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Ergotropy of Quantum Battery Controlled via Target Attractor Feedback

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Abstract:
***Model**: Quantum battery (QB) is a device that is capable to be charged efficiently and store the energy for a long period of time to be transferred to consumption centers. There are many different physical types of such devices and different charging schemes. Here we discuss the single-qubit based QB in the form of quantum oscillator in a Markovian bath environment. The charging of QB is performed via so-called 'coherent' control u(t) in the Hamiltonian and time dependent spectral density n(t) as an 'incoherent' control (number of excitations in the bath). Our goal is to drive the ergotropy of the stored qubit via the certain control algorithm.*
***Methods**: For the effective control we apply here Kolesnikov's 'target attractor' (TA) feedback algorithm. In the frame of this approach we form an attractor set targeting the evolution of the basic characteristics of quantum battery. TA method makes the effective design of the control fields charging the battery; the corresponding control signals could be restored explicitly from the dynamical equations. Interestingly, the proposed algorithm applied to our single qubit model of QB has an analytical solution.*
***Results and Discussion**: As a result for the control goal, we obtain an exponentially converting behavior for driving the quantum battery ergotopic characteristics. Our algorithm can be extended to the multi-qubit model of QB (for the parallel or collective charging scheme). It could be applied also for different physical realizations of QBs: Dicke QB, spin QB, harmonic oscillator QB; and for all working stages of the QB (charging, long time storage and the energy transfer to a consumption center or engine).*
***Conclusion:** Feedback algorithms, particularly in the form of target attractor approach, can be applied efficiently to control the set of fundamental characteristics of quantum batteries, including the ergotropy, charging power and others. The analytical study of the proposed model and its numerical simulations demonstrate the possibility to imply the developed mathematical algorithm experimentally for a single qubit system and the set of few qubits as well.*
***Key Word**: Quantum Battery; Ergotropy; Qubit; Target Attractor; Feedback Algorithm Control.*

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I. Introduction: Single Qubit Model for Quantum Battery

Quantum battery (QB) is a device that is capable to be charged efficiently and store the energy for a long period of time to be transferred to consumption centers[1,2]. There are many different types of such devices (Dicke QB, spin QB, harmonic oscillator QB)[3,4] and different charging schemes[5]. QB could be based on a single qubit, but the resent research also considers *N*-qubit arrays for the energy storage[6,7].

Here we discuss the single-qubit based QB in the form of quantum oscillator in a Markovian bath environment based on the Lindblad-type equation[8] for the density matrix ρ:

$$\frac{d\rho}{dt} = -i[H_0 + u(t)\hat{Q}, \rho] + \hat{L}[\rho] \, , \tag{1}$$

where we denote for the one-dimensional quantum harmonic oscillator with frequency ω_0:

$$H_0 = \left(\omega_0 + \frac{1}{2}\right) a^+ a \; ; \quad \hat{Q} = \frac{a^+ + a}{\sqrt{2\omega_0}} \; ; \quad \hat{P} = i\sqrt{\frac{\omega_0}{2}}(a^+ - a) \tag{2}$$

(the Planck constant is equal to 1); $u(t)$ is a real time-dependent driven function of so-called 'coherent' control. The Lindblad part of (1) could be expressed with the creation-annihilation operators as[9]:

$$\hat{L}[\rho] = \gamma \cdot (n(t)+1)\left(2a\rho a^{+} - \rho a^{+}a - a^{+}a\rho\right) + \gamma \cdot n(t)\left(2a^{+}\rho a - aa^{+}\rho - \rho aa^{+}\right), \tag{3}$$

where γ stands for the rate of relaxation, and the time dependent spectral density $n(t)$ as an 'incoherent' control is a number of the excitations in the bath. The bath itself is supposed to be non-thermal, with the arbitrary spectral density.

The model (1)-(3) can be reduced to the system of three differential equations. Indeed, considering the average quantum oscillator energy as $E(t) = Tr(H_0\rho)$, one can get[10]:

$$\begin{aligned} \frac{dE}{dt} &= 2\gamma \cdot (\omega_0 n - E) - uP; \\ \frac{dQ}{dt} &= P - \gamma \cdot Q; \\ \frac{dP}{dt} &= -\omega_0^2 Q - \gamma \cdot P - u, \end{aligned} \tag{4}$$

where we use the notation: $Q(t) = Tr(\hat{Q}\rho)$, $P(t) = Tr(\hat{P}\rho)$.

The qubit used for the energy storage has many characteristics: the ratio between the extractable work and the QB energy value, the charging power of the battery, and others. The main one is its ergotropy defined as[11,12]:

$$W(t) = E(t) - E_0, \tag{5}$$

with the lowest accessible passive battery state E_0. In this case the charging power corresponds to the derivative of Eq. (5):

$$p(t) = \frac{dW}{dt}\ . \tag{6}$$

Our goal is to investigate how we can manage the ergotropy of the energy storing qubit and its derivatives via the certain control algorithm.

II. Target Attractor Control Method

The system (4) is driven with two control fields: $u(t)$ and $n(t)$. The goal of control could be different: stabilization of the ergotropy (5) at the certain desired constant level E_*, tracking the charging power (6) with the certain function $p_*(t)$, or their combination.

Target Attractor Control: Among the variety of optimal and sub-optimal control algorithms we apply here the target attractor (TA) feedback ("synergetic control" in author's terminology) based on the "directed self-organization of the dynamical system"[13]. The *m*-parametric attracting invariant manifold, i.e. the subset referring the control target:

$$G_s(x_1,...,x_n) = 0\ ;\ s = 1...m \tag{7}$$

is defined as a functions of the state variables $x_1,\ldots,x_n$. The set of Eqs. (7) provides the asymptotic stability of the system dynamics with respect to the control goal. To do it, we chose:

$$T_s \frac{dG_s(t)}{dt} + G_s(t) = 0\cdot \tag{8}$$

where T_s are positive constants, i.e. time scales[14]. Tending to zeros exponentially, the goal set (7)-(8) leads the dynamical evolution of the system to the target attractor. TA control has been efficiently applied for quantum systems as well[15].

In such approach the dynamical equations become simplified, but the shape of control fields looks more complicated to compare with other feedback algorithms, for instance, with the speed gradient[10,16].

Feedback Control Algorithm and Its Analytical Solution: In the model (4) TA control has to be adopted for the combination of 'coherent' and 'incoherent' components. The control drives the first and the third Eqs. in (4). For that reason, the set of goal functions (7) has to be defined to express the 'coherent' and 'incoherent' components. We need two TA equations to obtain both fields, *u* and *n*, separately.

To do it, let's re-write Eq.(8) for the certain target stabilization constants E_* and P_* in the form:

$$\frac{dE}{dt} = -\frac{1}{T_1}(E - E_*) \ ; \qquad \frac{dP}{dt} = -\frac{1}{T_2}(P - P_*) \ . \tag{9}$$

Here T_1 and T_2 are positive constants defining the typical scales of TA control. The system (9) guarantees the exponential divergence of the system evolution towards the desired attractor set $\{E_*, P_*\}$. The solution to (9) is:

$$E(t) = E(0) \cdot e^{-t/T_1} + E_* \cdot (1 - e^{-t/T_1}) \ ; \qquad P(t) = P(0) \cdot e^{-t/T_2} + P_* \cdot (1 - e^{-t/T_2}) \ . \tag{10}$$

By the second Eq. of the system (4) we get:

$$Q(t) = Q(0) + \frac{P(0) + P_*}{1/T_2 - \gamma} \cdot \left(1 - e^{-t/T2}\right) .. \tag{11}$$

Combining (9)-(10) with the LHS (4), one can express the control fields via the dynamical variables:

$$u(t) = -\omega_0^2 Q(t) - \gamma \cdot P(t) + \frac{P(t) - P_*}{T_2} \ ; \cdot \tag{12}$$

and

$$n(t) = \frac{1}{2\gamma\omega_0}\left[u(t) \cdot P(t) + 2\gamma \cdot E(t) - \frac{E(t) - E_*}{T_1}\right] ; \cdot \tag{13}$$

The function $n(t)$ must be non-negative, thus, by (12)-(13):

$$\left[-\omega_0^2 Q(t) - \gamma \cdot P(t) + \frac{P(t) - P_*}{T_2}\right] \cdot P(t) + 2\gamma \cdot E(t) - \frac{E(t) - E_*}{T_1} \geq 0 \ . \tag{14}$$

That implies the constrain for the choice of the initial $Q(0)$ and P_*; such that in the asymptotic $t \to \infty$ the inequality (14) becomes:

$$P_* \leq \sqrt{\left|2E_* - \frac{\omega_0^2 Q(0)}{\gamma}\right|} \ . \tag{15}$$

Thus, the dynamical problem (4) under the target attractor control (12)-(13) has an analytical solution (10)-(13) under the constrain (14)-(15).

Controlled Ergotropy and Battery Charging Power: Correspondingly, by Eq.(9) the ergotropy (5) is given as:

$$W(t) = E(0) \cdot e^{-t/T_1} + E_* \cdot (1 - e^{-t/T_1}) - E_0 \ , \tag{16}$$

and the battery charging power (6) is represented with:

$$p(t) = \frac{E_* - E(0)}{T_1} \cdot e^{-t/T_1} \ . \tag{17}$$

Eq. (16) tends to the control target stabilization exponentially. The charging power also is decreasing exponentially in time. Thus, we can say that in the proposed algorithm the constant T_1 defines the time scale for the battery charging.

III. Numerical Simulations and Results

For the numerical simulations let's take the following set of parameters closed to a realistic experimental set: $E_0 = 0.5$; the initial set: $E(0) = 0.6$; $P(0) = 0$, $Q(0)=0.9$. The set of the model constants is: $\omega_0 = 1$; $\gamma = 0.5$; $T_1 = T_2 = 1$. The target stabilization constants are: $E_* = 1.2$; $P_* = -0.9$; the negative P_* provides the truthiness of the inequalities (14)-(15).

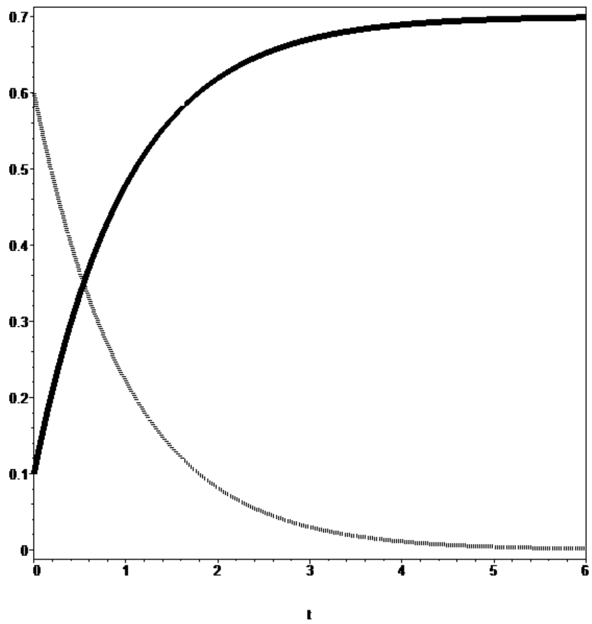

Figure 1. The ergotropy (solid line) and the charging power (thin dot line) for the single qubit quantum battery driven via target attractor algorithm.

On Figure 1 one can observe the exponential evolution of the ergotropy W and the charging power p for the single qubit quantum battery driven via target attractor algorithm. The typical time scale of the process is defined by the time constant T_1.

On Figure 2 we represent the control functions for the 'coherent' and 'incoherent' components. It is important to emphasize that the 'incoherent' part of control (the dashed line) does not become negative for the whole time interval of the system dynamics.

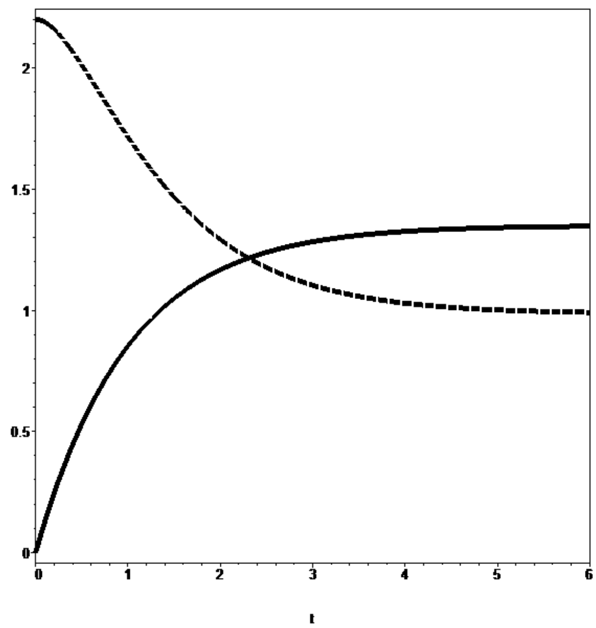

Figure 2. The 'coherent' control $u(t)$ (solid line) and 'incoherent' control $n(t)$ (dashed line) components for the single qubit quantum battery driven via target attractor algorithm.

Thus, as a result, we obtain the exponentially converging dynamical behavior for the control over the quantum battery ergotopic characteristics.

IV. Discussion and Further Considerations

Here we discuss few items for the further development of the proposed control algorithm.

First of all, the algorithm must be extended to the multi-qubit model. The key point will be to study how the architecture of the multi-qubit quantum battery (like the parallel vs collective charging scheme) will influence the efficiency of the algorithm.

The second point is to compare the efficiency of different alternative feedback schemes: the Pontryagin optimal control, the speed gradient and other algorithms. We need to investigate the robustness in the frame of different approaches, their energy costs and other parameters of the control efficiency.

The third point is to investigate the application of the proposed algorithm for different physical realizations of QBs: Dicke QB, spin QB, harmonic oscillator QB. We need to study in more details how the control process depends on the particular shape of the Hamiltonian for the energy storing system.

Finally, the details of all working stages for the QB (charging, long time storage and the energy transfer to a consumption center or engine) should be also modeled carefully from the perspective of the control algorithm efficiency.

V. Conclusion

Feedback algorithms, particularly in the form of target attractor approach, can be applied efficiently to control the set of fundamental characteristics of quantum batteries, including the ergotropy, charging power and others. The analytical study of the proposed model and its numerical simulations demonstrate the possibility to imply the developed mathematical algorithm experimentally for a single qubit system and for the set of few qubits as well.

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