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Snell's Law Lab Report

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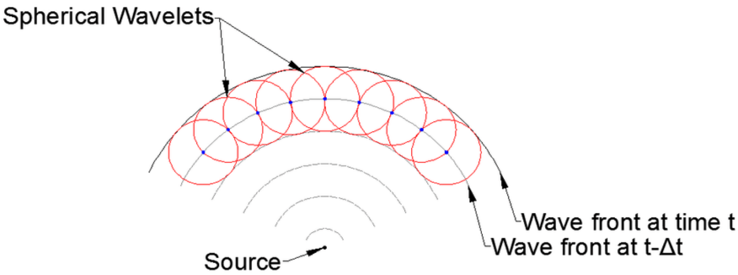
I. Research Question:

What is the relationship between the angle of incidence and angle of refraction of a red laser beam incident on an air-water surface?

II. Background

When a beam of light or any other wave passes through a surface separating two media of different refractive indexes, the light bends changing its direction. This is known as refraction. Snell's law relates the direction of the incident beam and the direction of the refracted beam. To understand the phenomena of refraction, we first refer to Huygen's principle. This states that a beam of light reaches a point in space. This point becomes a new source of secondary wavelets traveling in all directions. The figure below shows how each point on the wavefronts from the primary source (represented in blue) becomes a new source of secondary wavefronts (represented in red).

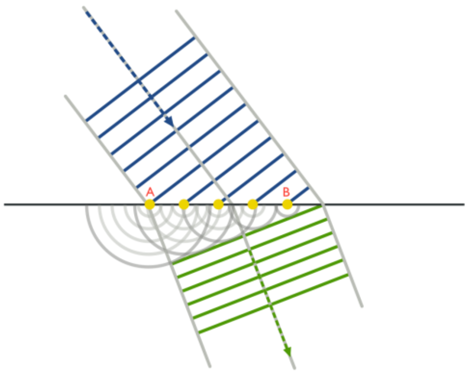
Figure 1: Huygens Principle



Citation in Bibliography

According to Huygen's principle, as the different rays of light reach the surface between the two media, the rays continue traveling along the direction where they are in phase.

Figure 2: Travelling Through Different Media



Citation in Bibliography

For the case shown in the figure above, the rays are traveling from the medium of lower refractive index to the medium of higher refractive index. The ray reaching point B has travelled a longer distance than the ray reaching point A. Therefore, there is a certain phase difference between the two. Thus, the two rays cannot continue like this as they need to remain in phase. They will compensate for the phase difference. If the refractive indexes are the same, the distance compensated would equal the difference between the distance originally travelled. However, in this lab the refractive indexes are not the same and therefore the distance will not be the same. This can be calculated using the refractive index formula.

$$n = \frac{c}{v}$$

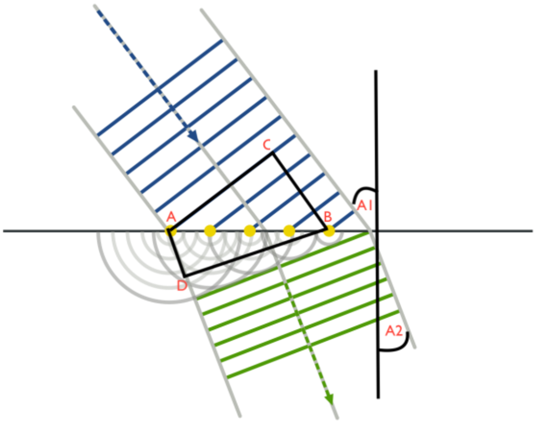
With n being the refractive index, c being the speed of light and v being the speed of the electromagnetic wave in a given medium. The speed of light C is always the same. Therefore, when the refractive index changes, v must also change. If the refractive index increases, v decreases and vice versa. We also know that:

$$v = f\lambda$$

With f being the frequency and λ being the wavelength. The frequency goes back to the source of the wave and therefore, is unchanged as it goes between different media. Thus, if the velocity varies the wavelength must change. If the velocity increases so will the wavelength and vice versa.

To go back to the description, since the refractive index of water is greater than that of air than the velocity must decrease and therefore the wavelength must decrease. Hence, the phase difference compensation of the rays, will be less than the original distance travelled. Quantitative demonstration is shown below.

Figure 3: Path Difference



Citation in Bibliography (Same as above)

The time taken to travel the given distance in the air is:

$$t_{air} = \frac{distance}{speed} = \frac{CB}{v_{air}}$$

The time taken to travel the given distance in the water is:

$$t_{w} = \frac{distance}{speed} = \frac{AD}{v_{water}}$$

We know that the time taken to travel the different distances between the rays will be the same therefore:

$$t_a = t_w$$

and

$$\frac{CB}{v_{air}} = \frac{AD}{v_{water}}$$

Distance AS can be calculated using trigonometry. Note that angle CAB is equal to angle A1 (will call it θ_1) and since angle ABD is equal to angle A2 (will call it θ_2). Therefore, from the right triangle CAB:

$$CB = AB\sin(CAB) = AB\sin(\theta_1)$$

And from the right angle triangle ABD:

$$AD = AB\sin(ABD) = AB\sin(\theta_2)$$

Substituting for CB and AD into the equation relating the times:

$$\frac{AB\sin(\theta_2)}{v_{water}} = \frac{AB\sin(\theta_1)}{v_{air}}$$

Or

$$\frac{□□□(□_2)}{□_{□□□□□}} = \frac{□□□(□_1)}{□_{□□□}}$$

This is known as Snell's law; relating the angles of incidence and refraction to the speeds of light in both media. This can be reworked to use other variables. For instance, since

$$□ = \frac{□}{□}$$

then

$$\frac{□_2□□□(□_2)}{□_{□□□□□}} = \frac{□_1□□□(□_1)}{□_{□□□}}$$

And therefore

$$□_2□□□(□_2) = □_1□□□(□_1)$$

III. Procedures

A. Variable Selection

1. Independent Variable: The incident angle is the independent variable for this lab. In this given procedure a wide range of angles was used, ranging from approximately 10 - 70 degrees. The range is important as it needs to have a difference large enough where there will be notable differences in the data, if the angles are close together it will serve as an unreliable independent variable, with data points very similar to one another. Also, the angle must be below 90° in order for refraction to occur. Subsequent to the beginning of each trial the tray will be positioned at the center of the protractor paper (°) +/-0.5 and measured as accurately as the naked eye possibly can. However, an assumption made here is that the tray is horizontally on the measuring sheet throughout.

2. Dependent Variable: The refracted angle is the dependent variable. To ensure accuracy, the pen listed in the materials will be used to precisely measure the refracted angle. A dot will be marked exactly where the incident light is refracted. However, the results will be as accurate as the human eye can be. The angle will be taken five times for seven incidents. There will be another observer who will measure the refracted beam on the protractor paper to ensure accuracy. An assumption made here is that the refractive index of air is exactly 1. This would be the case if we are in a vacuum, however, for now, we are assuming a refractive index of exactly one.

3. Controlled Variables:

a) The liquid in the tray: For each trial, the liquid in the tray remains constant. In order to keep the data consistent and accurate, this must be the case. H2O is the only substance used for this specific lab. Also, the same H2O should be used for the entirety of the process. An assumption made here is that the H2O is completely pure.

b) Laser: More specifically, the color of the laser. This must be controlled as different colors have different wavelengths and according to snell's law and as stated above: $\frac{□□□\ □_1}{□□□\ □_2} = \frac{□_1}{□_2}$. Therefore, by changing the wavelength of the incident angle, the refracted angle will be affected. This can easily be controlled by only using one laser throughout the entire process.

c) Position of Tray: It is very important for this to be controlled to ensure an accurate angle of refraction. If the tray is even moved slightly, it will cause an error in the data. The tray must be the same distance away from the laser and the tray must be perpendicular to the normal at all times. If the tray is at a different angle, the angle of refraction will be inaccurate. If the position of the tray is farther from the laser or closer it will alter the waist size of the refracted beam and can make it harder to read. An assumption made is that the refractive index of the tray is nonexistent. Plastic has a refractive index of about 1.5 but it varies due to its thickness. We are assuming that the thickness is small therefore it can be negligible.

Hypothesis:

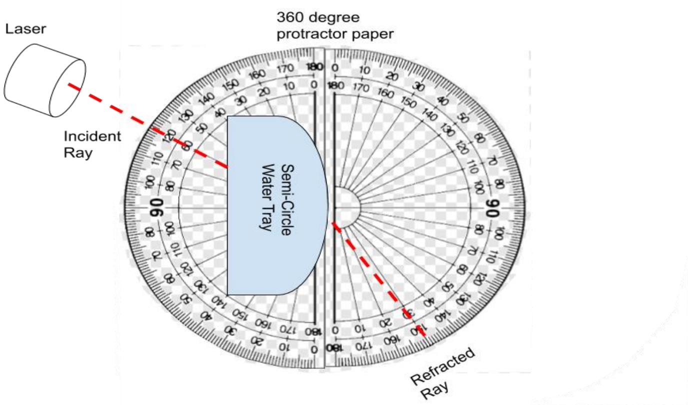
If the sin value of the angle of incidence is increasing then the sin value of the angle of refraction will increase proportionally because they have a directly proportional relationship. The relationship can be modelled by the equation derived in the background information: $□_{□□□□□} = \frac{□□□(□_{□□□})}{□□□(□_{□□□□□})} * □_{□□□}$. If sin($□_{□□□}$) increases by a certain value then sin($□_{□□□□□}$) will increase by the same value to keep $□_{□□□□□}$ the same.

Materials:

- Vernier Laser Pointer
  - Used to create the beam of light
  - Also bring two extra AAA batteries just in case the laser malfunctions
  - Power < 1mW should be okay

- Protractor Paper ± 0.5°
  - Either one 360° or two 180°
  - Everything will be placed on it
  - Used to record angle of incidence and refraction
- Pencil/Pen
  - Used to record the data
  - Also used to mark the refracted beam of light on the protractor for a more accurate reading
- Datasheet
  - Used to record the data
- Goggles/Protective Eyewear
  - To protect from the high intensity laser
  - Better to use goggles with an anti-reflective surface to help see better
- Water
  - Used to fill up the Semi-circle tray, amount is dependant on the volume of the tray you should fill the tray up
  - Preferably distilled water, this is more pure and will give a more accurate refractive index
- Semi-Circle Plastic Empty Tray
  - Used to hold the water
  - Try to have a small thickness as to not interfere with the actual refractive index
    - We are making an assumption that the refractive index of plastic is negligible
  - It is very important that the tray is a semi- circle as after the refraction in the surface of air and water, the ray must hit the water plastic surface normally in regard to the angle of incidence and a semi-circle will allow that to happen

Figure 4: Diagram of Apparatus Diagram of Apparatus Used Below
Created on google drawings By: Alaa El-Zayat



Steps:

1. Lay down your protractor paper on a table
2. Fill up semi-circle tray with water (preferably distilled water)
   a. Distilled water will work better as it is more pure and therefore will have a more accurate refractive index
3. Place the semi-circle water tray on the 0° axis with the straight side facing you
   a. Make sure that the center of the semicircle is at the origin
   b. Ensure that this is a placer where the semicircle can be placed at for all 35 trials of this lab as this needs to be controlled
4. Set up a place where you will shine your laser at the semi-circle to allow refraction to occur
5. Take the laser and place it just around the perimeter of the protractor paper
   a. Ensure that only one laser is used throughout to ensure that the same light of the same wavelength is used. This also must be controlled
6. Select seven different angles that you will complete your lab with
   a. It is better to have these angles beforehand as it helps reduce confusion
   b. All angles should be less than 90° a good range to use is anywhere from 10°-80°

7. Place your laser at your first angle and shine it directly at the origin
a. It must go through the origin in all trials to ensure accurate data
8. Record the refracted light on the other side of the semi-circle in your data table
a. You should see a complete refracted beam although, in most cases only a small dot will appear on the protractor paper, this is okay as well.
9. Repeat these steps for six more angles
a. Angles should be different from each other
b. Angles should not be very close together to have a good data set and prevent the results from being very close to each other
10. Repeat these steps 4 more times
a. This will give you a total of five results for each of the seven angles

Safety Considerations:

For the most part this lab is pretty safe and is a very low risk lab. However, for people who have sensitive eyes it is important that they wear protective eye wear to reduce the risk of damaging their eyes from the laser. A laser produces in phase highly monochromatic light that is directional and is concentrated on a very small spot, therefore it will have a high intensity and has the potential to damage sensitive eyes. Furthermore, since we are dealing with water in an open top tray, it is important not to spill the water on the laser as it can electrocute someone. These can be avoided by being very careful with the semi-circle tray and the laser as well as wearing protective eyewear.

IV. Data Collection

Raw Data:

Table 1A below shows the data collected from the refraction. The first column shows the angle selected. The second through sixth columns all show the resulting refracted angle from the Seven different angles that were used, and each was tried five times in an attempt to minimize random errors. However, fortunately, all data collected seemed to be within a consistent and accurate range of one another.

Table 1A: Raw Data

AIR Angle (°) +/-0.5	Refracted Angle 1 (°)	Refracted Angle 2 (°)	Refracted Angle 3 (°)	Refracted Angle 4 (°)	Refracted Angle 5 (°)
10.0	7.0	5.0	7.0	6.0	8.5
20.0	15.0	15.0	14.0	14.0	15.0
30.0	19.5	23.0	21.0	21.0	21.0
40.0	25.0	28.0	26.0	28.0	28.0
50.0	35.0	34.0	32.0	34.5	33.0
60.0	40.0	39.0	39.0	40.0	39.0
70.0	44.0	45.0	44.0	42.0	43.0

Processed Data:

To be able to plot the data points against each other, the average refracted angle must be calculated. To calculate the averages, the data from all five trials were added together and divided by five. As observed in the tables above, with an increase in the angle of incidence, there is also an increase in the refracted angle, ultimately showing a proportional relationship.

Table 1B: Refracted Data

AIR Angle (°) +/-0.5	Refracted Average (°)	Refracted Max (°)	Refracted Min (°)
10.0	6.7	8.5	5.0
20.0	14.6	15.0	14.0
30.0	21.1	23.0	19.5
40.0	27.0	28.0	25.0
50.0	33.7	35.0	32.0
60.0	39.4	40.0	39.0
70.0	43.6	45.0	42.0

Snell's law states that the incident angle is directly proportional to the refracted one. However, assuming that the two angles themselves have a proportional relationship is incorrect. Snell's law states that $n_1 \sin\theta_1 = n_2 \sin\theta_2$ meaning that the sin value of the two angles is what is proportional and not just the angles themselves. The angles were converted into radians and the sin value was taken.

Table 2B: Air Data

AIR Angle (°) +/-0.5	AIR sin(AIR) **Y-axis**	AIR sin(MAX)	AIR sin(MIN)
10.0	0.174	0.182	0.165
20.0	0.342	0.350	0.334
30.0	0.500	0.508	0.492
40.0	0.643	0.649	0.636
50.0	0.766	0.772	0.760
60.0	0.866	0.870	0.862
70.0	0.940	0.943	0.937

The same will be done to the average of the refracted angle in order to have a proper correlation between the two.

Table 3B: H2O Data

AIR Angle (°) +/-0.5	H2O sin(AVG) **X-Axis**	H2O sin(MAX)	H2O sin(MIN)
10.0	0.117	0.148	0.087
20.0	0.252	0.259	0.242
30.0	0.360	0.391	0.334
40.0	0.454	0.469	0.423
50.0	0.555	0.574	0.530
60.0	0.635	0.643	0.629
70.0	0.689	0.707	0.669

As labeled in the table above the x-axis will be presented by the H2O sin(AVG) and the Y-axis will be presented by the sin(AIR) values. This is different from what was stated above in the exploration. It was stated that the angle of incidence (angle in the air) was the independent variable, however, here it is the H2O sin(AVG) that lies on the x-axis. This is due to the fact that it leads to a more simple answer for the refractive index. Snell's law shows that $\frac{\sin(\theta_{air})}{\sin(\theta_{water})} = \frac{n_{water}}{n_{air}}$ and therefore $n_{water} = \frac{\sin(\theta_{air})}{\sin(\theta_{water})} * n_{air}$. Theoretically, n_{air} is equivalent to one, thus it will have no effect on the formula, hence $n_{water} = \frac{\sin(\theta_{air})}{\sin(\theta_{water})}$. Therefore by putting the H2O sin(AVG) on the x-axis and the sin(AIR) on the y-axis, the slope of the graph will be the refractive index of water.

Figure 5: Line of Best Fit

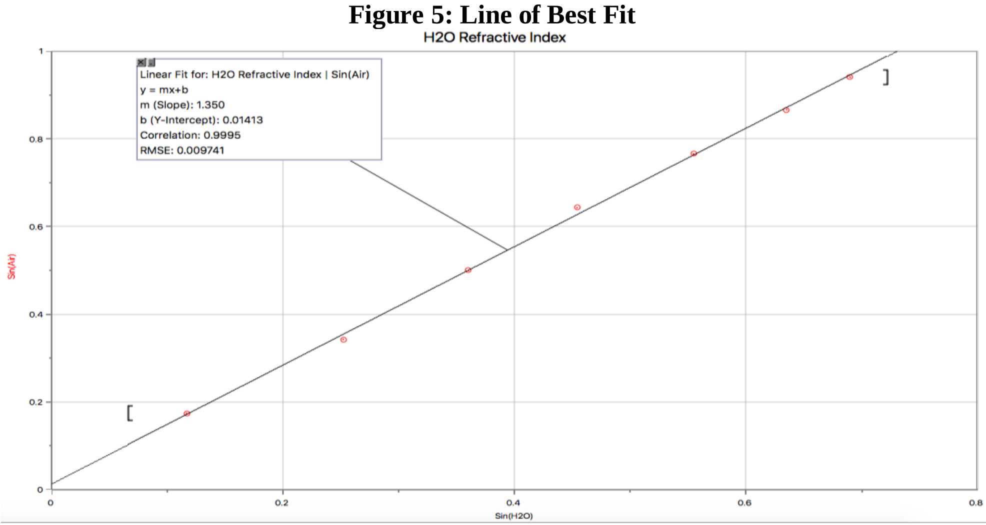


Figure 5 displays the line of best fit for the data plotted. Evidently, a linear fit was used to depict this data. This can be known from the relationship between the two variables. A directly proportional relationship means that it is a linear relationship that goes through the origin. Although, unfortunately this doesn't go through the origin, however, this is a source of error that will be stressed on later. Quantitatively, it can also be derived that the relationship will be linear. As stated above, snells law shows that $\frac{sin(\theta_{air})}{sin(\theta_{water})} = \frac{n_{water}}{n_{air}}$, and ultimately, it was concluded that $n_{water} = \frac{sin(\theta_{air})}{sin(\theta_{water})} * n_{air}$. To make this clearer it can be directly compared with our basic linear equation. Y = mx + B. In comparison with this equation, the n_{water} is the y value, the n_{air} is the x value, the $\frac{sin(\theta_{air})}{sin(\theta_{water})}$ is the m value (slope) with the B value being zero.

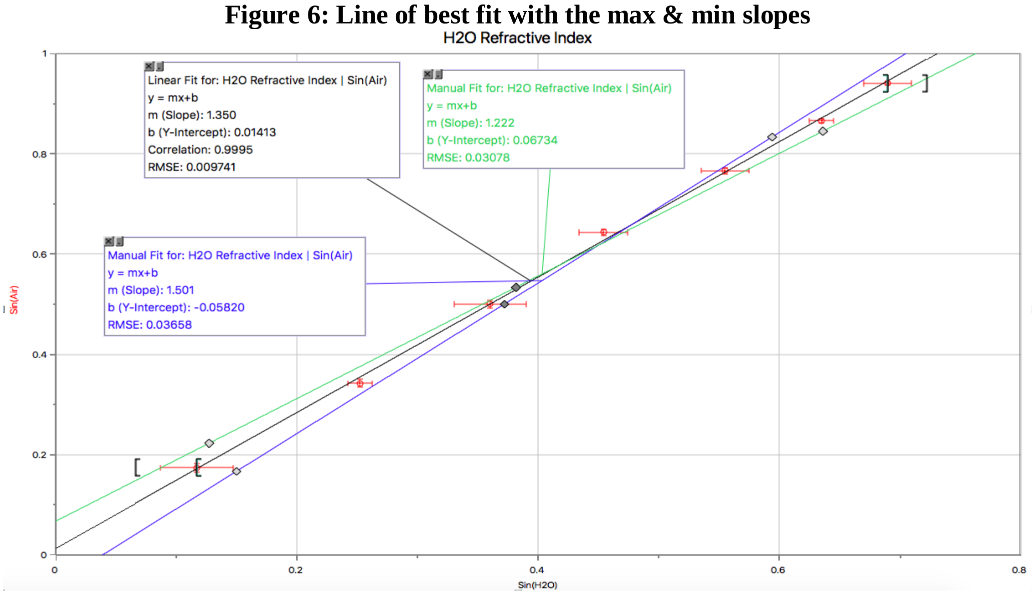
This is the theoretical case. When our data was tested it gave an equation of y=1.35x+.01413. The data has a b value when this shouldn't be the case. This leads back to the sources of error. The average slope was calculated to be 1.35, although this does, in fact, have uncertainty and therefore cannot be taken as 1.35 alone. There are two axes and therefore there are two uncertainties to calculate.

Table 4B: Uncertainties

AIR Angle (°) +/- 0.5	H2O sin(MAX)	H2O sin(MIN)	**Uncertainty H2O Sin(AVG)**	AIR sin(MAX)	AIR sin(MIN)	**Uncertainty Air Sin(AIR)**
10.0	0.148	0.087	0.030	0.182	0.165	0.009
20.0	0.259	0.242	0.008	0.350	0.334	0.008
30.0	0.391	0.334	0.028	0.508	0.492	0.008
40.0	0.469	0.423	0.023	0.649	0.636	0.007
50.0	0.574	0.530	0.022	0.772	0.760	0.006
60.0	0.643	0.629	0.007	0.870	0.862	0.004
70.0	0.707	0.669	0.019	0.943	0.937	0.003

The uncertainty for the angle of incidence (air) was calculated using the max and min values for the sin values of the refracted angles. The equation $\frac{max - min}{2}$ was used to achieve both sets of uncertainties. For instance, for the angle of *10* ∘for H2O, the sin value of the min .087 was subtracted from the sin value of the max .148 and was then divided by two to get .030 as an uncertainty. This was then done for all seven angles. The exact same was done for the uncertainty of the angle of incidence (air). These uncertainties are then used to create the max and min slopes.

Figure 6: Line of best fit with the max & min slopes



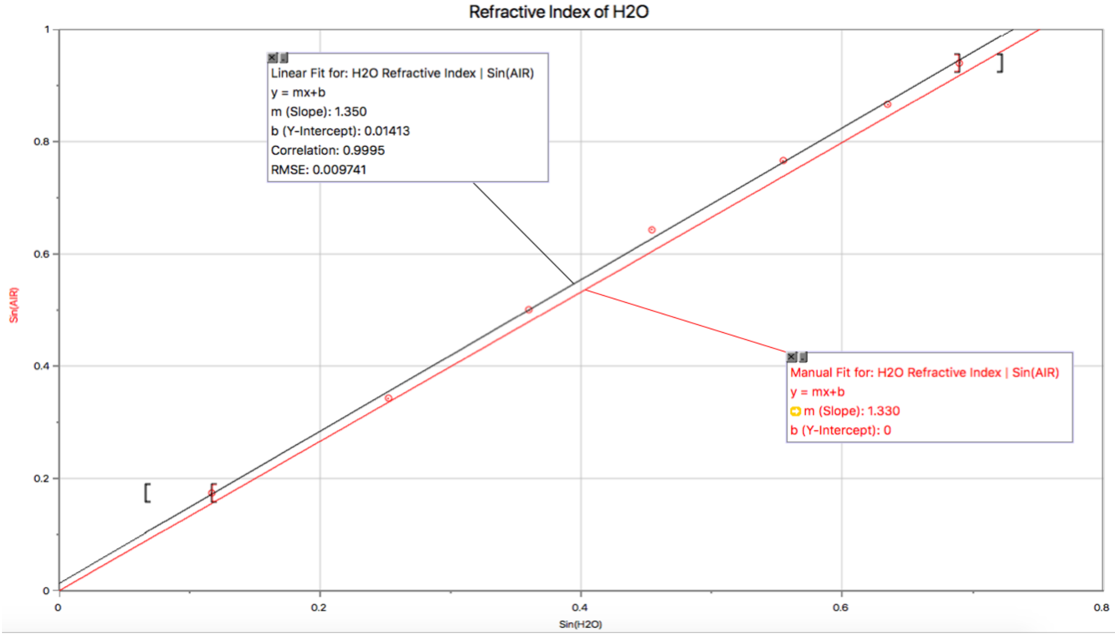
Max slope - Best fit slope = 1.501 - 1.350 = .151
Best fit slope - Min slope = 1.350 - 1.222 = .128
Slope with uncertainty = 1.350±.151

Shown above in figure 6, the max and min slopes were 1.501 and 1.222 respectively. In figure five the slope obtained was 1.350. The max and min slopes were illustrated with the use of the uncertainties. These slopes are crucial to be able to calculate the overall uncertainty for the refractive index of water. When the best-fit slope is subtracted from the max slope value of .151 is obtained. When the min slope is subtracted from the best-fit slope value of .128 is obtained. Therefore, a more specific representation for the refractive index of water according to this data is 1.350±.151.

Percent uncertainty = (1.501-1.222)/2 * 100 = 13.98% ~ 14%

Additionally, Percent uncertainty represents the margin of error in a data set. Since it's represented using a + or - this shows that the data is not consistent and is, therefore, random. The percent uncertainty is achieved through. The number achieved from this procedure was about 14%, which is not too high but is definitely not low. This is technically the case in our experiments as we do not conduct them in a full proof closed environment. These errors will be stressed on later on.

Figure 7: Comparison vs Actual Refractive Index of H2O



The universally known value for the refractive index of water is 1.33. After completing the necessary procedure we derived an experimental value, 1.35 which is fairly close to the actual value. The formula for calculating a percent error is $\frac{□□□□□□□□□□□□ - □□□□□□}{□□□□□□} * 100$. The percent error was about 1.5%, almost perfect. This means that the lab we conducted, featured very little systematic error. This type of error occurs when there is a constant source of error that is affecting the data in one way. Meaning that it adds to the value only or subtracts from the value only. These sources of error will be stressed on later.

V. Conclusion & Evaluation

All in all, to answer the question prompted in the beginning, "What is the relationship between the angle of incidence and angle of refraction of a red laser beam incident on an air-water surface?" Through the processing of the data collected in this procedure, we are able to verify that the relationship is directly proportional. Figure 6 shows the average sin value of the refracted angles plotted against the sin value of the incident angles. The best fit line is linear and fortunately, the points on the graph do agree with this. Through rearranging the equations introduced in the background, it does in fact show that the sign values of the two angles should be directly proportional. The trend in the data can be identified through a correlation coefficient of .9995, indicating a linear relationship between the two and confirming the snell law derivation. Furthermore, ff the value calculated for the refractive index of water in the lab is close to the universally known refractive index this will confirm the relationship between the two. This is because we are taking the relationship to be $□_{□□□□□} = \frac{□□□(□_{□□□})}{□□□(□_{□□□□□})} * □_{□□□}$ and therefore we are expecting a result for the refractive index of water from $\frac{□□□(□_{□□□})}{□□□(□_{□□□□□})}$ or the slope of the graph. With a calculated percent error between the two of 1.5% this shows a strong relationship between the two and that it is in fact the equation shown above, which suggests that they are directly proportional.

This strong correlation coefficient can be attributed to the many strengths of the procedure. The main strength of this is that we used a semi-circle tray to shine the laser at. As stated above, after the refraction in the surface of air and water, the ray must hit the water plastic surface normally in regard to the angle of incidence. A semi-circle tray ensures that this will happen and therefore gives more accurate data. Furthermore, another strength is that there were two people conducting this lab. There was one person shining the laser and another taking the readings off of the protractor paper. This increases the probability of an accurate reading and ultimately gives more reliable and accurate data.

Unfortunately though, the data was supposed to indicate a directly proportional relationship, as according to the background shown above, this is the case within snell’s law. This means that the line of best fit should have passed through the origin, however, it intercepted the Y-axis .014 units above the origin, indicating sources of error. This shows a systematic error, represented by a percent error, that needs to be addressed. However, it must not be regarded that this lab was pretty accurate, with a calculated percent error of 1.5%.Nonetheless, there are systematic errors that we simply cannot ignore. In addition, Percent uncertainty represents the margin of error in a data set. Since it’s represented using a + or - this shows that the data is not consistent and is, therefore, random. The percent uncertainty is achieved through subtracting the min slope from the max slope and dividing it by two. The number achieved from this procedure was about 14%, which is not too high but is definitely not low. This number indicates various random errors that will be addressed.

One of the main problems found when trying to read the angle to which the laser beam was refracted was related to the illuminated spot on the protractor measuring the angle of refraction. The laser beam traveled the full radius of the water tray after refracting at the surface, and therefore, did not keep its narrow beam waist. Instead, it kept on getting wider resulting in a larger spot at the scale of the protractor. This introduced random errors when reading the scale. Usually, an optical system would be used in such situations to refocus the beam to a single point. However, this solution would be too complicated and impractical to use in my setup. Instead, using a smaller cylindrical water container, in which the laser beam travels just a few cm instead of about 15 cm would be more practical. This way the beam waist of the laser beam would not spread as much as it did and the bright spot on the protractor scale would be much smaller.

Another main source of error in determining the refractive index of water, which should be 1.33, was related to the existence of impurities in the water. The deviation of the measured index of refraction from the actual one would increase with increasing concentration of these impurities. This represents a source of systematic error. To solve this problem, the experiment requires the use of distilled water, in addition to careful cleaning of the inner surface of the plastic container to avoid introducing any impurities to the distilled water. Furthermore, the water should be covered while not using it to prevent any external sources that could damage the purity of the water.

Furthermore, there was another source of random errors related to the laser beam not passing the center of both protractors. This introduces errors to the readings of both, the angle of incidence and the angle of refraction. To ensure that the laser beam can only pass through the center of both protractors. A setup should be used where the both protractors are fixed to a mask with a single hole. The mask is fixed such that its plane is perpendicular to the plane of both protractors. The hole of the mask should be aligned exactly at the centers of both protractors. This setup would avoid any light reaching the water surface unless it is exactly at the required point at the center of both protractors.

Extension:

As we already have a setup that can easily by used to measure the refractive index of water or any other liquid, it would be interesting to use the setup or a modified version of the setup, to measure the refractive index of other liquids or extend the experiment by measuring the refractive index of water with known impurities that are added to the water at determined concentrations. For example, one could measure the refractive index of a sugar-water solution or salt-water solution and record the values for different concentrations, which can then be used to determine the concentration of a solution of unknown concentration by measuring its refractive index and comparing the results with the recorded values.

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