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Wave Equations of Electrogravitodynamics

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Abstract: *A formal structure of Electrogravitodynamics (EGD) is presented from wave equation development in both Electromagnetism (EM) and Gyrogravitation (GG) respect of the transverse component fields, as well as the poynting vector of each radiation, generically in a material environment and with non-zero generating sources. Once the complete structure of each radiation type, EM and GG, has been obtained, the interrelation between both two can be incorporated. This interrelation will be based on equations between irrotational components, the same within solenoidal components and also irrotational with solenoidal component relation equations regarding different radiations, incorporating the corresponding sources for the material environment and, later simplifying, for the vacuum. The EM-GG interrelation includes the relationship between independent sources for EM and GG. Finally, the EGD is completed with an intrinsic impedances and specific energy intensities comparative study for EM radiation, GG radiation and GG radiation for EM-GG conversion.*

Keywords: *Electrogravitodynamics, Gyrogravitational Radiation, Gyrogravitational Poynting Vector, Wave Equation, Intrinsic Impedance, Specific Energy Intensity.*

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I. Introduction

Maxwell equations analysis is almost always performed in a free field sources wave propagation environment, that is, with both charge ρ and current $\vec{J}$ densities zero. For simplicity, in regard to electromagnetic radiation (EM) and, even if an environment with field sources is used, it is usually analyzed in a vacuum (non-material environment) [1]. EM radiation, as an electric $\vec{E}$ and magnetic induction $\vec{B}$ transverse field combination, transports energy through space with a non-zero mean value. However, such a statement is only really true in certain EM wave propagation regions [2]. Sometimes, even for relevant field strengths, with non-zero EM source distribution, the EM radiation mean power density can be zero.

The electric $\vec{E}$ and magnetic $\vec{H}$ fields behavior can differ hugely within different EM energy propagation zones, for the same EM radiation flux. However, it is common to use simplified Maxwell equations for vacuum (when we almost never work in a vacuum). We apply them considering an environment without sources, obtaining the electric $\vec{E}$ and magnetic $\vec{H}$ fields wave equations and subsequently, their solutions in a simple way. The mistake made when working in this way can be important in certain situations, especially when the so-called inductive near field is handled. The solution is to use generic initial conditions. In other words, it is about handling Maxwell equations in a material medium, incorporating its possible variability in change terms on the electric permittivity ϵ and the magnetic permeability μ. From this generic perspective, the wave equations for the EM radiation transverse fields are obtained and hence the solutions, depending on the conditions of the propagation environment considered. Results obtained for the wave equations treated in this generic way are not symmetric, offering solutions in the form of not null d'Alembertians.

It is curious that when we talk about the wave equation associated with EM radiation, it is expressed as a combination of its transversal components wave equations in simplified terms (vacuum and without field sources). A single compact expression is never used representing the EM radiation propagation, for example, through the poynting vector $\vec{S}$, which is more EM energy propagation representative, than its transverse components. The wave equation for the poynting vector $\vec{S}$ will be developed for a propagation environment in a material medium, introducing the non-zero field sources possibility and allowing electric permittivity and magnetic permeability with spatial variability.

Once the generic wave equation for the poynting vector $\vec{S}$ in a material medium has been obtained, with non-zero sources and electric and magnetic constants spatial variability, it will be simplified by applying the EM radiation conditions in vacuum free space, without sources. The result is that of a null d'Alembertian for the poynting vector $\vec{S}$.

As described in Electrogravitodynamics (EGD) defined in [3], the gyrogravitation field (GG) is, similar to the EM field, a field composed of two transverse components, the gravitational field $\vec{g}$ with an irrotational nature and the gravitation torsion field $\vec{\Omega}_\tau$ (gyrotation $\vec{\Omega}$) of solenoidal type. It is now a matter of writing the wave equations for the GG field transverse components. In a similar way to what has been done with the EM field, with respect to the GG field, the electric ϵ and magnetic μ constants are replaced by the gravitational permittivity ξ and the gravitation torsion permeability τ constants, respectively, where the latter is a function of the light propagation speed in material medium s and, therefore, spatially variable [4]. Results obtained for the GG field wave equations from transverse components are not symmetric, defining solutions in the form of non-zero d'Alembertians. If it is considered in the GG field propagation environment that sources (material density ρ_m and mass current density $\vec{J}_m$) are zero and the medium refractive index n is constant, the GG field components wave equations appear as symmetric in the form of null d'Alembertians.

It remains to describe the GG field propagation, expressed by the gyrogravitational poynting vector $\vec{S}_g$, in a compact way through a single generic wave equation, for a material medium, with the possibility of non-zero GG field sources and spatial variability of the gravitation torsion constant τ. The generic d'Alembertian obtained is non-zero, although if we apply the GG radiation conditions in vacuum free space, without sources, it is simplified into a null symmetric equation.

Electrogravitodynamics (EGD) begins from the formal structure that describes EM and GG radiations separately, incorporating the interrelation and influence in both directions, between gyrogravitation and electromagnetism component fields (gyrogravitation induction in electromagnetism and vice versa). Formal relationships will be obtained within irrotational type components and the same for solenoidal type components. At the end, we will propose relation equations of irrotational components with solenoidal ones between EM and GG radiation.

In the EM or GG radiation propagation, the relationship within transverse field components and the corresponding poynting vector through the intrinsic impedance η results important. On the other hand, the specific energy intensity S_η is introduced as the radiated energy intensity per unit of area and specific impedance of the propagation environment in the considered radiation. A comparative study between EM and GG radiation will be made based on intrinsic impedance relationships and specific energy intensity relationships.

II. Wave Equations in Electromagnetism

It is usual to analyze the solutions to Maxwell equations in a field sources free environment (null charge density ρ and current density $\vec{J}$) or, when talking about EM radiation, to use an environment with non-null field sources but, for simplicity, of a non-material type (in a vacuum) [5]. EM radiation as a combination of electric $\vec{E}$ and magnetic induction $\vec{B}$ auto-propagated fields in the form of transverse waves always in phase, transports energy through space. But this statement is only really valid in the radiation or far field region, where the EM wave carries non-zero average power. That is, the power density $\vec{P}_{avg}$, defined as the EM wave mean power per unit of transverse area respect to the propagation or temporal average of the poynting vector $\vec{S}$, is different from zero.

$$\vec{S}(\vec{r},t) = \vec{E}(\vec{r},t) \times \vec{H}(\vec{r},t) \qquad \text{and} \qquad \vec{P}_{avg} = \frac{1}{T}\int_0^T \left(\vec{E} \times \vec{H}\right) dt \tag{1}$$

In the far field zone the relationship between the electric field $\vec{E}$ and the magnetic field $\vec{H}$ (magnetic field strength), given by the intrinsic impedance η, is determined such that, for a vacuum it will be η_0,

$$\eta = \frac{\vec{E}}{\vec{H}} \text{ with } \vec{H} = \frac{\vec{B}}{\mu} \text{ and if } \mu = \mu_0 \ \Rightarrow \ \eta_0 = \mu_0 c \ , \tag{2}$$

Where μ_0 and c are the magnetic permeability and the speed of light propagation, respectively, for the vacuum particular case.

However, when speaking of the near field region and, more specifically, in the reactive or induction near field region, the energy transport is such that the power density $\vec{P}_{avg}$ is zero, even though there are relevant field values. In this region, the electric $\vec{E}$ and magnetic $\vec{H}$ fields values depend on the charge distribution ρ and the current density $\vec{J}$. On the other hand, it is usual to consider the transition region between inductive near field and radiative far field, named as radiant near field, as a zone where radiation fields already begin to predominate and the power density progressively ceases to be zero.

There is a great difference in the electric $\vec{E}$ and magnetic $\vec{H}$ fields behavior within the inductive field region and that of radiant field, as well as, respect to the EM energy transport itself that they represent in each case, described through the power density value. In the space regions where the current density $\vec{J}$ is not zero, the symmetry between EM radiation transverse fields is violated [6]. However, the usual when talking about EM radiation propagation is to use simplified Maxwell equations for vacuum and apply them in an environment without sources, obtaining the electric field $\vec{E}$ and magnetic field $\vec{H}$ wave equations and, at last, their solutions [7]. Although it is true that these conditions and results are valid for most of the cases, since almost always the

EM radiation is referring to the far field, the error made when applying them is important, at least for the inductive near field. To avoid this problem, one should start with generic conditions. That is, to use Maxwell equations in a material medium and also consider that the medium possible variability affects the electric permittivity ϵ and the magnetic permeability μ in the environment considered. From this generic perspective, the wave equations for the EM radiation transverse fields are obtained and hence, the solutions depending on the propagation environment conditions.

A material medium generic characteristics are given by the electric permittivity ϵ and the magnetic permeability μ in the environment considered, such that,

$$\epsilon = \epsilon_0 \epsilon_r \quad \text{and} \quad \mu = \mu_0 \mu_r \tag{3}$$

Where ϵ_0 and μ_0 are the electric permittivity and the magnetic permeability, respectively, in vacuum. And ϵ_r and μ_r are the electric permittivity and magnetic permeability relative values, respectively, in a material medium.

We are going to consider the possible material medium spatial variability with respect to electric permittivity and magnetic permeability, thus,

$$\vec{\nabla} \epsilon \neq 0 \;\; \text{and} \;\; \vec{\nabla} \mu \neq 0, \text{ though } \frac{\partial \epsilon}{\partial t} = 0 \text{ and } \frac{\partial \mu}{\partial t} = 0 \tag{4}$$

We will use Maxwell equations for material medium, so,

$$\vec{\nabla} . \vec{E} \Leftarrow \frac{\rho}{\epsilon} \tag{5}$$

$$\vec{\nabla} . \vec{H} = 0 \tag{6}$$

$$\vec{\nabla} \times \vec{E} \Leftarrow -\mu \frac{\partial \vec{H}}{\partial t} \tag{7}$$

$$\vec{\nabla} \times \vec{H} \Leftarrow \vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t} \tag{8}$$

The symbol $\Leftarrow$ in the above equations represents that one term in the equation is inducer and the other one is induced (where the arrow points to). Cause-effect principle applied in terms of inducer-induced concept [8].

Incorporating nabla operator $\vec{\nabla}$ on (8) and, considering conditions in (4), the following generic continuity equation is obtained,

$$\vec{\nabla} . \vec{J} \Leftarrow -\frac{\partial \rho}{\partial t} - \vec{\nabla} \epsilon . \frac{\partial \vec{E}}{\partial t} \tag{9}$$

Wave equations for electric field $\vec{E}$ and magnetic field $\vec{H}$ are obtained by determining the Laplace operator Δ on each field. Generically, a field $\vec{A}$ Laplacian is equal to,

$$\Delta \vec{A} = \vec{\nabla}(\vec{\nabla} . \vec{A}) - \vec{\nabla} \times (\vec{\nabla} \times \vec{A}) \tag{10}$$

If we apply the rotational operator $\vec{\nabla} \times$ on (7), we obtain the following,

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) \Leftarrow \vec{\nabla} \times \left(-\mu \frac{\partial \vec{H}}{\partial t}\right) = -\mu \frac{\partial}{\partial t}(\vec{\nabla} \times \vec{H}) - \vec{\nabla} \mu \times \frac{\partial \vec{H}}{\partial t} \tag{11}$$

Taking into account (8) and (10) over (11),

$$\vec{\nabla}(\vec{\nabla} . \vec{E}) - (\vec{\nabla} . \vec{\nabla})\vec{E} \Leftarrow -\mu \frac{\partial}{\partial t}\left(\vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t}\right) - \vec{\nabla} \mu \times \frac{\partial \vec{H}}{\partial t} \tag{12}$$

Applying (5) in (12) and developing,

$$\vec{\nabla} \frac{\rho}{\epsilon} - \vec{\nabla}^2 \vec{E} \Leftarrow -\mu \frac{\partial \vec{J}}{\partial t} - \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} - \vec{\nabla} \mu \times \frac{\partial \vec{H}}{\partial t} \tag{13}$$

Using the Laplacian definition on the electric field $\vec{E}$ in (13), we obtain,

$$\Delta \vec{E} \Leftarrow \frac{1}{s^2} \frac{\partial^2 \vec{E}}{\partial t^2} + \mu \frac{\partial \vec{J}}{\partial t} + \vec{\nabla} \frac{\rho}{\epsilon} + \vec{\nabla} \mu \times \frac{\partial \vec{H}}{\partial t} \tag{14}$$

Where *s* is the speed of light propagation in a material medium,

$$s = \frac{c}{n} = \frac{1}{\sqrt{\mu \epsilon}} = \frac{c}{\sqrt{\mu_r \epsilon_r}} \tag{15}$$

Being *n* the material medium absolute refractive index,

$$n = \sqrt{\mu_r \epsilon_r} \tag{16}$$

Taking into account the d'Alembertian operator $\Box$ definition,

$$\Box = \frac{1}{s^2} \frac{\partial^2}{\partial t^2} - \Delta \tag{17}$$

Applying (17) over (14), the wave equation for the electric field component $\vec{E}$ is definitely obtained,

$$\Box \vec{E} \Leftarrow -\mu \frac{\partial \vec{J}}{\partial t} - \vec{\nabla} \frac{\rho}{\epsilon} - \vec{\nabla} \mu \times \frac{\partial \vec{H}}{\partial t} \tag{18}$$

We will use the same procedure as above for the magnetic field $\vec{H}$.

If we apply the rotational operator on (8) we obtain,

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{H}) \Leftarrow \vec{\nabla} \times \left(\vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t}\right) = \vec{\nabla} \times \vec{J} + \epsilon \frac{\partial}{\partial t}(\vec{\nabla} \times \vec{E}) + \vec{\nabla} \epsilon \times \frac{\partial \vec{E}}{\partial t} \tag{19}$$

Taking into account (7) and (10) over (19),

$$\vec{\nabla}\left(\vec{\nabla}.\vec{H}\right)-\left(\vec{\nabla}.\vec{\nabla}\right)\vec{H}\Leftarrow\vec{\nabla}\times\vec{J}-\epsilon\mu\frac{\partial^2\vec{H}}{\partial t^2}+\vec{\nabla}\epsilon\times\frac{\partial\vec{E}}{\partial t} \quad (20)$$

Applying (6) in (20) and developing,

$$-\vec{\nabla}^2\vec{H}\Leftarrow\vec{\nabla}\times\vec{J}-\epsilon\mu\frac{\partial^2\vec{H}}{\partial t^2}+\vec{\nabla}\epsilon\times\frac{\partial\vec{E}}{\partial t} \quad (21)$$

Using the Laplacian definition on the magnetic field $\vec{H}$ and (15) in (21), we get,

$$\Delta\vec{H}\Leftarrow\frac{1}{s^2}\frac{\partial^2\vec{H}}{\partial t^2}-\vec{\nabla}\times\vec{J}-\vec{\nabla}\epsilon\times\frac{\partial\vec{E}}{\partial t} \quad (22)$$

Applying the d'Alembertian operator definition in (17) over (22), the wave equation for the magnetic field component $\vec{H}$ is definitely achieved,

$$\Box\vec{H}\Leftarrow\vec{\nabla}\times\vec{J}+\vec{\nabla}\epsilon\times\frac{\partial\vec{E}}{\partial t} \quad (23)$$

Observe that the results obtained through (18) and (23) are not null, as it is traditionally described when simplifying, using a vacuum as medium and electric ρ and magnetic $\vec{J}$ null sources in the considered propagation environment.

III. Wave Equation Based on Poynting Vector in Electromagnetism

Traditionally, it is usual when talking about EM radiation to express the propagation conditions in simplified terms (vacuum and null field sources) through the field components wave equations, that is,

$$\Box\vec{E}=0 \quad (24)$$

$$\Box\vec{H}=0 \quad (25)$$

A single compact expression is never used to represent the EM radiation propagation, for example, through the poynting vector $\vec{S}$, which also implies the EM field longitudinal character. The aim is to develop the wave equation for the poynting vector in a generic propagation environment, for a material medium, with the possibility of non-zero field sources and electric permittivity and magnetic permeability both with spatial variability.

We start by setting the Laplacian operator Δ on the poynting vector $\vec{S}$, applying (A6),

$$\Delta\left(\vec{E}\times\vec{H}\right)=\vec{\nabla}^2\left(\vec{E}\times\vec{H}\right)=\vec{\nabla}\left(\vec{\nabla}.\left(\vec{E}\times\vec{H}\right)\right)-\vec{\nabla}\times\left(\vec{\nabla}\times\left(\vec{E}\times\vec{H}\right)\right) \quad (26)$$

Developing the previous equation in parts, through (A1) and (A5), we have that,

$$\vec{\nabla}.\left(\vec{E}\times\vec{H}\right)=\vec{H}.\vec{\nabla}\times\vec{E}-\vec{E}.\vec{\nabla}\times\vec{H}=-\mu\vec{H}.\frac{\partial\vec{H}}{\partial t}-\epsilon\vec{E}.\frac{\partial\vec{E}}{\partial t}-\vec{E}.\vec{J} \quad \text{(scalar)} \quad (27)$$

$$\vec{\nabla}\times\left(\vec{E}\times\vec{H}\right)=\vec{E}\left(\cancel{\vec{\nabla}.\vec{H}}\right)-\vec{H}\left(\vec{\nabla}.\vec{E}\right)+\left(\vec{H}.\vec{\nabla}\right)\vec{E}-\left(\vec{E}.\vec{\nabla}\right)\vec{H} \quad \text{(vector)} \quad (28)$$

Using (27) and (28) over (26), we get,

$$\Delta\left(\vec{E}\times\vec{H}\right)=\vec{\nabla}\left(\vec{H}.\vec{\nabla}\times\vec{E}\right)-\vec{\nabla}\left(\vec{E}.\vec{\nabla}\times\vec{H}\right)+\vec{\nabla}\times\left(\vec{H}\left(\vec{\nabla}.\vec{E}\right)\right)-\vec{\nabla}\times\left(\left(\vec{H}.\vec{\nabla}\right)\vec{E}\right)+\vec{\nabla}\times\left(\left(\vec{E}.\vec{\nabla}\right)\vec{H}\right) \quad (29)$$

Applying vector calculus with (A3) and (A8) over (29), we obtain,

$$\Delta\left(\vec{E}\times\vec{H}\right)=\left(\left(\vec{\nabla}\times\vec{E}\right).\vec{\nabla}\right)\vec{H}+\cancel{\left(\vec{H}.\vec{\nabla}\right)\left(\vec{\nabla}\times\vec{E}\right)}+\left(\vec{\nabla}\times\vec{E}\right)\times\left(\vec{\nabla}\times\vec{H}\right)+\vec{H}\times\left(\vec{\nabla}\times\left(\vec{\nabla}\times\vec{E}\right)\right)-$$
$$-\left(\left(\vec{\nabla}\times\vec{H}\right).\vec{\nabla}\right)\vec{E}-\cancel{\left(\vec{E}.\vec{\nabla}\right)\left(\vec{\nabla}\times\vec{H}\right)}-\left(\vec{\nabla}\times\vec{H}\right)\times\left(\vec{\nabla}\times\vec{E}\right)-\vec{E}\times\left(\vec{\nabla}\times\left(\vec{\nabla}\times\vec{H}\right)\right)+\frac{\rho}{\epsilon}\left(\vec{\nabla}\times\vec{H}\right)+\vec{\nabla}\frac{\rho}{\epsilon}\times\vec{H}-$$
$$-\cancel{\left(\vec{H}.\vec{\nabla}\right)\left(\vec{\nabla}\times\vec{E}\right)}-\vec{\nabla}\left(\vec{H}.\vec{\nabla}\right)\times\vec{E}+\cancel{\left(\vec{E}.\vec{\nabla}\right)\left(\vec{\nabla}\times\vec{H}\right)}+\vec{\nabla}\left(\vec{E}.\vec{\nabla}\right)\times\vec{H} \quad (30)$$

To simplify (30), we are going to develop the following procedure.

Taking into account (A1), it can be put respect the magnetic field $\vec{H}$ that,

$$-\left(\vec{\nabla}\times\vec{H}\right).\vec{\nabla}=\vec{\nabla}.\left(\vec{\nabla}\times\vec{H}\right)-\left(\vec{\nabla}\times\vec{\nabla}\right).\vec{H}=\vec{\nabla}.\left(\vec{\nabla}\times\vec{H}\right) \quad (31)$$

Then, using (31) and applying (8), we have that,

$$-\left(\left(\vec{\nabla}\times\vec{H}\right).\vec{\nabla}\right)\vec{E}=\left(\vec{\nabla}.\left(\vec{\nabla}\times\vec{H}\right)\right)\vec{E}=\left(\vec{\nabla}.\vec{J}\right)\vec{E}+\left(\vec{\nabla}.\epsilon\frac{\partial\vec{E}}{\partial t}\right)\vec{E} \quad (32)$$

Substituting the EM continuity equation (9) in (32) and developing, we have,

$$-\left(\left(\vec{\nabla}\times\vec{H}\right).\vec{\nabla}\right)\vec{E}=-\frac{\partial\rho}{\partial t}\vec{E}-\cancel{\left(\vec{\nabla}\epsilon.\frac{\partial\vec{E}}{\partial t}\right)\vec{E}}+\epsilon\frac{\partial}{\partial t}\left(\vec{\nabla}.\vec{E}\right)\vec{E}+\cancel{\left(\vec{\nabla}\epsilon.\frac{\partial\vec{E}}{\partial t}\right)\vec{E}} \quad (33)$$

Substituting (5) in (33) and simplifying, the result is,

$$-\left(\left(\vec{\nabla}\times\vec{H}\right).\vec{\nabla}\right)\vec{E}=-\frac{\partial\rho}{\partial t}\vec{E}+\epsilon\frac{\partial}{\partial t}\left(\frac{\rho}{\epsilon}\right)\vec{E}=0 \quad (34)$$

On the other hand, taking into account (A1), it can be put respect the electric field $\vec{E}$ that,

$$\left(\vec{\nabla}\times\vec{E}\right).\vec{\nabla}=-\vec{\nabla}.\left(\vec{\nabla}\times\vec{E}\right)+\left(\vec{\nabla}\times\vec{\nabla}\right).\vec{E}=-\vec{\nabla}.\left(\vec{\nabla}\times\vec{E}\right) \quad (35)$$

Then, using (35) and applying (7), we have,

$$\left(\left(\vec{\nabla}\times\vec{E}\right).\vec{\nabla}\right)\vec{H}=-\left(\vec{\nabla}.\left(\vec{\nabla}\times\vec{E}\right)\right)\vec{H}=\left(\vec{\nabla}.\left(\mu\frac{\partial\vec{H}}{\partial t}\right)\right)\vec{H} \quad (36)$$

Applying (A7) in (36), we get,

$$\left(\left(\vec{\nabla}\times\vec{E}\right).\vec{\nabla}\right)\vec{H}=\left(\mu\frac{\partial}{\partial t}\left(\vec{\nabla}.\vec{H}\right)\right)\vec{H}+\left(\vec{\nabla}\mu.\frac{\partial\vec{H}}{\partial t}\right)\vec{H} \quad (37)$$

Also, inserting (6) into (37), the result is,

$$\left(\left(\vec{\nabla}\times\vec{E}\right).\vec{\nabla}\right)\vec{H}=\left(\vec{\nabla}\mu.\frac{\partial\vec{H}}{\partial t}\right)\vec{H} \quad (38)$$

Applying (34) and (38) in (30) and developing, it remains the following,

$$\Delta\left(\vec{E}\times\vec{H}\right)=\left(\vec{\nabla}\mu.\frac{\partial\vec{H}}{\partial t}\right)\vec{H}+2\vec{J}\times\mu\frac{\partial\vec{H}}{\partial t}+2\mu\epsilon\left(\frac{\partial\vec{E}}{\partial t}\times\frac{\partial\vec{H}}{\partial t}\right)+\cancel{\vec{H}\times\left(\vec{\nabla}\times\left(\vec{\nabla}\times\vec{E}\right)\right)}-\vec{E}\times\cancel{\left(\vec{\nabla}\times\left(\vec{\nabla}\times\vec{H}\right)\right)}+\frac{\rho}{\epsilon}\vec{J}+$$
$$+\rho\frac{\partial\vec{E}}{\partial t}+\vec{\nabla}\frac{\rho}{\epsilon}\times\vec{H}-\left(\left(\vec{\nabla}.\vec{\nabla}\right)\vec{H}\right)\times\vec{E}-\left(\left(\vec{H}.\vec{\nabla}\right)\vec{\nabla}\right)\times\vec{E}-\cancel{\left(\vec{\nabla}\times\left(\vec{\nabla}\times\vec{H}\right)\right)\times}\vec{E}-\left(\vec{H}\times\left(\vec{\nabla}\times\vec{\nabla}\right)\right)\times\vec{E}+$$
$$+\left(\left(\vec{\nabla}.\vec{\nabla}\right)\vec{E}\right)\times\vec{H}+\left(\left(\vec{E}.\vec{\nabla}\right)\vec{\nabla}\right)\times\vec{H}+\cancel{\left(\vec{\nabla}\times\left(\vec{\nabla}\times\vec{E}\right)\right)}\times\vec{H}+\left(\vec{E}\times\left(\vec{\nabla}\times\vec{\nabla}\right)\right)\times\vec{H} \quad (39)$$

To simplify (39), we are going to use the following expressions.

Applying (A3), we have that,

$$\vec{\nabla}\left(\vec{\nabla}.\vec{H}\right)=\left(\vec{H}.\vec{\nabla}\right)\vec{\nabla}+\left(\vec{\nabla}.\vec{\nabla}\right)\vec{H}+\vec{H}\times\left(\vec{\nabla}\times\vec{\nabla}\right)+\vec{\nabla}\times\left(\vec{\nabla}\times\vec{H}\right) \quad (40)$$

Therefore, using (40) we can rewrite,

$$\left[\left(\vec{H}.\vec{\nabla}\right)\vec{\nabla}+\left(\vec{H}\times\left(\vec{\nabla}\times\vec{\nabla}\right)\right)\right]\times\vec{E}=\left[\vec{\nabla}\left(\vec{\nabla}.\vec{H}\right)-\left(\vec{\nabla}.\vec{\nabla}\right)\vec{H}-\vec{\nabla}\times\left(\vec{\nabla}\times\vec{H}\right)\right]\times\vec{E}=0 \quad (41)$$

Since, according to (A6), applied to the magnetic field $\vec{H}$,

$$\left(\vec{\nabla}.\vec{\nabla}\right)\vec{H}=\vec{\nabla}^2\vec{H}=\vec{\nabla}\left(\vec{\nabla}.\vec{H}\right)-\vec{\nabla}\times\left(\vec{\nabla}\times\vec{H}\right) \quad (42)$$

On the other hand, using (A3) again, we have to,

$$\vec{\nabla}\left(\vec{\nabla}.\vec{E}\right)=\left(\vec{E}.\vec{\nabla}\right)\vec{\nabla}+\left(\vec{\nabla}.\vec{\nabla}\right)\vec{E}+\vec{E}\times\left(\vec{\nabla}\times\vec{\nabla}\right)+\vec{\nabla}\times\left(\vec{\nabla}\times\vec{E}\right) \quad (43)$$

Therefore, using (43) we can rewrite,

$$\left[\left(\vec{E}.\vec{\nabla}\right)\vec{\nabla}+\left(\vec{E}\times\left(\vec{\nabla}\times\vec{\nabla}\right)\right)\right]\times\vec{H}=\left[\vec{\nabla}\left(\vec{\nabla}.\vec{E}\right)-\left(\vec{\nabla}.\vec{\nabla}\right)\vec{E}-\vec{\nabla}\times\left(\vec{\nabla}\times\vec{E}\right)\right]\times\vec{H}=0 \quad (44)$$

Since, according to (A6), applied to the electric field $\vec{E}$,

$$\left(\vec{\nabla}.\vec{\nabla}\right)\vec{E}=\vec{\nabla}^2\vec{E}=\vec{\nabla}\left(\vec{\nabla}.\vec{E}\right)-\vec{\nabla}\times\left(\vec{\nabla}\times\vec{E}\right) \quad (45)$$

Using the results in (41) and (44) over (39), the following is achieved,

$$\Delta\left(\vec{E}\times\vec{H}\right)=\left(\vec{\nabla}\mu.\frac{\partial\vec{H}}{\partial t}\right)\vec{H}+2\vec{J}\times\mu\frac{\partial\vec{H}}{\partial t}+2\mu\epsilon\left(\frac{\partial\vec{E}}{\partial t}\times\frac{\partial\vec{H}}{\partial t}\right)+\frac{\rho}{\epsilon}\vec{J}+\rho\frac{\partial\vec{E}}{\partial t}+\vec{\nabla}\frac{\rho}{\epsilon}\times\vec{H}+\vec{E}\times\Delta\vec{H}+\Delta\vec{E}\times\vec{H} \quad (46)$$

Now, let's consider the next procedure.

$$\frac{\partial^2}{\partial t^2}\left(\vec{E}\times\vec{H}\right)=\frac{\partial}{\partial t}\left[\frac{\partial}{\partial t}\left(\vec{E}\times\vec{H}\right)\right]=\frac{\partial}{\partial t}\left[\left(\frac{\partial\vec{E}}{\partial t}\times\vec{H}\right)+\left(\vec{E}\times\frac{\partial\vec{H}}{\partial t}\right)\right]=\left(\frac{\partial^2\vec{E}}{\partial t^2}\times\vec{H}\right)+2\left(\frac{\partial\vec{E}}{\partial t}\times\frac{\partial\vec{H}}{\partial t}\right)+\left(\vec{E}\times\frac{\partial^2\vec{H}}{\partial t^2}\right) \quad (47)$$

Applying the EM wave equations (14) and (22) in (47), it can be rewritten,

$$\frac{1}{s^2}\frac{\partial^2}{\partial t^2}\left(\vec{E}\times\vec{H}\right)=\Delta\vec{E}\times\vec{H}-\mu\frac{\partial\vec{J}}{\partial t}\times\vec{H}-\vec{\nabla}\frac{\rho}{\epsilon}\times\vec{H}-\left(\vec{\nabla}\mu\times\frac{\partial\vec{H}}{\partial t}\right)\times\vec{H}+2\left(\vec{\nabla}\times\vec{E}\right)\times\left(\vec{\nabla}\times\vec{H}\right)-2\vec{J}\times\mu\frac{\partial\vec{H}}{\partial t}+$$
$$+\vec{E}\times\Delta\vec{H}+\vec{E}\times\left(\vec{\nabla}\times\vec{J}\right)+\vec{E}\times\left(\vec{\nabla}\epsilon\times\frac{\partial\vec{E}}{\partial t}\right) \quad (48)$$

Observe that using (7) and (8), we can put that,

$$2\left(\vec{\nabla}\times\vec{E}\right)\times\left(\vec{\nabla}\times\vec{H}\right)=2\mu\epsilon\left(\frac{\partial\vec{E}}{\partial t}\times\frac{\partial\vec{H}}{\partial t}\right)+2\vec{J}\times\mu\frac{\partial\vec{H}}{\partial t} \quad (49)$$

Therefore, using (48) and (49) in (46), we obtain,

$$\Delta\left(\vec{E}\times\vec{H}\right)=\frac{1}{s^2}\frac{\partial^2}{\partial t^2}\left(\vec{E}\times\vec{H}\right)+\mu\frac{\partial\vec{J}}{\partial t}\times\vec{H}+\left(\vec{\nabla}\mu.\frac{\partial\vec{H}}{\partial t}\right)\vec{H}+\left(\vec{\nabla}\mu\times\frac{\partial\vec{H}}{\partial t}\right)\times\vec{H}+2\vec{J}\times\mu\frac{\partial\vec{H}}{\partial t}-\vec{E}\times\left(\vec{\nabla}\times\vec{J}\right)-$$
$$-\vec{E}\times\left(\vec{\nabla}\epsilon\times\frac{\partial\vec{E}}{\partial t}\right)+2\vec{\nabla}\frac{\rho}{\epsilon}\times\vec{H}+\frac{\rho}{\epsilon}\vec{J}+\rho\frac{\partial\vec{E}}{\partial t} \quad (50)$$

If we consider (A8), the following can be developed,

$$\vec{\nabla}\frac{\rho}{\epsilon}\times\vec{H}=\vec{\nabla}\times\frac{\rho}{\epsilon}\vec{H}-\frac{\rho}{\epsilon}\vec{\nabla}\times\vec{H}=\vec{\nabla}\times\frac{\rho}{\epsilon}\vec{H}-\frac{\rho}{\epsilon}\vec{J}-\rho\frac{\partial\vec{E}}{\partial t} \quad (51)$$

Using the result in (51) over (50), it allows obtaining,

$$\Delta\left(\vec{E}\times\vec{H}\right)=\frac{1}{s^2}\frac{\partial^2}{\partial t^2}\left(\vec{E}\times\vec{H}\right)+\left(\vec{\nabla}\mu.\frac{\partial\vec{H}}{\partial t}\right)\vec{H}+\vec{\nabla}\times\frac{\rho}{\epsilon}\vec{H}+2\vec{J}\times\mu\frac{\partial\vec{H}}{\partial t}+\mu\frac{\partial\vec{J}}{\partial t}\times\vec{H}+\left(\vec{\nabla}\mu\times\frac{\partial\vec{H}}{\partial t}\right)\times\vec{H}+\vec{\nabla}\frac{\rho}{\epsilon}\times\vec{H}-$$
$$-\vec{E}\times\left(\vec{\nabla}\times\vec{J}\right)-\vec{E}\times\left(\vec{\nabla}\epsilon\times\frac{\partial\vec{E}}{\partial t}\right) \quad (52)$$

Considering the wave equations for the electric field $\vec{E}$ (18) and for the magnetic field $\vec{H}$ (23), it can be stated that,

$$-\square\vec{E}\times\vec{H}=\mu\frac{\partial\vec{J}}{\partial t}\times\vec{H}+\vec{\nabla}\frac{\rho}{\epsilon}\times\vec{H}+\left(\vec{\nabla}\mu\times\frac{\partial\vec{H}}{\partial t}\right)\times\vec{H} \quad (53)$$

$$-\vec{E}\times\square\vec{H}=\square\vec{H}\times\vec{E}=\left(\vec{\nabla}\times\vec{J}\right)\times\vec{E}+\left(\vec{\nabla}\epsilon\times\frac{\partial\vec{E}}{\partial t}\right)\times\vec{E}=-\vec{E}\times\left(\vec{\nabla}\times\vec{J}\right)-\vec{E}\times\left(\vec{\nabla}\epsilon\times\frac{\partial\vec{E}}{\partial t}\right) \quad (54)$$

If we take into account the d'Alembertian operator definition in (17) on the poynting vector $\vec{S}$,

$$\square\vec{S}=\frac{1}{s^2}\frac{\partial^2\vec{S}}{\partial t^2}-\Delta\vec{S}\text{ , that is, }\square\left(\vec{E}\times\vec{H}\right)=\frac{1}{s^2}\frac{\partial^2\left(\vec{E}\times\vec{H}\right)}{\partial t^2}-\Delta\left(\vec{E}\times\vec{H}\right) \quad (55)$$

Therefore, introducing (53), (54) and (55) in (52) and, applying the inducer-induced concept [8], we obtain,

$$\Box\vec{S} \Leftarrow \vec{E} \times \Box\vec{H} + \Box\vec{E} \times \vec{H} - \left(\vec{\nabla}\mu . \frac{\partial \vec{H}}{\partial t}\right)\vec{H} - \vec{\nabla} \times \frac{\rho}{\epsilon}\vec{H} - 2\vec{J} \times \mu\frac{\partial \vec{H}}{\partial t} \qquad (56)$$

Formalism in (56) is the wave equation for the poynting vector $\vec{S}$ in a material medium, with non-zero sources and electric permittivity and magnetic permeability both with spatial variability.

Developed, the solution (56) can also be expressed as,

$$\Box\vec{S} \Leftarrow -\left(\vec{\nabla}\mu . \frac{\partial \vec{H}}{\partial t}\right)\vec{H} - \vec{\nabla} \times \frac{\rho}{\epsilon}\vec{H} - 2\vec{J} \times \mu\frac{\partial \vec{H}}{\partial t} - \mu\frac{\partial \vec{J}}{\partial t} \times \vec{H} - \left(\vec{\nabla}\mu \times \frac{\partial \vec{H}}{\partial t}\right) \times \vec{H} - \vec{\nabla}\frac{\rho}{\epsilon} \times \vec{H} + \vec{E} \times \left(\vec{\nabla} \times \vec{J}\right) + \vec{E} \times \left(\vec{\nabla}\epsilon \times \frac{\partial \vec{E}}{\partial t}\right) \qquad (57)$$

If in (56) we apply the EM radiation conditions in vacuum free space without sources, it is simplified in,

$$\Box\vec{S} = 0 \qquad (58)$$

IV. Wave Equations in Gyrogravitation

The gyrogravitation field (GG) is, similarly to the EM field, a field composed of two transverse components, the gravitational field $\vec{g}$ with an irrotational nature and the gravitation torsion field $\vec{\Omega}_\tau$ of solenoidal type, as defined in Electrogravitodynamics (EGD) described in [3].

Let's try to find the wave equations for the indicated transverse components of the gyrogravitational field (GG). To do this, in an equivalent way done with the EM field, we will use the equations that relate the GG field components, as well as their sources, material density ρ_m and mass current density $\vec{J}_m$ [4]. Here we do not find the EM field problem, where it is different to consider a material medium or a vacuum, in addition to the possible spatial variability in the electric ϵ and magnetic μ constants. Regarding the field GG, it does not affect the environment considered, since the gravitation universal constant *G* value is always the same. For this reason, the gravitational permittivity constant ξ is not variable, neither spatially nor temporally. However, the gravitation torsion permeability constant τ, being a function of the speed of light propagation in the medium *s*, can at least be spatially variable. This situation can be verified through the expressions of the gravitational constants ξ and τ,

$$\xi = \frac{-1}{4\pi G} \qquad \text{and} \qquad \tau = \frac{4\pi G}{s^2} \qquad (59)$$

Where *s* is the speed of light propagation in a material medium (15), but defined now from its relationship with the gravitational constants as,

$$s = \sqrt{\frac{-1}{\xi\tau}} = \frac{c}{n} \qquad (60)$$

Being *n* the absolute refractive index of the material medium described in (16) and *c* is the speed of light in vacuum.

Taking into account both the spatial and temporal constancy of the gravitational permittivity ξ and the possible spatial variability of the gravitation torsion permeability τ, regardless of the medium in which the GG field propagation occurs, we are going to consider that,

$$\vec{\nabla}\xi = 0 \text{ and } \vec{\nabla}\tau \neq 0, \text{ besides } \frac{\partial \xi}{\partial t} = 0 \text{ and } \frac{\partial \tau}{\partial t} = 0 \qquad (61)$$

The EGD equations that relate GG field components and their sources are,

$$\vec{\nabla} . \vec{g} \Leftarrow \frac{\rho_m}{\xi} \qquad (62)$$

$$\vec{\nabla} . \vec{\Omega}_\tau = 0 \qquad (63)$$

$$\vec{\nabla} \times \vec{g} \Leftarrow -\tau\frac{\partial \vec{\Omega}_\tau}{\partial t} \qquad (64)$$

$$\vec{\nabla} \times \vec{\Omega}_\tau \Leftarrow -\vec{J}_m - \xi\frac{\partial \vec{g}}{\partial t} \qquad (65)$$

Where $\vec{\Omega}_\tau$ is the gravitation torsion field, proportional to the gyrotation field $\vec{\Omega}$, similar to the induction field $\vec{B}$ in EM, such that,

$$\vec{\Omega}_\tau = \frac{\vec{\Omega}}{\tau} \qquad (66)$$

Remember that, according to data from [3], for a homogeneous sphere of mass *m*, radius *R*, rotating around an axis that passes through its center with angular velocity $\vec{w}$ and, therefore, with angular momentum $\vec{L}$, we have as gravitation torsion field $\vec{\Omega}_\tau$,

$$\vec{\Omega}_{\tau_{Ext}}\left(\vec{r}, \vec{L}\right) = \frac{L}{8\pi r^3}\left[\vec{u}_L - 3cos\theta\vec{u}_r\right] \qquad \text{with } r \geq R \qquad (67)$$

$$\vec{\Omega}_{\tau_{Int}}\left(\vec{r}, \vec{L}\right) = \frac{L}{16\pi R^3}\left[\left(5 - \frac{3r^2}{R^2}\right)\vec{u}_L + \left(\frac{9r^2}{R^2} - 15\right)cos\theta\vec{u}_r\right] \quad \text{with } r \leq R \qquad (68)$$

Where,

$L = \frac{2}{5} m R^2 w$ and $\vec{L} = L \vec{u}_L$ (69)

$\vec{u}_L = \frac{\vec{L}}{L} = \vec{u}_w = \frac{\vec{w}}{w}$, $w = |\vec{w}|$, $\vec{u}_r = \frac{\vec{r}}{r}$ and $\theta = \left(\widehat{\vec{L},\vec{r}}\right) = \left(\widehat{\vec{w},\vec{r}}\right)$ (70)

Applying nabla operator $\vec{\nabla}$ on (65) and, considering the conditions in (61), the following gravitational continuity equation is obtained,

$\vec{\nabla}.\vec{J}_m \Leftarrow -\frac{\partial \rho_m}{\partial \mathrm{t}}$ (71)

The wave equations for gravitational $\vec{g}$ and gravitation torsion $\vec{\Omega}_\tau$ fields are obtained by determining Laplace operator Δ on each field.

If we apply the rotational operator $\vec{\nabla} \times$ on (64) we obtain,

$\vec{\nabla} \times \left(\vec{\nabla} \times \vec{g}\right) \Leftarrow \vec{\nabla} \times \left(-\tau \frac{\partial \vec{\Omega}_\tau}{\partial \mathrm{t}}\right) = -\tau \frac{\partial}{\partial \mathrm{t}} \left(\vec{\nabla} \times \vec{\Omega}_\tau\right) - \vec{\nabla}\tau \times \frac{\partial \vec{\Omega}_\tau}{\partial \mathrm{t}}$ (72)

Taking into account (65) and (A6) over (72),

$\vec{\nabla}\left(\vec{\nabla}.\vec{g}\right) - \left(\vec{\nabla}.\vec{\nabla}\right)\vec{g} \Leftarrow \tau \frac{\partial}{\partial \mathrm{t}} \left(\vec{J}_m + \xi \frac{\partial \vec{g}}{\partial \mathrm{t}}\right) - \vec{\nabla}\tau \times \frac{\partial \vec{\Omega}_\tau}{\partial \mathrm{t}}$ (73)

Applying (62) in (73) and developing,

$\vec{\nabla} \frac{\rho_m}{\xi} - \vec{\nabla}^2 \vec{g} \Leftarrow \tau \frac{\partial \vec{J}_m}{\partial \mathrm{t}} + \tau\xi \frac{\partial^2 \vec{g}}{\partial \mathrm{t}^2} - \vec{\nabla}\tau \times \frac{\partial \vec{\Omega}_\tau}{\partial \mathrm{t}}$ (74)

Using the Laplacian definition on the gravitational field $\vec{g}$ in (74), we obtain,

$\Delta \vec{g} \Leftarrow \frac{1}{s^2} \frac{\partial^2 \vec{g}}{\partial \mathrm{t}^2} + \vec{\nabla} \frac{\rho_m}{\xi} - \tau \frac{\partial \vec{J}_m}{\partial \mathrm{t}} + \vec{\nabla}\tau \times \frac{\partial \vec{\Omega}_\tau}{\partial \mathrm{t}}$ (75)

Where *s* is the speed of light propagation in a material medium, described in (60).

Applying the definition of the d'Alembertian operator $\Box$ (17) on (75), the wave equation for the gravitational field component $\vec{g}$ is definitely obtained,

$\Box\vec{g} \Leftarrow \tau \frac{\partial \vec{J}_m}{\partial \mathrm{t}} - \vec{\nabla} \frac{\rho_m}{\xi} - \vec{\nabla}\tau \times \frac{\partial \vec{\Omega}_\tau}{\partial \mathrm{t}}$ (76)

We will use the same procedure as above for the gravitation torsion field $\vec{\Omega}_\tau$.

If we apply the rotational operator $\vec{\nabla} \times$ on (65) we obtain,

$\vec{\nabla} \times \left(\vec{\nabla} \times \vec{\Omega}_\tau\right) \Leftarrow \vec{\nabla} \times \left(-\vec{J}_m - \xi \frac{\partial \vec{g}}{\partial \mathrm{t}}\right) = -\vec{\nabla} \times \vec{J}_m - \xi \frac{\partial}{\partial \mathrm{t}} \left(\vec{\nabla} \times \vec{g}\right) - \vec{\nabla}\xi \times \frac{\partial \vec{g}}{\partial \mathrm{t}}$ with $\vec{\nabla}\xi \times \frac{\partial \vec{g}}{\partial \mathrm{t}} = 0$ (77)

Taking into account (64) and (A6) over (77),

$\vec{\nabla}\left(\vec{\nabla}.\vec{\Omega}_\tau\right) - \left(\vec{\nabla}.\vec{\nabla}\right)\vec{\Omega}_\tau \Leftarrow -\vec{\nabla} \times \vec{J}_m + \xi\tau \frac{\partial^2 \vec{\Omega}_\tau}{\partial \mathrm{t}^2}$ (78)

Applying (63) in (78) and developing,

$-\vec{\nabla}^2 \vec{\Omega}_\tau \Leftarrow -\vec{\nabla} \times \vec{J}_m + \xi\tau \frac{\partial^2 \vec{\Omega}_\tau}{\partial \mathrm{t}^2}$ (79)

Using the Laplacian definition on the gravitation torsion field $\vec{\Omega}_\tau$ and (60) in (79), we obtain,

$\Delta \vec{\Omega}_\tau \Leftarrow \frac{1}{s^2} \frac{\partial^2 \vec{\Omega}_\tau}{\partial \mathrm{t}^2} + \vec{\nabla} \times \vec{J}_m$ (80)

Applying the definition of the d'Alembertian operator in (17) over (80), the wave equation for the gravitation torsion field component $\vec{\Omega}_\tau$ is definitely achieved,

$\Box\vec{\Omega}_\tau \Leftarrow -\vec{\nabla} \times \vec{J}_m$ (81)

Observe that the results obtained through (76) and (81) are not null, as would be the case when simplifying, using gravitational source ρ_m and gravitation torsion source $\vec{J}_m$ with values zero, in addition to spatial variability in the gravitation torsion permeability constant τ also null, for the propagation environment considered.

V. Wave Equation for Gyrogravitational Poynting Vector

If the gyrogravitational field propagation conditions are such that the mass density ρ_m and mass current density $\vec{J}_m$, sources in the environment considered, are zero and the material medium refractive index *n* is constant, the field components wave equations can be simplified, that is,

$\Box\vec{g} = 0$ (82)

$\Box\vec{\Omega}_\tau = 0$ (83)

In a similar way to how it is done with EM radiation, we are going to describe the GG field propagation in the form of GG radiation, expressed through the gyrogravitational poynting vector $\vec{S}_g$ [9], since it implies the GG field longitudinal character, formally described as follows,

$\vec{S}_g(\vec{r},t) = \vec{g}(\vec{r},t) \times \vec{\Omega}_\tau(\vec{r},t)$ (84)

Therefore, the gyrogravitational power density $\vec{P}_{g_{avg}}$ is defined as the GG wave average power per transverse area unit to the propagation or temporal average of the gyrogravitational poynting vector $\vec{S}_g$, that is,

$$\vec{P}_{g_{avg}} = \frac{1}{T}\int_0^T \left(\vec{g} \times \vec{\Omega}_\tau\right) dt \tag{85}$$

It is intended to develop the wave equation for the gyrogravitational poynting vector $\vec{S}_g$ in a generic propagation environment, for a material medium, with the possibility of non-zero GG field sources and spatial variability in the gravitation torsion permeability τ.

We begin incorporating the Laplace operator Δ on the poynting vector $\vec{S}_g$, applying (A6),

$$\Delta\left(\vec{g} \times \vec{\Omega}_\tau\right) = \vec{\nabla}^2\left(\vec{g} \times \vec{\Omega}_\tau\right) = \vec{\nabla}\left(\vec{\nabla}.\left(\vec{g} \times \vec{\Omega}_\tau\right)\right) - \vec{\nabla} \times \left(\vec{\nabla} \times \left(\vec{g} \times \vec{\Omega}_\tau\right)\right) \tag{86}$$

Developing the previous equation in parts, through (A1) and (A5), we have that,

$$\vec{\nabla}.\left(\vec{g} \times \vec{\Omega}_\tau\right) = \vec{\Omega}_\tau.\left(\vec{\nabla} \times \vec{g}\right) - \vec{g}.\left(\vec{\nabla} \times \vec{\Omega}_\tau\right) = -\vec{\Omega}_\tau.\tau\frac{\partial\vec{\Omega}_\tau}{\partial t} + \vec{g}.\xi\frac{\partial\vec{g}}{\partial t} + \vec{g}.\vec{J}_m \text{ (scalar)} \tag{87}$$

$$\vec{\nabla} \times \left(\vec{g} \times \vec{\Omega}_\tau\right) = \vec{g}\cancel{\left(\vec{\nabla}.\vec{\Omega}_\tau\right)} - \vec{\Omega}_\tau\left(\vec{\nabla}.\vec{g}\right) + \left(\vec{\Omega}_\tau.\vec{\nabla}\right)\vec{g} - \left(\vec{g}.\vec{\nabla}\right)\vec{\Omega}_\tau \qquad \text{(vector)} \tag{88}$$

Using (87) and (88) over (86), we get,

$$\Delta\left(\vec{g} \times \vec{\Omega}_\tau\right) = \vec{\nabla}\left(\vec{\Omega}_\tau.\left(\vec{\nabla} \times \vec{g}\right)\right) - \vec{\nabla}\left(\vec{g}.\left(\vec{\nabla} \times \vec{\Omega}_\tau\right)\right) + \vec{\nabla} \times \left(\vec{\Omega}_\tau\left(\vec{\nabla}.\vec{g}\right)\right) - \vec{\nabla} \times \left(\left(\vec{\Omega}_\tau.\vec{\nabla}\right)\vec{g}\right) + \vec{\nabla} \times \left(\left(\vec{g}.\vec{\nabla}\right)\vec{\Omega}_\tau\right) \tag{89}$$

Applying vector calculus with (A3) and (A8) over (89), we obtain,

$$\begin{aligned}\Delta\left(\vec{g} \times \vec{\Omega}_\tau\right) = &\left(\left(\vec{\nabla} \times \vec{g}\right).\vec{\nabla}\right)\vec{\Omega}_\tau + \cancel{\left(\vec{\Omega}_\tau.\vec{\nabla}\right)\left(\vec{\nabla} \times \vec{g}\right)} + \left(\vec{\nabla} \times \vec{g}\right) \times \left(\vec{\nabla} \times \vec{\Omega}_\tau\right) + \vec{\Omega}_\tau \times \left(\vec{\nabla} \times \left(\vec{\nabla} \times \vec{g}\right)\right) - \\ &- \left(\left(\vec{\nabla} \times \vec{\Omega}_\tau\right).\vec{\nabla}\right)\vec{g} - \cancel{\left(\vec{g}.\vec{\nabla}\right)\left(\vec{\nabla} \times \vec{\Omega}_\tau\right)} - \left(\vec{\nabla} \times \vec{\Omega}_\tau\right) \times \left(\vec{\nabla} \times \vec{g}\right) - \vec{g} \times \left(\vec{\nabla} \times \left(\vec{\nabla} \times \vec{\Omega}_\tau\right)\right) + \left(\vec{\nabla}.\vec{g}\right)\left(\vec{\nabla} \times \vec{\Omega}_\tau\right) + \\ &+ \vec{\nabla}\left(\vec{\nabla}.\vec{g}\right) \times \vec{\Omega}_\tau - \cancel{\left(\vec{\Omega}_\tau.\vec{\nabla}\right)\left(\vec{\nabla} \times \vec{g}\right)} - \vec{\nabla}\left(\vec{\Omega}_\tau.\vec{\nabla}\right) \times \vec{g} + \cancel{\left(\vec{g}.\vec{\nabla}\right)\left(\vec{\nabla} \times \vec{\Omega}_\tau\right)} + \vec{\nabla}\left(\vec{g}.\vec{\nabla}\right) \times \vec{\Omega}_\tau\end{aligned} \tag{90}$$

To simplify (90), we are going to develop the following procedure.

Taking into account (A1), it can be put that,

$$-\left(\vec{\nabla} \times \vec{\Omega}_\tau\right).\vec{\nabla} = \vec{\nabla}.\left(\vec{\nabla} \times \vec{\Omega}_\tau\right) - \left(\vec{\nabla} \times \vec{\nabla}\right).\vec{\Omega}_\tau = \vec{\nabla}.\left(\vec{\nabla} \times \vec{\Omega}_\tau\right) \tag{91}$$

Then, using (91) and applying (65), we have,

$$-\left(\left(\vec{\nabla} \times \vec{\Omega}_\tau\right).\vec{\nabla}\right)\vec{g} = \left(\vec{\nabla}.\left(\vec{\nabla} \times \vec{\Omega}_\tau\right)\right)\vec{g} = \left(\vec{\nabla}.\left(-\vec{J}_m - \xi\frac{\partial\vec{g}}{\partial t}\right)\right)\vec{g} \tag{92}$$

Expanding (92) and substituting (62), we get,

$$-\left(\left(\vec{\nabla} \times \vec{\Omega}_\tau\right).\vec{\nabla}\right)\vec{g} = -\left(\vec{\nabla}.\vec{J}_m\right)\vec{g} - \left(\vec{\nabla}\xi.\frac{\partial\vec{g}}{\partial t}\right)\vec{g} - \xi\frac{\partial\left(\frac{\rho_m}{\xi}\right)}{\partial t}\vec{g} \tag{93}$$

Applying the GG continuity equation (71) and (61) in (93) and simplifying, the result is,

$$-\left(\left(\vec{\nabla} \times \vec{\Omega}_\tau\right).\vec{\nabla}\right)\vec{g} = \frac{\partial\rho_m}{\partial t}\vec{g} - \xi\frac{\partial\rho_m}{\partial t}\frac{1}{\xi}\vec{g} = 0 \tag{94}$$

On the other hand, taking into account (A1), it can be put that,

$$\left(\vec{\nabla} \times \vec{g}\right).\vec{\nabla} = -\vec{\nabla}.\left(\vec{\nabla} \times \vec{g}\right) + \left(\vec{\nabla} \times \vec{\nabla}\right).\vec{g} = -\vec{\nabla}.\left(\vec{\nabla} \times \vec{g}\right) \tag{95}$$

Then, using (95) and applying (64), we have,

$$\left(\left(\vec{\nabla} \times \vec{g}\right).\vec{\nabla}\right)\vec{\Omega}_\tau = -\left(\vec{\nabla}.\left(\vec{\nabla} \times \vec{g}\right)\right)\vec{\Omega}_\tau = \left(\vec{\nabla}.\left(\tau\frac{\partial\vec{\Omega}_\tau}{\partial t}\right)\right)\vec{\Omega}_\tau \tag{96}$$

Applying (A7) in (96), we get,

$$\left(\left(\vec{\nabla} \times \vec{g}\right).\vec{\nabla}\right)\vec{\Omega}_\tau = \left(\tau\frac{\partial}{\partial t}\left(\vec{\nabla}.\vec{\Omega}_\tau\right)\right)\vec{\Omega}_\tau + \left(\vec{\nabla}\tau.\frac{\partial\vec{\Omega}_\tau}{\partial t}\right)\vec{\Omega}_\tau \tag{97}$$

Also, introducing (63) into (97), the result is,

$$\left(\left(\vec{\nabla} \times \vec{g}\right).\vec{\nabla}\right)\vec{\Omega}_\tau = \left(\vec{\nabla}\tau.\frac{\partial\vec{\Omega}_\tau}{\partial t}\right)\vec{\Omega}_\tau \tag{98}$$

Applying (94) and (98) in (90) and developing, it remains,

$$\begin{aligned}\Delta\left(\vec{g} \times \vec{\Omega}_\tau\right) = &\left(\vec{\nabla}\tau.\frac{\partial\vec{\Omega}_\tau}{\partial t}\right)\vec{\Omega}_\tau + 2\tau\frac{\partial\vec{\Omega}_\tau}{\partial t} \times \vec{J}_m - 2\tau\xi\left(\frac{\partial\vec{g}}{\partial t} \times \frac{\partial\vec{\Omega}_\tau}{\partial t}\right) + \cancel{\vec{\Omega}_\tau \times \left(\vec{\nabla} \times \left(\vec{\nabla} \times \vec{g}\right)\right)} - \cancel{\vec{g} \times \left(\vec{\nabla} \times \left(\vec{\nabla} \times \vec{\Omega}_\tau\right)\right)} - \\ &- \frac{\rho_m}{\xi}\vec{J}_m - \rho_m\frac{\partial\vec{g}}{\partial t} + \vec{\nabla}\frac{\rho_m}{\xi} \times \vec{\Omega}_\tau - \left(\left(\vec{\nabla}.\vec{\nabla}\right)\vec{\Omega}_\tau\right) \times \vec{g} - \left(\left(\vec{\Omega}_\tau.\vec{\nabla}\right)\vec{\nabla}\right) \times \vec{g} - \cancel{\left(\vec{\nabla} \times \left(\vec{\nabla} \times \vec{\Omega}_\tau\right)\right) \times \vec{g}} - \left(\vec{\Omega}_\tau \times \left(\vec{\nabla} \times \vec{\nabla}\right)\right) \times \vec{g} + \\ &+ \left(\left(\vec{\nabla}.\vec{\nabla}\right)\vec{g}\right) \times \vec{\Omega}_\tau + \left(\left(\vec{g}.\vec{\nabla}\right)\vec{\nabla}\right) \times \vec{\Omega}_\tau + \cancel{\left(\vec{\nabla} \times \left(\vec{\nabla} \times \vec{g}\right)\right) \times \vec{\Omega}_\tau} + \left(\vec{g} \times \left(\vec{\nabla} \times \vec{\nabla}\right)\right) \times \vec{\Omega}_\tau\end{aligned} \tag{99}$$

To simplify (99), we are going to use the following expressions.

Applying (A3), we have that,

$$\vec{\nabla}\left(\vec{\nabla}.\vec{\Omega}_\tau\right) = \left(\vec{\Omega}_\tau.\vec{\nabla}\right)\vec{\nabla} + \left(\vec{\nabla}.\vec{\nabla}\right)\vec{\Omega}_\tau + \vec{\Omega}_\tau \times \left(\vec{\nabla} \times \vec{\nabla}\right) + \vec{\nabla} \times \left(\vec{\nabla} \times \vec{\Omega}_\tau\right) \tag{100}$$

Therefore, using (100) we can rewrite,

$$\left[\left(\vec{\Omega}_\tau.\vec{\nabla}\right)\vec{\nabla} + \left(\vec{\Omega}_\tau \times \left(\vec{\nabla} \times \vec{\nabla}\right)\right)\right] \times \vec{g} = \left[\vec{\nabla}\left(\vec{\nabla}.\vec{\Omega}_\tau\right) - \left(\vec{\nabla}.\vec{\nabla}\right)\vec{\Omega}_\tau - \vec{\nabla} \times \left(\vec{\nabla} \times \vec{\Omega}_\tau\right)\right] \times \vec{g} = 0 \tag{101}$$

Since, according to (A6), applied to the gravitation torsion field $\vec{\Omega}_\tau$,

$$(\vec{\nabla}.\vec{\nabla})\vec{\Omega}_\tau = \vec{\nabla}^2\vec{\Omega}_\tau = \vec{\nabla}(\vec{\nabla}.\vec{\Omega}_\tau) - \vec{\nabla}\times(\vec{\nabla}\times\vec{\Omega}_\tau) \tag{102}$$

On the other hand, using (A3) again, we have to,

$$\vec{\nabla}(\vec{\nabla}.\vec{g}) = (\vec{g}.\vec{\nabla})\vec{\nabla} + (\vec{\nabla}.\vec{\nabla})\vec{g} + \vec{g}\times(\vec{\nabla}\times\vec{\nabla}) + \vec{\nabla}\times(\vec{\nabla}\times\vec{g}) \tag{103}$$

Therefore, using (103) we can rewrite,

$$\left[(\vec{g}.\vec{\nabla})\vec{\nabla} + \left(\vec{g}\times(\vec{\nabla}\times\vec{\nabla})\right)\right]\times\vec{\Omega}_\tau = \left[\vec{\nabla}(\vec{\nabla}.\vec{g}) - (\vec{\nabla}.\vec{\nabla})\vec{g} - \vec{\nabla}\times(\vec{\nabla}\times\vec{g})\right]\times\vec{\Omega}_\tau = 0 \tag{104}$$

Since, according to (A6), applied to the gravitational field $\vec{g}$,

$$(\vec{\nabla}.\vec{\nabla})\vec{g} = \vec{\nabla}^2\vec{g} = \vec{\nabla}(\vec{\nabla}.\vec{g}) - \vec{\nabla}\times(\vec{\nabla}\times\vec{g}) \tag{105}$$

Using the results in (60), (101) and (104) over (99), the following is achieved,

$$\Delta(\vec{g}\times\vec{\Omega}_\tau) = \left(\vec{\nabla}\tau.\frac{\partial\vec{\Omega}_\tau}{\partial t}\right)\vec{\Omega}_\tau + 2\tau\frac{\partial\vec{\Omega}_\tau}{\partial t}\times\vec{J}_m + \frac{2}{s^2}\left(\frac{\partial\vec{g}}{\partial t}\times\frac{\partial\vec{\Omega}_\tau}{\partial t}\right) - \frac{\rho_m}{\xi}\vec{J}_m - \rho_m\frac{\partial\vec{g}}{\partial t} + \vec{\nabla}\frac{\rho_m}{\xi}\times\vec{\Omega}_\tau + \vec{g}\times\Delta\vec{\Omega}_\tau + \Delta\vec{g}\times\vec{\Omega}_\tau \tag{106}$$

Now, let's consider the next procedure.

$$\frac{\partial^2}{\partial t^2}(\vec{g}\times\vec{\Omega}_\tau) = \frac{\partial}{\partial t}\left[\frac{\partial}{\partial t}(\vec{g}\times\vec{\Omega}_\tau)\right] = \frac{\partial}{\partial t}\left[\left(\frac{\partial\vec{g}}{\partial t}\times\vec{\Omega}_\tau\right) + \left(\vec{g}\times\frac{\partial\vec{\Omega}_\tau}{\partial t}\right)\right] = \left(\frac{\partial^2\vec{g}}{\partial t^2}\times\vec{\Omega}_\tau\right) + 2\left(\frac{\partial\vec{g}}{\partial t}\times\frac{\partial\vec{\Omega}_\tau}{\partial t}\right) + \left(\vec{g}\times\frac{\partial^2\vec{\Omega}_\tau}{\partial t^2}\right) \tag{107}$$

Applying the GG wave equations (75) and (80) in (107), it can be rewritten,

$$\frac{1}{s^2}\frac{\partial^2}{\partial t^2}(\vec{g}\times\vec{\Omega}_\tau) = \Delta\vec{g}\times\vec{\Omega}_\tau + \tau\frac{\partial\vec{J}_m}{\partial t}\times\vec{\Omega}_\tau - \vec{\nabla}\frac{\rho_m}{\xi}\times\vec{\Omega}_\tau - \left(\vec{\nabla}\tau\times\frac{\partial\vec{\Omega}_\tau}{\partial t}\right)\times\vec{\Omega}_\tau + 2(\vec{\nabla}\times\vec{g})\times(\vec{\nabla}\times\vec{\Omega}_\tau) -$$
$$-2\tau\frac{\partial\vec{\Omega}_\tau}{\partial t}\times\vec{J}_m + \vec{g}\times\Delta\vec{\Omega}_\tau - \vec{g}\times(\vec{\nabla}\times\vec{J}_m) \tag{108}$$

Observe that incorporating (64) and (65), we can put that,

$$2(\vec{\nabla}\times\vec{g})\times(\vec{\nabla}\times\vec{\Omega}_\tau) = 2\tau\frac{\partial\vec{\Omega}_\tau}{\partial t}\times\vec{J}_m - 2\xi\tau\left(\frac{\partial\vec{g}}{\partial t}\times\frac{\partial\vec{\Omega}_\tau}{\partial t}\right) \tag{109}$$

Then, applying (60) in (109), we have that,

$$\frac{2}{s^2}\left(\frac{\partial\vec{g}}{\partial t}\times\frac{\partial\vec{\Omega}_\tau}{\partial t}\right) = -2\xi\tau\left(\frac{\partial\vec{g}}{\partial t}\times\frac{\partial\vec{\Omega}_\tau}{\partial t}\right) = 2(\vec{\nabla}\times\vec{g})\times(\vec{\nabla}\times\vec{\Omega}_\tau) - 2\tau\frac{\partial\vec{\Omega}_\tau}{\partial t}\times\vec{J}_m \tag{110}$$

Introducing (110) in (108), we get,

$$\frac{1}{s^2}\frac{\partial^2}{\partial t^2}(\vec{g}\times\vec{\Omega}_\tau) = \Delta\vec{g}\times\vec{\Omega}_\tau + \tau\frac{\partial\vec{J}_m}{\partial t}\times\vec{\Omega}_\tau - \vec{\nabla}\frac{\rho_m}{\xi}\times\vec{\Omega}_\tau - \left(\vec{\nabla}\tau\times\frac{\partial\vec{\Omega}_\tau}{\partial t}\right)\times\vec{\Omega}_\tau + \frac{2}{s^2}\left(\frac{\partial\vec{g}}{\partial t}\times\frac{\partial\vec{\Omega}_\tau}{\partial t}\right) + \vec{g}\times\Delta\vec{\Omega}_\tau - \vec{g}\times(\vec{\nabla}\times\vec{J}_m) \tag{111}$$

Therefore, using (111) in (106), we obtain,

$$\Delta(\vec{g}\times\vec{\Omega}_\tau) = \frac{1}{s^2}\frac{\partial^2}{\partial t^2}(\vec{g}\times\vec{\Omega}_\tau) + 2\tau\frac{\partial\vec{\Omega}_\tau}{\partial t}\times\vec{J}_m - \frac{\rho_m}{\xi}\vec{J}_m - \rho_m\frac{\partial\vec{g}}{\partial t} + 2\vec{\nabla}\frac{\rho_m}{\xi}\times\vec{\Omega}_\tau - \tau\frac{\partial\vec{J}_m}{\partial t}\times\vec{\Omega}_\tau + \vec{g}\times(\vec{\nabla}\times\vec{J}_m) +$$
$$+\left(\vec{\nabla}\tau.\frac{\partial\vec{\Omega}_\tau}{\partial t}\right)\vec{\Omega}_\tau + \left(\vec{\nabla}\tau\times\frac{\partial\vec{\Omega}_\tau}{\partial t}\right)\times\vec{\Omega}_\tau \tag{112}$$

If we consider (A8), the following can be developed,

$$\vec{\nabla}\frac{\rho_m}{\xi}\times\vec{\Omega}_\tau = \vec{\nabla}\times\frac{\rho_m}{\xi}\vec{\Omega}_\tau - \frac{\rho_m}{\xi}\vec{\nabla}\times\vec{\Omega}_\tau = \vec{\nabla}\times\frac{\rho_m}{\xi}\vec{\Omega}_\tau + \frac{\rho_m}{\xi}\vec{J}_m + \rho_m\frac{\partial\vec{g}}{\partial t} \tag{113}$$

Using the result in (113) over (112) allows obtaining,

$$\Delta(\vec{g}\times\vec{\Omega}_\tau) = \frac{1}{s^2}\frac{\partial^2}{\partial t^2}(\vec{g}\times\vec{\Omega}_\tau) + 2\tau\frac{\partial\vec{\Omega}_\tau}{\partial t}\times\vec{J}_m + \vec{\nabla}\frac{\rho_m}{\xi}\times\vec{\Omega}_\tau + \vec{\nabla}\times\frac{\rho_m}{\xi}\vec{\Omega}_\tau - \tau\frac{\partial\vec{J}_m}{\partial t}\times\vec{\Omega}_\tau + \vec{g}\times(\vec{\nabla}\times\vec{J}_m) +$$
$$+\left(\vec{\nabla}\tau.\frac{\partial\vec{\Omega}_\tau}{\partial t}\right)\vec{\Omega}_\tau + \left(\vec{\nabla}\tau\times\frac{\partial\vec{\Omega}_\tau}{\partial t}\right)\times\vec{\Omega}_\tau \tag{114}$$

Considering the wave equation for the gravitational field $\vec{g}$ (76) and for the gravitation torsion field $\vec{\Omega}_\tau$ (81), it can be stated that,

$$-\Box\vec{g}\times\vec{\Omega}_\tau = -\tau\frac{\partial\vec{J}_m}{\partial t}\times\vec{\Omega}_\tau + \vec{\nabla}\frac{\rho_m}{\xi}\times\vec{\Omega}_\tau + \left(\vec{\nabla}\tau\times\frac{\partial\vec{\Omega}_\tau}{\partial t}\right)\times\vec{\Omega}_\tau \tag{115}$$

$$-\vec{g}\times\Box\vec{\Omega}_\tau = \Box\vec{\Omega}_\tau\times\vec{g} = -(\vec{\nabla}\times\vec{J}_m)\times\vec{g} = \vec{g}\times(\vec{\nabla}\times\vec{J}_m) \tag{116}$$

If we take into account the d'Alembertian operator definition in (17) on the gyrogravitational poynting vector $\vec{S}_g$,

$$\Box\vec{S}_g = \frac{1}{s^2}\frac{\partial^2\vec{S}_g}{\partial t^2} - \Delta\vec{S}_g \quad \text{, that is,} \quad \Box(\vec{g}\times\vec{\Omega}_\tau) = \frac{1}{s^2}\frac{\partial^2(\vec{g}\times\vec{\Omega}_\tau)}{\partial t^2} - \Delta(\vec{g}\times\vec{\Omega}_\tau) \tag{117}$$

Therefore, introducing (115), (116) and (117) in (114) and, incorporating the inducer-induced concept [8], we obtain,

$$\Box\vec{S}_g \Leftarrow \vec{g}\times\Box\vec{\Omega}_\tau + \Box\vec{g}\times\vec{\Omega}_\tau - \left(\vec{\nabla}\tau.\frac{\partial\vec{\Omega}_\tau}{\partial t}\right)\vec{\Omega}_\tau - \vec{\nabla}\times\frac{\rho_m}{\xi}\vec{\Omega}_\tau - 2\tau\frac{\partial\vec{\Omega}_\tau}{\partial t}\times\vec{J}_m \tag{118}$$

Equation (118) is the wave equation for the poynting vector $\vec{S}_g$ in a material medium, with non-zero sources and spatial variability in gravitation torsion permeability.

Developed, the solution (118) can also be expressed as,

$$\Box\vec{S}_g \Leftarrow -\left(\vec{\nabla}\tau.\frac{\partial\vec{\Omega}_\tau}{\partial t}\right)\vec{\Omega}_\tau - \vec{\nabla}\times\frac{\rho_m}{\xi}\vec{\Omega}_\tau - 2\tau\times\frac{\partial\vec{\Omega}_\tau}{\partial t}\vec{J}_m + \tau\frac{\partial\vec{J}_m}{\partial t}\times\vec{\Omega}_\tau - \left(\vec{\nabla}\tau\times\frac{\partial\vec{\Omega}_\tau}{\partial t}\right)\times\vec{\Omega}_\tau - \vec{\nabla}\frac{\rho_m}{\xi}\times\vec{\Omega}_\tau - \vec{g}\times\left(\vec{\nabla}\times\vec{J}_m\right) \quad (119)$$

If on (118) we apply the GG radiation conditions in vacuum free space, without sources, it is simplified in,

$$\Box\vec{S}_g = 0 \quad (120)$$

VI. Electrogravitodynamics (EGD): Electromagnetism and Gyrogravitation Interrelation Equations

Electrogravitodynamics (EGD) aims to find the relationship between EM radiation and GG radiation. For this, interrelation equations will be proposed within components of each radiation variant, first between the same type components and then between different type ones. That is, starting from the equation set describing each radiation (see Appendix IX), we are going to write formal relationships between irrotational type components and the same for solenoidal type ones; then, we will propose relation equations of irrotational with solenoidal components in different radiations.

We begin by considering a vacuum environment, where the equations relating the electric $\vec{E}$ and gravitational $\vec{g}$ irrotational fields and, on the other hand, the magnetic $\vec{H}$ and the gravitation torsion $\vec{\Omega}_\tau$ solenoidal fields, are the following,

$$\vec{E} \Leftarrow -\vec{g}\sqrt{\frac{K_0}{G}} \qquad \text{with} \qquad K_0 = \frac{1}{4\pi\epsilon_0} \quad (121)$$

$$\vec{B} \Leftarrow -\vec{\Omega}\sqrt{\frac{K_0}{G}} \qquad \text{with} \qquad \vec{\Omega} = \tau\vec{\Omega}_\tau \quad \text{and} \quad \vec{B} = \mu_0\vec{H} \quad (122)$$

In a material medium, the above relationships can be expressed as,

$$\vec{D} = \epsilon\vec{E} \Leftarrow -\vec{g}\sqrt{\frac{\epsilon}{4\pi G}} \qquad \text{or} \qquad \vec{D} = -\vec{g}\sqrt{-\epsilon\xi} = -\frac{\vec{g}}{s}\sqrt{\frac{\epsilon}{\tau}} \quad (123)$$

$$\vec{E} \Leftarrow -\vec{g}\sqrt{\frac{K}{G}} \qquad \text{or} \qquad \vec{E} \Leftarrow -\vec{g}\sqrt{\frac{-\xi}{\epsilon}} = -\frac{\vec{g}}{s} \quad (124)$$

$$\vec{H} = \frac{\vec{B}}{\mu} \Leftarrow -\frac{\tau\vec{\Omega}_\tau}{\mu}\sqrt{\frac{K}{G}} = -\vec{\Omega}_\tau\sqrt{\frac{G}{K}} \qquad \text{or} \qquad \vec{H} \Leftarrow -\vec{\Omega}_\tau\sqrt{\frac{\tau}{\mu}} = -\frac{\vec{\Omega}_\tau}{s}\sqrt{-\frac{1}{\xi\mu}} \quad (125)$$

Where,

$$K = \frac{1}{4\pi\epsilon} \quad (126)$$

The electric permittivity ϵ and the magnetic permeability μ are defined in (3), the gravitational permittivity ξ and the gravitation torsion permeability τ in (59) and the propagation speed s in (15) for EM radiation and in (60) for GG radiation.

To find the relationship between solenoidal and irrotational components which allows one type of radiation to be converted into another, it is first necessary to know the relation between the same type sources and different radiation. Therefore, first of all, we are going to look for the relationship between the electric $\vec{J}$ and mass $\vec{J}_m$ current densities, that is,

$$\vec{J} = \rho\vec{v} \quad (127)$$

$$\vec{J}_m = \rho_m\vec{v} \quad (128)$$

Where $\vec{v}$ is the charged particles velocity considered in motion.

Combining (127) and (128) and applying (5), we obtain,

$$\vec{J} = \frac{\rho}{\rho_m}\vec{J}_m = \frac{\rho}{\epsilon}\frac{\epsilon}{\rho_m}\vec{J}_m = \left(\vec{\nabla}.\vec{E}\right)\frac{\epsilon}{\rho_m}\vec{J}_m \quad (129)$$

Introducing (124) in (129), we get,

$$\vec{J} = \left(\vec{\nabla}.\left(-\vec{g}\sqrt{\frac{K}{G}}\right)\right)\frac{\epsilon}{\rho_m}\vec{J}_m \quad (130)$$

Using the rule (A7) in (130), it develops as,

$$\vec{J} = \left(-\vec{g}.\vec{\nabla}\sqrt{\frac{1}{4\pi G\epsilon}} - \sqrt{\frac{K}{G}}\left(\vec{\nabla}.\vec{g}\right)\right)\frac{\epsilon}{\rho_m}\vec{J}_m \quad (131)$$

Now, applying (59) and (62) in (131) and, simplifying, it turns out that,

$$\vec{J} = \left(\frac{\vec{g}.\vec{\nabla}\epsilon}{2\rho_m} - \frac{\epsilon}{\xi}\right)\sqrt{\frac{K}{G}}\vec{J}_m \quad (132)$$

Going from (131) to (132), it has been taken into account that,

$$\vec{\nabla}\sqrt{\epsilon^{-1}} = -\frac{\epsilon^{-2}}{2\sqrt{\epsilon^{-1}}}\vec{\nabla}\epsilon = -\frac{\vec{\nabla}\epsilon}{2\sqrt{\epsilon^3}} \quad (133)$$

Using (59) and (126) in (132), the permittivity constants relationship is suppressed and, we get,

$$\vec{\mathrm{J}} = \left(\frac{\vec{g}.\vec{\nabla}\epsilon}{2\rho_m}\sqrt{\frac{K}{G}} + \sqrt{\frac{G}{K}}\right)\vec{J}_m \qquad (134)$$

Secondly, we are going to look for the relation between the electric charge ρ and the mass ρ_m densities. For this, reordering (129) and considering the result in (134), we have that,

$$\frac{\rho}{\rho_m} = \frac{\vec{\mathrm{J}}}{\vec{J}_m} = \frac{\vec{g}.\vec{\nabla}\epsilon}{2\rho_m}\sqrt{\frac{K}{G}} + \sqrt{\frac{G}{K}} \qquad (135)$$

And therefore, (135) can be put as,

$$\rho = \frac{\vec{g}.\vec{\nabla}\epsilon}{2}\sqrt{\frac{K}{G}} + \rho_m\sqrt{\frac{G}{K}} \qquad (136)$$

Note that in order to arrive at the results of (134) and (136), spatial variability in material medium has been considered respect to the non null electric permittivity ϵ. In case that in the considered medium $\vec{\nabla}\epsilon = 0$ then,

$$\frac{\vec{\mathrm{J}}}{\vec{J}_m} = \frac{\rho}{\rho_m} = \sqrt{\frac{G}{K}} \quad \text{with} \quad \vec{\nabla}\epsilon = 0 \qquad (137)$$

Now, applying (124) in (8), we obtain the solenoidal magnetic field $\vec{\mathrm{H}}$ with the irrotational gravitational field $\vec{\mathrm{g}}$ relationship.

$$\vec{\nabla} \times \vec{H} \Leftarrow \vec{J} - \epsilon\sqrt{\frac{K}{G}}\frac{\partial\vec{g}}{\partial \mathrm{t}} \qquad (138)$$

If we want the magnetic field $\vec{\mathrm{H}}$ only depends on gravitational field parameters, we must apply (134) in (138), obtaining,

$$\vec{\nabla} \times \vec{H} \Leftarrow \sqrt{\frac{G}{K}}\vec{J}_m + \left(\frac{\vec{g}.\vec{\nabla}\epsilon}{2\rho_m}\vec{J}_m - \epsilon\frac{\partial\vec{g}}{\partial \mathrm{t}}\right)\sqrt{\frac{K}{G}} \qquad (139)$$

If we combine (59) and (126) to obtain the relationship between permittivities, we get,

$$\frac{\epsilon}{\xi} = -\frac{G}{K} \qquad (140)$$

Introducing (140) in (139), we obtain the relation of the magnetic field $\vec{\mathrm{H}}$ with the gravitational field $\vec{\mathrm{g}}$ and the gravitational sources, material density ρ_m and mass current density $\vec{J}_m$ below,

$$\vec{\nabla} \times \vec{H} \Leftarrow \sqrt{\frac{G}{K}}\left(\vec{J}_m + \xi\frac{\partial\vec{g}}{\partial \mathrm{t}}\right) + \sqrt{\frac{K}{G}}\frac{\vec{g}.\vec{\nabla}\epsilon}{2\rho_m}\vec{J}_m \qquad (141)$$

Using (65), the expression (141) can also be put finally as,

$$\vec{\nabla} \times \vec{H} \Leftarrow -\sqrt{\frac{G}{K}}\vec{\nabla} \times \vec{\Omega}_\tau + \sqrt{\frac{K}{G}}\frac{\vec{g}.\vec{\nabla}\epsilon}{2\rho_m}\vec{J}_m \qquad (142)$$

On the other hand, using (124) over (65), the solenoidal gravitation torsion field $\vec{\Omega}_\tau$ with the irrotational electric field $\vec{\mathrm{E}}$ relation is obtained,

$$\vec{\nabla} \times \vec{\Omega}_\tau \Leftarrow -\vec{J}_m + \xi\sqrt{\frac{G}{K}}\frac{\partial\vec{E}}{\partial \mathrm{t}} \qquad (143)$$

For (143) depends on EM sources, expressions (134) and (136), relationships between current densities and material densities, respectively, must be arranged.

The density relationship (mass-charge) in (136) can also be expressed as,

$$\rho_m = \rho\sqrt{\frac{K}{G}} - \frac{\vec{g}.\vec{\nabla}\epsilon}{2}\frac{K}{G} \qquad (144)$$

By introducing (144) into (134), the current density relationship is set as a function of the charge density ρ,

$$\vec{\mathrm{J}} = \left(\frac{\vec{g}.\vec{\nabla}\epsilon}{2\rho - \vec{g}.\vec{\nabla}\epsilon\sqrt{\frac{K}{G}}} + \sqrt{\frac{G}{K}}\right)\vec{J}_m \qquad (145)$$

Thus, inverting the current densities in (145), it can also be put as,

$$\vec{J}_m = \left(\frac{2\rho - \vec{g}.\vec{\nabla}\epsilon\sqrt{\frac{K}{G}}}{2\rho\sqrt{\frac{G}{K}}}\right)\vec{\mathrm{J}} \qquad (146)$$

Introducing in (146) the expression (124), it remains,

$$\vec{J}_m = \left(\frac{2\rho + \vec{E}.\vec{\nabla}\epsilon}{2\rho}\sqrt{\frac{K}{G}}\right)\vec{\mathrm{J}} = \left(1 + \frac{\vec{E}.\vec{\nabla}\epsilon}{2\rho}\right)\sqrt{\frac{K}{G}}\vec{\mathrm{J}} \qquad (147)$$

Now, if we want the gravitation torsion field $\vec{\Omega}_\tau$ only depends on electromagnetical type field parameters, we must apply (147) in (143), obtaining,

$$\vec{\nabla} \times \vec{\Omega}_\tau \Leftarrow -\left(1 + \frac{\vec{E}.\vec{\nabla}\epsilon}{2\rho}\right)\sqrt{\frac{K}{G}}\vec{\mathrm{J}} + \xi\sqrt{\frac{G}{K}}\frac{\partial\vec{E}}{\partial \mathrm{t}} \quad (148)$$

Incorporating (140) in (148), we obtain the relationship of the gravitation torsion field $\vec{\Omega}_\tau$ with the electric field $\vec{\mathrm{E}}$ and the EM sources (charge density ρ and electric current density $\vec{\mathrm{J}}$) below,

$$\vec{\nabla} \times \vec{\Omega}_\tau \Leftarrow -\left(\frac{\vec{E}.\vec{\nabla}\epsilon}{2\rho}\vec{\mathrm{J}} + \vec{\mathrm{J}} + \epsilon\frac{\partial\vec{E}}{\partial \mathrm{t}}\right)\sqrt{\frac{K}{G}} \quad (149)$$

Applying (8) to (149), the desired relationship is obtained,

$$\vec{\nabla} \times \vec{\Omega}_\tau \Leftarrow -\sqrt{\frac{K}{G}}\left(\vec{\nabla} \times \vec{\mathrm{H}} + \frac{\vec{E}.\vec{\nabla}\epsilon}{2\rho}\vec{\mathrm{J}}\right) \quad (150)$$

In short, from (142) and (150), it follows that the rotational of a given field radiation solenoidal component can be set as a rotational function of the opposite field radiation solenoidal component and, of the sources and irrotational component in the opposite field radiation, if there is spatial variability in the electric permittivity of the medium used.

If the considered medium does not allow spatial variability in electric permittivity, (142) and (150) simplify into the next bidirectional inducer-induced equation,

$$\vec{\nabla} \times \vec{\Omega}_\tau \Leftrightarrow -\sqrt{\frac{K}{G}}\vec{\nabla} \times \vec{\mathrm{H}} \qquad \text{with} \qquad \vec{\nabla}\epsilon = 0 \quad (151)$$

VII. Poynting Vectors and Intrinsic Impedances in EGD

In EM radiation, transverse field components $\vec{\mathrm{E}}$ and $\vec{\mathrm{H}}$ are related to the poynting vector $\vec{\mathrm{S}}$ through the intrinsic impedance η. In a generic way, in a material medium, the intrinsic impedance η is defined as,

$$\eta = \frac{\left|\frac{\vec{\mathrm{S}}}{|\vec{\mathrm{S}}|}\times\vec{\mathrm{E}}\right|}{|\vec{\mathrm{H}}|} \quad \text{or} \quad \eta = \sqrt{\frac{\mu}{\epsilon}} = \frac{4\pi K}{s} = \frac{1}{\epsilon s} = \mu s \quad , \quad \text{with } s = c/n \quad (152)$$

In vacuum, we have the intrinsic impedance η_0, with a known concrete value,

$$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = \frac{4\pi K_0}{c} = \frac{1}{\epsilon_0 c} = \mu_0 c \approx 377\Omega \quad (153)$$

Therefore, relating the intrinsic impedance η_0 in vacuum with the intrinsic impedance η in any material medium,

$$\eta = \eta_0\frac{\mu_r}{n} = \eta_0\frac{n}{\epsilon_r} \approx 377\frac{n}{\epsilon_r}\Omega \quad (154)$$

For GG radiation, we are going to use a definition similar to (152) to relate the transverse field components $\vec{\mathrm{g}}$ and $\vec{\Omega}_\tau$ with the gyrogravitational poynting vector $\vec{\mathrm{S}}_g$, giving rise to the intrinsic gyrogravitational impedance η_g,

$$\eta_g = \frac{\left|\frac{\vec{\mathrm{S}}_g}{|\vec{\mathrm{S}}_g|}\times\vec{\mathrm{g}}\right|}{|\vec{\Omega}_\tau|} \quad \text{or} \quad \eta_g = \sqrt{-\frac{\tau}{\xi}} = \frac{4\pi G}{s} = -\frac{1}{\xi s} = \tau s \quad (155)$$

That is, numerically the intrinsic gyrogravitational impedance η_g can be expressed as,

$\eta_g = 8.905.10^{-19}\pi n = 2.797510^{-18} n\Omega\mathrm{C}^2\mathrm{kg}^{-2}$, n is the medium absolute refractive index (156)

Note that the gyrogravitational intrinsic impedance η_g can be normalized expressed in ohms, as the EM intrinsic impedance η, without more than considering the system mass-charge relationship where it is applied. In case the system considered consists of the gyrogravitational relationship maintained by two masses m_{i1} and m_{i2} charged with q_{i1} and q_{i2}, respectively, we will have the normalized GG impedance $\eta'_{g_{12}}$, such that,

$$\eta'_{g_{12}} = \eta_g\left(\frac{m_{i1}}{q_{i1}}\right)\left(\frac{m_{i2}}{q_{i2}}\right) \quad (157)$$

If, for example, we have a GG system in a vacuum defined by two masses of values $m_{i1} = 5kg$ and $m_{i2} = 10kg$ charged with $q_{i1} = 10^{-9}C$ and $q_{i2} = \frac{10^{-9}}{2}C$, respectively, applying (157) we obtain,

$$\eta'_{g_{12}} = 70\Omega \quad (158)$$

On the other hand, the normalized gyrogravitational intrinsic impedance η'_g can also describe the characteristic impedance of a mass m_i charged with q_i with respect to the environment considered, that is,

$$\eta'_{g_i} = \eta_g\left(\frac{m_i}{q_i}\right)^2 \quad (159)$$

If, for example, we have a GG system in a vacuum given by a mass of $m_i = 5kg$ charged with $q_i = 10^{-9}C$, applying (159) we obtain,

$$\eta'_{g_i} = 70\Omega \quad (160)$$

Furthermore, the relationship between intrinsic impedances in EM η and in GG η_g can be obtained from (152) and (155), such that,

$\frac{\eta}{\eta_g} = \frac{\mu}{\tau} = \frac{K}{G} = \frac{1.3485}{\epsilon_r} 10^{20} kg^2 C^{-2}$, ϵ_r is the electric permittivity relative to the medium (161)

When we speak of EM energy conversion into GG energy and vice versa [10] we must consider that for the electric $\vec{E}$ and magnetic $\vec{H}$ transverse fields generation, the gravitational $\vec{g}$ and gravitation torsion $\vec{\Omega}_\tau$ transverse fields are not such as those to keep in mind. Actually, for the electric $\vec{E}$ and magnetic $\vec{H}$ fields generation from GG fields or on the contrary, it is necessary to consider the equivalent conversion fields $\vec{G}_E$ and $\vec{G}_H$, respectively, as demonstrated in the previous study indicated, so that,

$$\vec{G}_E = \vec{\Omega}_\tau \sqrt{\frac{K}{G}} \tag{162}$$

$$\vec{G}_H = \frac{\vec{g}}{s^2} \sqrt{\frac{K}{G}} \tag{163}$$

In this way, the EM-GG conversion gyrogravitational poynting vector $\vec{S}_{g_c}$ is defined as,

$$\vec{S}_{g_c} = \vec{G}_H \times \vec{G}_E = \frac{K}{s^2 G} \left(\vec{g} \times \vec{\Omega}_\tau \right) \tag{164}$$

So, for the conversion energy GG, we have the intrinsic gyrogravitational conversion impedance η_{g_c},

$$\eta_{g_c} = \frac{\left| \frac{\vec{S}_{g_c}}{|\vec{S}_{g_c}|} \times \vec{g} \right| \frac{1}{s^2}}{|\vec{\Omega}_\tau|} = \frac{\tau}{s} \tag{165}$$

The specific energy intensity S_η is established as the radiated energy intensity per unit of area and specific impedance in the radiation considered propagation environment. It is the relationship between the energy flow modulus and the impedance, in the radiation environment where the propagation occurs. So, we have to,

$$S_\eta = \frac{|\vec{S}|}{\eta} \tag{166}$$

Therefore, three different types of specific energy intensity will be considered, for EM radiation S_η, for GG radiation S_{η_g} and for conversion GG radiation $S_{\eta_{g_c}}$.

$$S_\eta = \frac{|\vec{E} \times \vec{H}|}{\mu s} = \left| \vec{H} \right|^2 \tag{167}$$

$$S_{\eta_g} = \frac{|\vec{g} \times \vec{\Omega}_\tau|}{\tau s} = \left| \vec{\Omega}_\tau \right|^2 \tag{168}$$

$$S_{\eta_{g_c}} = \frac{K |\vec{g} \times \vec{\Omega}_\tau|}{G \tau s} = \frac{K}{G} \left| \vec{\Omega}_\tau \right|^2 \tag{169}$$

Using the specific energies indicated in (167), (168) and (169), together with the intrinsic impedances associated with EM radiation, GG radiation and GG radiation for EM-GG conversion, respectively, the intrinsic impedance relationships are obtained between radiations and, on the other hand, the relationships between specific energy intensities for them. See results in Table 1.

Table 1: Poynting vectors and intrinsic impedance relationships

Radiation	Poynting Vector	Poynting Vector Relationship	Intrinsic Impedance	Specific Energy Intensity $S_\eta = \frac{\|\vec{S}\|}{\eta}$	Intrinsic Impedance Relationship	Specific Energy Intensity Relationship
EM	$\vec{S} = \vec{E} \times \vec{H}$	$\frac{\|\vec{S}\|}{\|\vec{S}_g\|} = 1$	$\eta = \frac{\left\| \frac{\vec{S}}{\|\vec{S}\|} \times \vec{E} \right\|}{\|\vec{H}\|} = \mu s$	$S_\eta = \frac{\|\vec{E} \times \vec{H}\|}{\mu s} = \|\vec{H}\|^2$	$\frac{\eta}{\eta_g} = \frac{K}{G}$	$\frac{S_\eta}{S_{\eta_g}} = \frac{G}{K}$
GG	$\vec{S}_g = \vec{g} \times \vec{\Omega}_\tau$	$\frac{\|\vec{S}_g\|}{\|\vec{S}_{g_c}\|} = \frac{s^2 G}{K}$	$\eta_g = \frac{\left\| \frac{\vec{S}_g}{\|\vec{S}_g\|} \times \vec{g} \right\|}{\|\vec{\Omega}_\tau\|} = \tau s$	$S_{\eta_g} = \frac{\|\vec{g} \times \vec{\Omega}_\tau\|}{\tau s} = \|\vec{\Omega}_\tau\|^2$	$\frac{\eta_g}{\eta_{g_c}} = s^2$	$\frac{S_{\eta_g}}{S_{\eta_{g_c}}} = \frac{G}{K}$
Conversion GG	$\vec{S}_{g_c} = \frac{K}{s^2 G} (\vec{g} \times \vec{\Omega}_\tau)$	$\frac{\|\vec{S}\|}{\|\vec{S}_{g_c}\|} = \frac{s^2 G}{K}$	$\eta_{g_c} = \frac{\left\| \frac{\vec{S}_{g_c}}{\|\vec{S}_{g_c}\|} \times \vec{g} \right\| \frac{1}{s^2}}{\|\vec{\Omega}_\tau\|} = \frac{\tau}{s}$	$S_{\eta_{g_c}} = \frac{K \|\vec{g} \times \vec{\Omega}_\tau\|}{G \tau s} = \frac{K}{G} \|\vec{\Omega}_\tau\|^2$	$\frac{\eta}{\eta_{g_c}} = \frac{K}{G} s^2$	$\frac{S_\eta}{S_{\eta_{g_c}}} = \left(\frac{G}{K}\right)^2$

VIII. Conclusions

Electrogravitodynamics (EGD) formal structure has been presented in this work, summarized in:

- Table 5, Table 6 and Table 7 in Appendix IX: EM and GG basic structural equations. Description of the permittivity ϵ and ξ and permeability μ and τ constants, continuity equations and force generated by the transversal component field combination. Maxwell equations for EM and GG. Transverse components conversion within EM and GG, applied through the radiation propagation speed.

- Table 2: Wave equations development in the EM, both of the electric $\vec{\mathrm{E}}$ and magnetic $\vec{\mathrm{H}}$ transverse field components, as well as the poynting vector $\vec{\mathrm{S}}$, in a material medium with non-zero sources and later, in a vacuum without sources.
- Table 3: Wave equations development for the GG, both of the gravitational $\vec{\mathrm{g}}$ and gravitation torsion $\vec{\Omega}_\tau$ transverse field components, as well as the gravitational poynting vector $\vec{\mathrm{S}}_g$, in a material medium with non-zero sources and after, in vacuum without sources.
- Table 4: Interrelation EM and GG. Equations between irrotational type components and the same for solenoidal type components. Irrotational with solenoidal components relation equations between different radiations are also proposed, incorporating the corresponding sources for material medium and, later simplifying, for the vacuum. Relationship between sources for EM and GG.
- Table 1: Specific energy intensity definitions, together with intrinsic impedances for EM radiation, GG radiation, and GG radiation for EM-GG conversion. Poynting vector and intrinsic impedances relationships between radiations and, on the other hand, relationships between specific energy intensities.

Table 2: Generic wave equations in electromagnetism.

Component	Wave Equation (Generic)	Wave Equation (In vacuum, null sources)
Electric Field $\vec{\mathrm{E}}$	$\square\vec{E} \Leftarrow -\mu\frac{\partial\vec{J}}{\partial t} - \vec{\nabla}\frac{\rho}{\epsilon} - \vec{\nabla}\mu \times \frac{\partial\vec{H}}{\partial t}$	$\square\vec{E} = 0$
Magnetic Field $\vec{\mathrm{H}}$	$\square\vec{H} \Leftarrow \vec{\nabla}\times\vec{J} + \vec{\nabla}\epsilon \times \frac{\partial\vec{E}}{\partial t}$	$\square\vec{H} = 0$
Poynting Vector $\vec{\mathrm{S}}$	$\square\vec{S} \Leftarrow \vec{E}\times\square\vec{H} + \square\vec{E}\times\vec{H} - \left(\vec{\nabla}\mu.\frac{\partial\vec{H}}{\partial t}\right)\vec{H} - \vec{\nabla}\times\frac{\rho}{\epsilon}\vec{H} - 2\vec{J}\times\mu\frac{\partial\vec{H}}{\partial t}$	$\square\vec{S} = 0$

Table 3: Generic wave equations in gyrogravitation.

Component	Wave Equation (Generic)	Wave Equation (In vacuum, null sources)
Gravitational Field $\vec{\mathrm{g}}$	$\square\vec{g} \Leftarrow \tau\frac{\partial\vec{J}_m}{\partial t} - \vec{\nabla}\frac{\rho_m}{\xi} - \vec{\nabla}\tau \times \frac{\partial\vec{\Omega}_\tau}{\partial t}$	$\square\vec{g} = 0$
Gravitation torsion field $\vec{\Omega}_\tau$	$\square\vec{\Omega}_\tau \Leftarrow -\vec{\nabla}\times\vec{J}_m$	$\square\vec{\Omega}_\tau = 0$
Gravitational Poynting Vector $\vec{\mathrm{S}}_g$	$\square\vec{\mathrm{S}}_g \Leftarrow \vec{g}\times\square\vec{\Omega}_\tau + \square\vec{\mathrm{g}}\times\vec{\Omega}_\tau - \left(\vec{\nabla}\tau.\frac{\partial\vec{\Omega}_\tau}{\partial t}\right)\vec{\Omega}_\tau - \vec{\nabla}\times\frac{\rho_m}{\xi}\vec{\Omega}_\tau - 2\tau\frac{\partial\vec{\Omega}_\tau}{\partial t}\times\vec{J}_m$	$\square\vec{\mathrm{S}}_g = 0$

Table 4: Electrogravitodynamics: EM and GG interrelation equations.

Inducer Component	Induced Component	Component Interrelation (Generic)	Component Interrelation (In vacuum, null sources)
$\vec{\mathrm{g}}$	$\vec{\mathrm{E}}$	$\vec{\mathrm{E}} \Leftarrow -\vec{\mathrm{g}}\sqrt{\frac{K}{G}} = -\vec{\mathrm{g}}\sqrt{\frac{-\xi}{\epsilon}}$	$\vec{\mathrm{E}} \Leftarrow -\vec{\mathrm{g}}\sqrt{\frac{K_0}{G}}$
	$\vec{\mathrm{H}}$	$\vec{\nabla}\times\vec{H} \Leftarrow \sqrt{\frac{G}{K}}\left(\vec{J}_m + \xi\frac{\partial\vec{g}}{\partial t}\right) + \sqrt{\frac{K}{G}}\frac{\vec{g}.\vec{\nabla}\epsilon}{2\rho_m}\vec{J}_m$	$\vec{\nabla}\times\vec{H} \Leftarrow \xi\frac{\partial\vec{g}}{\partial t}\sqrt{\frac{G}{K_0}}$
$\vec{\Omega}_\tau$	$\vec{\mathrm{E}}$	$\vec{\nabla}\times\vec{\Omega}_\tau \Leftarrow -\left(\frac{\vec{E}.\vec{\nabla}\epsilon}{2\rho}\vec{\mathrm{J}} + \vec{\mathrm{J}} + \epsilon\frac{\partial\vec{E}}{\partial t}\right)\sqrt{\frac{K}{G}}$	$\vec{\nabla}\times\vec{\Omega}_\tau \Leftarrow -\epsilon_0\frac{\partial\vec{E}}{\partial t}\sqrt{\frac{K_0}{G}}$
	$\vec{\mathrm{H}}$	$\vec{\mathrm{H}} \Leftarrow -\vec{\Omega}_\tau\sqrt{\frac{G}{K}} = -\vec{\Omega}_\tau\sqrt{\frac{\tau}{\mu}}$	$\vec{\mathrm{H}} \Leftarrow -\vec{\Omega}_\tau\sqrt{\frac{G}{K_0}}$
Sources		$\frac{\rho}{\rho_m} = \frac{\vec{\mathrm{J}}}{\vec{J}_m} = \frac{\vec{g}.\vec{\nabla}\epsilon}{2\rho_m}\sqrt{\frac{K}{G}} + \sqrt{\frac{G}{K}}$	$\frac{\rho}{\rho_m} = \frac{\vec{\mathrm{J}}}{\vec{J}_m} = \sqrt{\frac{G}{K_0}}$ (In vacuum)

IX. Appendix: Vectorial Calculus

Generic vector analysis equations used in this article are summarized below.

$\vec{a}$, $\vec{b}$ and $\vec{c}$ are vectors, while φ is a scalar. The dot product is defined using the dot notation. The cross product is defined by the symbol $\times$.

$$\vec{\nabla}.\left(\vec{a}\times\vec{b}\right) = \vec{b}.\left(\vec{\nabla}\times\vec{a}\right) - \vec{a}.\left(\vec{\nabla}\times\vec{b}\right) = \left(\vec{\nabla}\times\vec{a}\right).\vec{b} - \left(\vec{\nabla}\times\vec{b}\right).\vec{a} \quad \text{(A1)}$$

$$\left(\vec{a}\times\vec{b}\right)\times\vec{c} = \vec{b}\left(\vec{c}\times\vec{a}\right) - \vec{a}\left(\vec{c}\times\vec{b}\right) \quad \text{(A2)}$$

$$\vec{\nabla}\left(\vec{a}.\vec{b}\right) = \left(\vec{b}.\vec{\nabla}\right)\vec{a} + \left(\vec{a}.\vec{\nabla}\right)\vec{b} + \vec{b}\times\left(\vec{\nabla}\times\vec{a}\right) + \vec{a}\times\left(\vec{\nabla}\times\vec{b}\right) \quad \text{(A3)}$$

$$\frac{1}{2}\vec{\nabla}\left(\vec{a}.\vec{a}\right) = \left(\vec{a}.\vec{\nabla}\right)\vec{a} + \vec{a}\times\left(\vec{\nabla}\times\vec{a}\right) \quad \text{(A4)}$$

$$\vec{\nabla} \times (\vec{a} \times \vec{b}) = \vec{a}(\vec{\nabla}.\vec{b}) - \vec{b}(\vec{\nabla}.\vec{a}) + (\vec{b}.\vec{\nabla})\vec{a} - (\vec{a}.\vec{\nabla})\vec{b} \quad \text{(A5)}$$
$$\Delta\vec{a} = \vec{\nabla}^2\vec{a} = \vec{\nabla}(\vec{\nabla}.\vec{a}) - \vec{\nabla} \times (\vec{\nabla} \times \vec{a}) \quad \text{(A6)}$$
$$\vec{\nabla}.(\varphi\,\vec{a}) = \varphi(\vec{\nabla}.\vec{a}) + (\vec{\nabla}\,\varphi).\vec{a} \quad \text{(A7)}$$
$$\vec{\nabla}\times(\varphi\,\vec{a}) = \varphi(\vec{\nabla} \times \vec{a}) + (\vec{\nabla}\,\varphi) \times \vec{a} \quad \text{(A8)}$$

X. Appendix: Structural Equations in Electromagnetism and Gyrogravitation

Equations formal structure for the electromagnetic field in a material medium and the gyrogravitation field [3][11], is summarized below defined through,

- Table 5, initial condition parameters, describing the permittivity constants ϵ and ξ, the permeability constants μ and τ, the continuity equations for the charge and mass conservation and the force around a charge q and a mass m generated by the component transversal fields combination of the corresponding radiation.
- Table 6, Maxwell equations and equivalents for gyrogravitation, as well as interrelation between gyrogravitational field components and their relationship with the generating sources.
- Table 7, transverse component conversion equations for the electromagnetic field and for the gyrogravitational field, relating them through their propagation speed.

Table 5: Definitions and parameters of electromagnetism (in material medium) and gyrogravitation

Nomenclature	Electromagnetism (EM)	Girogravitation (GG)
Lorentz Force	$\vec{F} \Leftarrow q\,(\vec{E} + \vec{v} \times \mu\vec{H})$	$\vec{F} \Leftarrow m\,(\vec{g} + \vec{v} \times \tau\vec{\Omega}_\tau)$
Field Propagation Speed	$s = \left(\frac{1}{\mu\epsilon}\right)^{1/2} = \frac{c}{\sqrt{\mu_r\epsilon_r}}$	$s = \left(\frac{-1}{\xi\tau}\right)^{1/2}$
Permittivity Constant	$\epsilon = \epsilon_r\epsilon_0 = \frac{1}{4\pi K}$ (electric)	$\xi = -\frac{1}{4\pi G}$ (gravitational)
Permeability Constant	$\mu = \mu_r\mu_0 = \frac{4\pi K}{s^2}$ (magnetic)	$\tau = \frac{4\pi G}{s^2}$ (gravitation torsion)
Flux/Density Relationship (Continuity Equation)	$\vec{\nabla}.\vec{J} \Leftarrow -\frac{\partial\rho}{\partial t} - \vec{\nabla}\,\epsilon.\frac{\partial\vec{E}}{\partial t}$ (charge conservation)	$\vec{\nabla}.\vec{J}_m \Leftarrow -\frac{\partial\rho_m}{\partial t}$ (mass conservation)

Table 6: Relation characteristic equations between generating sources and associated fields, and interrelation within electromagnetism (in a material medium) and gyrogravitation components

Nomenclature	Electromagnetism (EM)	Girogravitation (GG)
Irrotacional Field and Associated Source	$\vec{\nabla}.\vec{E} \Leftarrow \frac{\rho}{\epsilon}$	$\vec{\nabla}.\vec{g} \Leftarrow \frac{\rho_m}{\xi}$
Solenoidal Field	$\vec{\nabla}.\vec{H} = 0$	$\vec{\nabla}.\vec{\Omega}_\tau = 0$
Lenz's Law	$\vec{\nabla} \times \vec{E} \Leftarrow -\mu\frac{\partial\vec{H}}{\partial t}$	$\vec{\nabla} \times \vec{g} \Leftarrow -\tau\frac{\partial\vec{\Omega}_\tau}{\partial t}$
Ampere's Law	$\vec{\nabla} \times \vec{H} \Leftarrow \vec{J} + \epsilon\frac{\partial\vec{E}}{\partial t}$	$\vec{\nabla} \times \vec{\Omega}_\tau \Leftarrow -\vec{J}_m - \xi\frac{\partial\vec{g}}{\partial t}$

Table 7: Transversal components conversion and their relationship with the propagation speed of electromagnetism (in a material medium) and gyrogravitation fields

Nomenclature	Electromagnetism (EM)	Girogravitation (GG)
Irrotacional Field Composition	$\vec{E} \Leftarrow \mu\vec{H} \times \vec{s}$	$\vec{g} \Leftarrow \tau\vec{\Omega}_\tau \times \vec{s}$
Solenoidal Field Composition	$\frac{\vec{H}}{\epsilon} \Leftarrow \vec{s} \times \vec{E}$	$-\frac{\vec{\Omega}_\tau}{\xi} \Leftarrow \vec{s} \times \vec{g}$

References

[1]. N. Torres López and F. Racedo Niebles, *Teoría Electromagnética* (Barranquilla, Colombia: Universidad del Atlántico, 2010), ISBN: 978-958-8123-82.

[2]. B. García Olmedo, *Fundamentos del Electromagnetismo* (Granada, España: Universidad de Granada, 2005).

[3]. J. Joglar Alcubilla, Electrogravitodynamics Principles (V2), https://www.researchgate.net/publication/339044382_Electrogravitodynamics_Principles, 2020.

[4]. J. Joglar Alcubilla, Electrogravitodynamics Principles, *IOSR Journal of Applied Physics*, 12 (1), 2020, 45-57, doi: 10.9790/4861-1201024557.

[5]. A. Gamero Rojas, *Ondas Electromagnéticas. Conceptos Básicos* (Córdoba, España: Universidad de Córdoba, 2005).

[6]. B.G. Levich, Teoría general del campo electromagnético, in Reverté S.A. (Ed.), *Física Teórica 1: Teoría del campo electromagnético y teoría de la relatividad*, 1 (Barcelona, 1974), 1-43, ISBN 842914061.

[7]. J. Joglar Alcubilla, Light Velocity Quantization and Harmonic Spectral Analysis, *IOSR Journal of Applied Physics*, 10 (4II), 2018, 57-71, doi: 10.9790/4861-1004025771.

[8]. O. D. Jefimenko, Causality Electromagnetic Induction and Gravitation, 2nd ed.: Electret Scientific (Star City - 2000) Chapter 1, Sec. 1-4, pag. 16, ISBN 0-917406-23-0.

[9]. C. J. de Matos and M. Tajmar, Gravitational Poynting Vector and Gravitational Larmor Theorem in Rotating Bodies with Angular Acceleration, *ArXiv: gr-qc/0107014v1*, 2001.
[10]. J. Joglar Alcubilla, On Subquantum Electromagnetic and Gravitational Interactions, *IOSR Journal of Applied Physics*, 11 (2), 2019, 01-09, doi: 10.9790/4861-1102010109.
[11]. S. G. Fedosin, Electromagnetic and Gravitational Pictures of the World, *Apeiron*, 14 (4), 2007, 385-413.

Javier Joglar Alcubilla. "Wave Equations of Electrogravitodynamics." *IOSR Journal of Applied Physics (IOSR-JAP)*, 12(5), 2020, pp. 34-49.