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Nonlinearly Trapped Particle Dynamics in TurbulentPlasma Systems

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Abstract
Conditions under which a density gradient of wave energy can lead to particle trapping in a plasma, have been discussed. It has been shown that such trapping can give rise to a new series of side band instabilities with a frequency separation proportional to the wave field amplitude in contrast to the usual side band instabilities due to phase trapping (where the frequency gap is proportional to $\sqrt{\mathbf{E}}$ *)*
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I. Introduction

The nonlinear effects of plasma are extremely diverse and it is important to access the various types of effects in a plasma wherever the collective fields act on the individual particles. One of the important problem is the nonlinear trapping of the particles by waves.

The effect of phase trapping of resonant particles leads to nonlinear saturation in damping of waves [1,2].The excitation of side band instabilities is due to coherent oscillations of these particles [3]. Effect of such particles trapped in an electromagnetic wave is responsible for a series of resonances for an electrostatic wave [4]. Similar resonances are possible for a whistler wave by the particles trapped in an electrostatic wave [5].However, phase trapping of particles may not be possible if the waves are incoherent and also in the presence of wave turbulence.

On the other hand, the density gradient of RF field has been used to increase the confinement of hot plasma in a mirror machine [6].Following the similar argument we have shown here that even in the absence of phase trapping, the particles can be trapped due to the density gradient of the wave energy under different conditions. The general expression for the bounce frequency is obtained.

II. Mathematical model

Let us consider the motion of an electron in a plasma containing high frequency electric field in equilibrium. This equilibrium electric field may be due to an externally applied pump as for example in laser produced plasmas or due to the presence of turbulence. Westart with magnetic field free plasma, for simplicity, and effect of presence of magnetic field will be discussed thereafter. The equation for motion of an electron is

$$\mathbf{m}\frac{\mathbf{d}^2\mathbf{x}}{\mathbf{dt}^2} = -\mathbf{eE}(\mathbf{x})\mathbf{e}^{\mathbf{i}(\omega\mathbf{t}-\mathbf{kx})} \quad [1]$$

where $\mathbf{m}$ and $-\mathbf{e}$ denote mass and charge of electron, $\mathbf{x}$ is its position co-ordinate at time $\mathbf{t}$, ω and $\mathbf{k}$ are the frequency and wave number of the oscillating field. The variation of wave amplitude is supposed to be weak compared to the fast oscillations, so that $\mathbf{E}(\mathbf{x})$ can be expanded in terms of Taylor series around $\mathbf{x}_0$ as

$$\mathbf{E}(\mathbf{x}) = \mathbf{E}(\mathbf{x}_0) + (\mathbf{x}-\mathbf{x}_0)\left(\frac{\partial\mathbf{E}}{\partial\mathbf{x}}\right)_{\mathbf{x}=\mathbf{x}_0} + \ldots\ldots\ldots \quad [2]$$

It is easy to show that the averaged orbit of electron is described by

$$\frac{\mathbf{d}^2}{\mathbf{dt}^2}\mathbf{x} = \frac{-\mathbf{e}^2}{2\mathbf{m}^2\omega^2}\frac{\mathbf{d}}{\mathbf{dx}}\left|\mathbf{E}(\mathbf{x})\right|^2 \quad [3]$$

where the amplitude $\mathbf{E}(\mathbf{x})$ has been assumed almost constant in the process of averaging over the fastoscillations. The zero order $(\mathbf{E}=0)$ velocity of the particle is assumed to be zero. Without any loss ofgenerality $\boldsymbol{\omega}$ can be put as a Doppler shifted frequency to account the initial translatory motions.
The force on the right hand side of equation [2]is well known ponderomotive force and is in the direction of negative gradient of $|\mathbf{E}(\mathbf{x})|^2$. This force is responsible for driving the particle out of the region of high field energy density [7-9]. However, in general, the situation may be different as discussed later in this manuscript.
Using the definition of energy density[10]

$$\mathbf{W}=\frac{1}{\boldsymbol{\omega}}\frac{\partial}{\partial\boldsymbol{\omega}}\left[\boldsymbol{\omega}^2\mathbf{D}(\mathbf{k},\boldsymbol{\omega})\right]\frac{\mathbf{E}^2}{8\boldsymbol{\pi}},$$

we can write equation [3] as

$$\boldsymbol{\mu}\frac{\mathbf{d}^2\mathbf{x}}{\mathbf{dt}^2}=-\boldsymbol{\sigma}\frac{\mathbf{dW}}{\mathbf{dx}} \quad [4]$$

where $\boldsymbol{\mu}=\left(\frac{\mathbf{m}}{\mathbf{e}}\right)^2$ and

$$\boldsymbol{\sigma}=4\boldsymbol{\pi}\left[\boldsymbol{\omega}\frac{\partial}{\partial\boldsymbol{\omega}}\left[\boldsymbol{\omega}^2\mathbf{D}(\mathbf{k},\boldsymbol{\omega})\right]\right]^{-1} \quad [5]$$

where $\mathbf{D}(\mathbf{k},\boldsymbol{\omega})$ is the dielectric constant of the medium.
We immediately notice that equation [4] is nothing but description of motion of a particle of mass $\boldsymbol{\mu}$ and charge $\boldsymbol{\sigma}$, where the wave energy density $\mathbf{W}$ plays the role of effective potential. Evidently for a positive effective charge, the particle is accelerated towards the decreasing field energy density $\mathbf{W}$ and decelerated towards increasing $\mathbf{W}$. Under proper conditions some of the particles can get trapped inside a potential well. Similarly for a negative charge, some particles will be trapped on the potential hill. The explicit form of the energy conservation law

$$\frac{1}{2}\boldsymbol{\mu}\left(\frac{\mathbf{dx}}{\mathbf{dt}}\right)^2+\boldsymbol{\sigma}\mathbf{W}=\boldsymbol{\varepsilon} \quad [6]$$

($\boldsymbol{\varepsilon}$ is a constant of motion and represents the total particle energy), shows it more clearly. The condition for the trapping turns out to be $-|\boldsymbol{\sigma}\mathbf{W}|<\boldsymbol{\varepsilon}<|\boldsymbol{\sigma}\mathbf{W}|$.

In a turbulent plasma are can expect effective wells and hills of wave energy. The wells can be the regions inside cavitons [11], envelope holes [12] and intermediate regions of series of solitons. The region of confined hot plasma by RF field in reference [6] is also an example of such a well. The hills are the envelope soliton like structures. Trapped particles with a finite kinetic energy will start oscillating back and forth and can give rise to a series of resonances. We illustrate this point in the following manner: Consider a particle with positive charge. Such a particle can be trapped in the bottom of the potential well $\mathbf{W}(\mathbf{x})$. To the lowest order, we write $\mathbf{W}(\mathbf{x})=-\mathbf{W}_0\left(1-\frac{\boldsymbol{\beta}^2}{2}\mathbf{x}^2\right)$ (where $\mathbf{W}_0$ is the depth of the well at $\mathbf{x}=0$, is a constant and represents the characteristic width of the well). Now equation [4] takes the form of a harmonic oscillator given by

$$\boldsymbol{\mu}\frac{\mathbf{d}^2\mathbf{x}}{\mathbf{dt}^2}+\boldsymbol{\sigma}\boldsymbol{\beta}^2\mathbf{W}_0\mathbf{x}=0 \quad [7]$$

which has a solution

$$\mathbf{x}=\mathbf{A}\,\mathbf{Sin}(\boldsymbol{\omega}_\mathbf{B}\mathbf{t}+\boldsymbol{\theta}) \quad [8]$$

where

$$\mathbf{A}=\sqrt{\mathbf{x}_0^2+\frac{\mathbf{v}_0^2}{\boldsymbol{\omega}_\mathbf{B}^2}},\ \boldsymbol{\theta}=\tan^{-1}\left(\frac{\mathbf{x}_0\boldsymbol{\omega}_\mathbf{B}}{\mathbf{v}_0}\right)$$

and $\mathbf{x}_0$, $\mathbf{v}_0$ are the constants of integration of equation [7].The bounce frequency of the trapped particle is

$$\omega_B = \sqrt{\frac{\sigma\beta^2 W_0}{\mu}} \quad [9]$$

Thus we see that the particle oscillates back and forth about its mean position inside the potential well. The bounce frequency of these oscillations is proportional to the square root of field energy density $\mathbf{W}_0 \propto |\mathbf{E}(\mathbf{x})|^2$ in contrast to the case of phase trapping where it is proportion to the square root of the wave amplitude. The average trajectory of the particle is defined in laboratory frame by

$$\mathbf{x}(\mathbf{t}) = \mathbf{x}_0 + \mathbf{v}_0\mathbf{t} + \mathbf{A}\,\mathbf{Sin}(\omega_B \mathbf{t} + \theta) \quad [10]$$

3. Distribution

Let us now consider the propagation of a small amplitude test wave in a warm plasma with initial distribution $\mathbf{f}_0$, and the particle orbits given by equation [10]. The Vlasov equation gives the change in the particle distribution as

$$\mathbf{f} = \frac{\mathbf{e}}{\mathbf{m}} \int_{-\infty}^{t} \mathbf{E}(\Omega, \mathbf{q}) \mathbf{e}^{i(\Omega t' - q x(t'))} \frac{\partial \mathbf{f}_0}{\partial \mathbf{v}} \mathbf{dt'} \quad [11]$$

Using equation [11] and the Poisson equation $\nabla.\mathbf{E} = -4\pi \mathbf{e}\int \mathbf{f}\mathbf{dv}$, one can arrive at a self consistent dispersion relation

$$\mathbf{q} = \frac{\omega_p^2}{\mathbf{n}_0} \int \mathbf{dv} \sum_{N=-\infty}^{\infty} \frac{\mathbf{J}_N^2(\mathbf{q}, \mathbf{A})}{(\Omega - \mathbf{N}\omega_B - \mathbf{q}\mathbf{v}_0)} \frac{\partial \mathbf{f}_0}{\partial \mathbf{v}} \quad [12]$$

where $\mathbf{dv}$ refers to velocity integration $\mathbf{N} = 0, \pm1, \pm2, ...$ and $\mathbf{J}_N(\mathbf{q}, \mathbf{A})$ are Bessel functions of the first kind, $(\Omega, \mathbf{q})$ represents the test wave and $\mathbf{n}_0$ the plasma density, ω_p is the plasma frequency.In obtaining equation[10], a large time average is taken and phase mixing is accounted.

For initial distribution $\mathbf{f}_0$ which is sum of large number of relatively cold particles and a bunch of streaming energetic and resonant particles, it can be shown that the perturbation is supported by the former and obeys the dispersion relation

$$\mathbf{D}(\Omega, \mathbf{q}) = \Omega^2 - \omega_p^2 - 3\mathbf{q}^2\mathbf{v}_T^2 = 0 \quad [13]$$

where $\mathbf{v}_T$ is the thermal velocity of the moderately cold plasma electrons. Here the contribution of energetic particles to the real part of dispersion is assumed to be negligible.

Using Landau prescription for the velocity integration of equation [12] around the singular points i.e., $\Omega - \mathbf{q}\mathbf{v}_0 = \mathbf{N}\omega_B$ the growth rate γ , of the test wave can be obtained,

$$\gamma = \frac{\pi(\omega - \Omega)^2 \omega}{\mathbf{n}_\gamma \mathbf{q}\Omega} \sum_{N=-\infty}^{\infty} \mathbf{J}_N^2(\mathbf{q}, \mathbf{A}) \int_0^{\infty} \left[\frac{\partial \mathbf{f}_0}{\partial \mathbf{v}} + \frac{\mathbf{q}}{\Omega}\left(\mathbf{v}\frac{\partial \mathbf{f}_0}{\partial \mathbf{v}_r} - \mathbf{v}_r \frac{\partial \mathbf{f}_0}{\partial \mathbf{v}} \right) \right]_{v=v_r} \mathbf{v}^2\mathbf{dv} \quad [14]$$

where , $\mathbf{v}_r = \dfrac{\Omega - \mathbf{N}\omega_B}{\mathbf{q}}$ and $\mathbf{n}_\gamma$ is the density of streaming energetic electrons.

The growth rate γ clearly shows that the test wave experiences resonances at frequencies which are separated by $\mathbf{N}\omega_B$. This situation is similar to the case of particles which are phase trapped. The trapped particles oscillate with frequency ω_B and any wave that Doppler shifts to this frequency can resonate with them. Depending upon the particle population, wave grows or decays. Therefore, we expect, in the presence of a large amplitude turbulence, a beam of energetic particles may trigger electrostatic fluctuations at the side band frequencies separated by $\mathbf{N}\omega_B$.

Let us now turn to a magnetoplasma where the field $\mathbf{E}(\mathbf{x})$ of the turbulence is normal to the ambient magnetic field. The equation of motion [4] is still valid but with a modified effective charge σ as

$$\sigma_{eff} = 4\pi\left[\omega\left(1-\frac{\omega_c^2}{\omega^2}\right)\frac{\partial}{\partial\omega}\omega^2 D(\omega,k)\right]^{-1} \quad [15]$$

where $\omega_c = \frac{eB_0}{mc}$ is the cyclotron frequency of the particles and c is the velocity of light. For $\omega^2 > \omega_c^2$, the charge σ remains positive and the above mentioned analysis still goes through. But different situation arises for $\omega^2 < \omega_c^2$. The sign of σ now inverted. Actually even for $\omega^2 > \omega_c^2$, σ can assume a negative sign if the background turbulence is due to negative energy waves (in that case $\left[\frac{1}{\omega}\frac{\partial}{\partial\omega}\omega^2 D(\omega,k)\right] < 0$). The direction of ponderomotive force term is then inverted and the particle are pushed out of the potential well and will be confined around the peak of the potential hill. So that we should write $W(x) = W_0\left(1-\frac{\beta^2 x^2}{2}\right)$ (W_0 is now the height of the hill), and it is easy to show that the particle trajectory can be defined in the same way as earlier with the new definition of equation [15]. In fact, it has been shown self-consistently in the case of whistlers that the plasma particles indeed tend to be confined in high energy density regions of the turbulence. The growth and dispersion of a test wave can be obtained in the same way and following the procedure of reference [5]. It is important to note that the direction of the ponderomotive force is independent of the sign of the electric charge, therefore, both electrons and ions tend to be trapped in the same region for high frequency turbulence $(\omega > \omega_p)$ .However, for low frequency turbulence $\omega_{ci}^2 < \omega^2 < \omega_{ce}^2$ (ω_{ce}, ω_{ci} electron and ion gyrofrequencies), electrons and ions feel the force in opposite direction. The difference, however, is not due to the sign of the charge but due to the different charge to mass ratio.

Thus whole picture of trapping process due to wave turbulence becomes very simple when the wave energy W and the effective charge σ_{eff} are appropriately known. The bounce frequency of trapped particle defined in equation [9] can be easily written in the explicitly form as

$$\omega_B^2 = \pm\frac{\omega_p^4}{\omega^2-\omega_c^2}(\beta\lambda_D)^2\frac{W_0}{n_0 T}\left[\frac{1}{\omega}\frac{\partial}{\partial\omega}\omega^2 D(\omega,k)\right]^{-1} \quad [16]$$

where appropriate sign is chosen in such a way that the bounce frequency is real. λ_D is the Debye length. $n_0 T$ is the thermal energy density of the plasma .From equation [16], it is easy to obtain the bounce frequency for any mode by choosing proper $D(k,\omega)$ and the energy density gradient. In case of electrostatic waves in the absence of magnetic field, the bounce frequency will be

$$\omega_B^2 = \frac{\omega_p^2}{2}(\beta\lambda_D)^2\frac{W_0}{n_0 T} \quad [17]$$

And for high frequency electromagnetic wave turbulence, the corresponding frequency is

$$\omega_B^2 = \frac{\omega_p^4}{2c^2k^2}(\beta\lambda_D)^2\frac{W_0}{n_0 T}, \quad \left(c^2k^2 >> \omega_p^2\right) \quad [18]$$

In case of magnetoplasma containing whistler turbulence $(\omega_{ci} << \omega < \omega_{ce})$, the bounce frequency is given by

$$\omega_B^2 = \omega_p^2(\beta\lambda_D)^2\frac{\omega}{\omega_{ce}}\frac{\omega_{ce}-\omega}{\omega_{ce}+\omega}\frac{W_0}{n_0 T} \quad [19]$$

III. Conclusion

In conclusion, by appropriate choice and the sign of the potential $W(x)$, one can study the effect of trapping which may lead to growth of waves at side band frequencies, with frequencies gaps proportional to square root of turbulent energy density. Due to this, there may be selective amplification of the waves which may look like spikes in the wave noise spectrum. The coherent harmonic generation of the waves is possible even from a turbulent plasma.

The trapping discussed in this paper is different from the phase trapping that we come across quite often in the literature. The phase trapping is possible in case of waves which are essentially monochromatic or where auto correlation time is greater than cross correlation. In that case, the frequency of trapped oscillation is $\sqrt{\frac{\mathbf{eEk}}{\mathbf{m}}}$ ($\mathbf{k}$ is wave number) which is proportional to the square root of the amplitude of the wave. The regions of trapping also depends upon the sign of the charge.

Whereas the trapping discussed here, is independent of the phases of the waves. For electromagnetic waves $\left(\omega > \omega_p\right)$, the phase trapping is not possible even when the waves are coherent, but the density gradient trapping is possible. The trapping frequency [equation 9] is proportional to square root of the energy density. The regions of trapping do not depend upon the electric charge of the particleand the particles of opposite charge can be trapped in the same region. However, the parameter σ [defined in equation 15] plays the role of effective charge and can distinguish the regions of trapping depending upon its sign.

The results obtained in our paper are quite general but the effect of anharmonicity should be taken into account if the realistic wave energy density $\mathbf{W}$ differs appreciably from $\mathbf{W} = \pm \mathbf{W}_0 \left(1 - \frac{\beta^2 \mathbf{x}^2}{2}\right)$.

Finally, we would like to say, that the effect of the turbulence discussed here, plays an important role in the study of detailed dynamics of plasma.

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