- μ_0 is the magnetic constant or permeability in vacuum (or free space).
- k_e is the Coulomb constant.
- R_K is the von Klitzing constant.
- Z_0 is the vacuum impedance or impedance in free space.
- v_e is the velocity of the electron in the first circular orbit of the Bohr model of the atom.
- q_p is the Planck charge that we have calculated previously.
- $r_e$is the classical electron radius, $\lambda_e$is the Compton wavelength, &a_0 is the Bohr radius.

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